Lec-17

Friday, May 8, 2020 8:16 AM

Open Circuit Transmission Line - Standing Wave Pattern.

$$V(\ell) = V^+ e^{j\beta\ell} + V^- e^{-j\beta\ell}$$

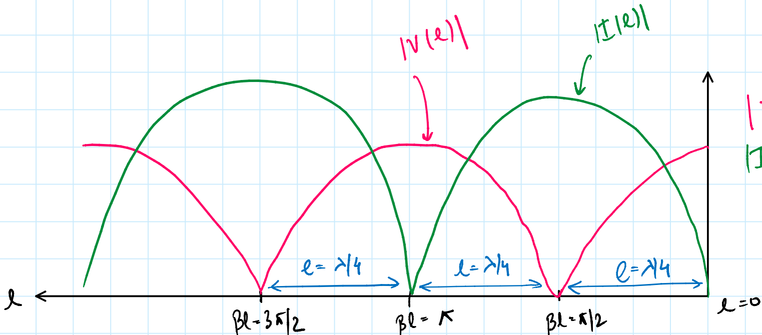
$$I(\ell) = \frac{V^+}{Z_0} e^{j\beta\ell} - \frac{V^-}{Z_0} e^{-j\beta\ell}$$

Open circuit $V^- = V^+$ $\{\Gamma_L = 1\}$

$$V(\ell) = 2V^+ \cos(\beta\ell)$$

$$I(\ell) = j\frac{2V^+}{Z_0} \sin(\beta\ell)$$

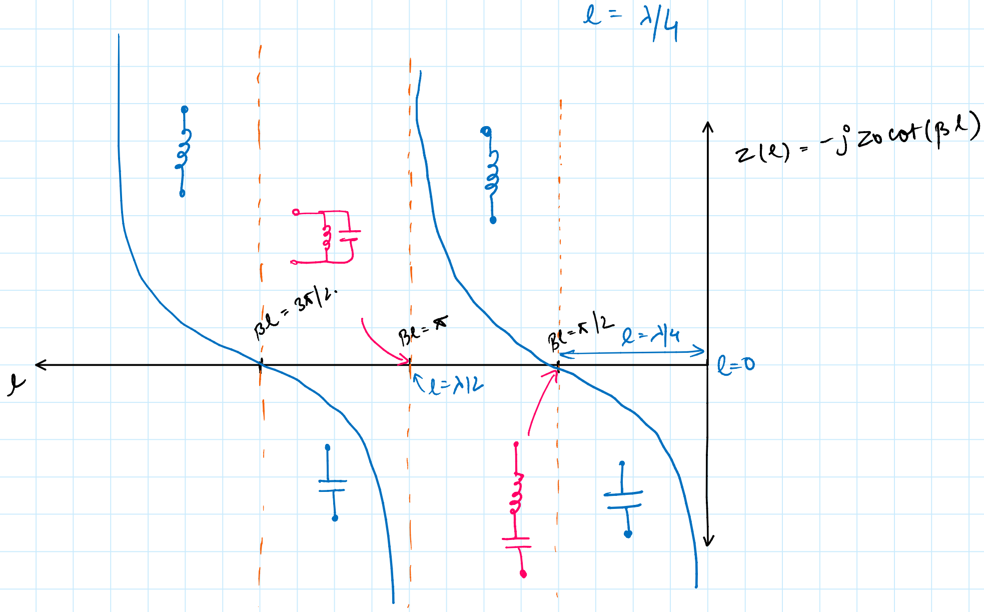
$$\boxed{Z(\ell) = \frac{V(\ell)}{I(\ell)} = -jZ_0 \cot(\beta\ell)}$$

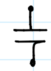
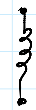
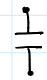


Note: Point of $|V(\ell)|_{max}$ = Point of $|I(\ell)|_{min}$

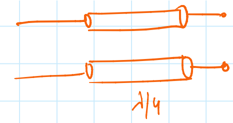
$$\frac{2\cancel{\pi}}{\lambda} \cdot \ell = \frac{\cancel{\pi}}{2}$$

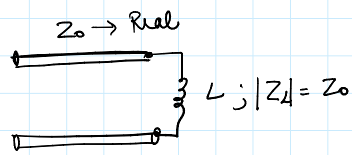
$$\ell = \lambda/4$$



- $0 \le l < \lambda/4 \equiv$ 
- $\frac{\lambda}{4} < l < \frac{\lambda}{2} \equiv$ 
- $\frac{\lambda}{2} < l < 3\lambda/4 \equiv$ 

$\vdots$



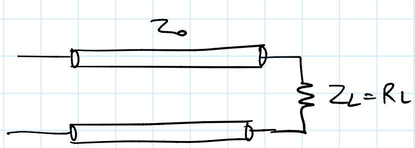


$|\Gamma_L| = 1$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{j\omega L - Z_0}{j\omega L + Z_0}$$

$$\left| \frac{a - jb}{a + jb} \right| \overset{?}{=} 1$$

Voltage & Current Standing Wave on a Pure Resistive Load



$$V(l) = V^+ e^{j\beta l} \left(1 + |\Gamma_L| e^{j(\phi_L - 2\beta l)}\right)$$

$$I(l) = \frac{V^+}{Z_0} e^{j\beta l} \left(1 - |\Gamma_L| e^{j(\phi_L - 2\beta l)}\right)$$

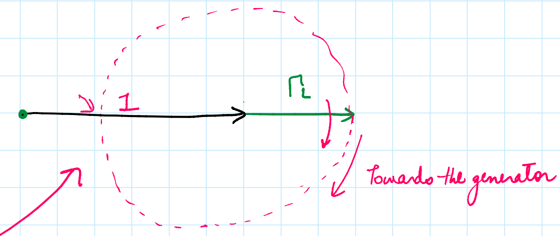
$$\left.\begin{array}{l} Z_L = R_L \rightarrow \text{Pure Resistive} \\ Z_0 \rightarrow \text{Real (loss less line)} \end{array}\right\} \Rightarrow \begin{array}{l} \Gamma_L = \text{Real} \\ \phi_L = 0 \end{array}$$

$$V(l) = V^+ e^{j\beta l} \left(1 + \Gamma_L e^{-j2\beta l}\right)$$

$$I(l) = \frac{V^+}{Z_0} e^{j\beta l} \left(1 - \Gamma_L e^{-j2\beta l}\right)$$

$l = 0$ (at the load) $\rightarrow$ $\boxed{\begin{aligned} V(l) &= V^+ (1 + \Gamma_L) \\ I(l) &= \frac{V^+}{Z_0} (1 - \Gamma_L) \end{aligned}}$

$$V(l) = V^+ (1 + \Gamma_L)$$

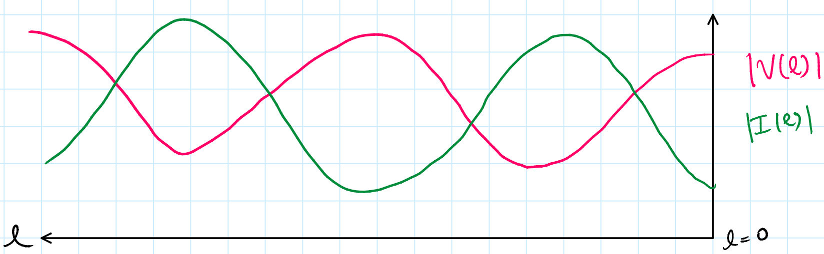


Case 1 : $R_L > Z_0$; $\Gamma_L > 0$

$V(l)$ is maximum at the load.
$I(l)$ is minimum at the load

Observations

- As $l \uparrow$ $V(l)$ reduces & $I(l)$ increases.
- $|V(l)|$ does not go to 0 because $|\Gamma_L| < 1$
- ρ (VSWR) is not infinite.



Case 2 : $R_L < Z_0$; $\Gamma_L < 0$

@ $l = 0$ $V(l)$ is minimum
$I(l)$ is maximum.