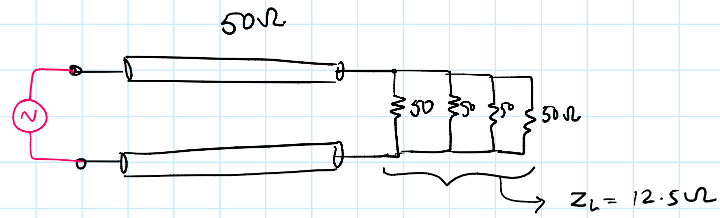
Lec-18

Monday, May 11, 2020 8:41 AM

HW#4 has been posted

Example 1: Resistive Load

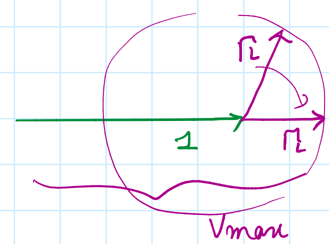


(a) $$\Gamma_L = \frac{12.5-50}{12.5+50} = -0.6 = \underline{0.6}\, e^{j\pi}$$

(b) Calculate Vmax, Vmin, Imax, Imin ; Assume $V^+ = 1V$

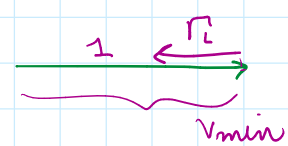
$$V_{max} = |V(\ell)|_{max} = |V^+|(1+\Gamma_L) = 1.6\,V$$

$$V_{min} = |V^+|(1-\Gamma_L) = 0.4\,V$$



$$I_{max} = \frac{|V^+|}{Z_0}(1+\Gamma_L) = \frac{1.6}{50} = \underline{32\,mA}$$

$$I_{min} = \frac{|V^+|}{Z_0}(1-\Gamma_L) = \frac{0.4}{50} = \underline{8\,mA}.$$



(c) Sketch $|V(\ell)|$ & $|I(\ell)|$ as a function of $\underline{\ell/\lambda}$ starting with $\ell=0$

$$V(\ell) = V^+ e^{j\beta\ell}\left(1 + |\Gamma_L| e^{j(\phi_L - 2\beta\ell)}\right)$$

$$\Rightarrow I(\ell) = \frac{V^+ e^{j\beta\ell}}{Z_0}\left(1 - |\Gamma_L| e^{j(\phi_L - 2\beta\ell)}\right)$$

$$V(\ell) = 1 \cdot e^{j\frac{2\pi}{\lambda}\cdot\ell}\left(1 + 0.6\, e^{j\left(\pi - 2\cdot\frac{2\pi}{\lambda}\ell\right)}\right)$$

$$|V(\ell)| = 1\left|\left(1 + 0.6\, e^{j\pi(1-4\ell/\lambda)}\right)\right|$$

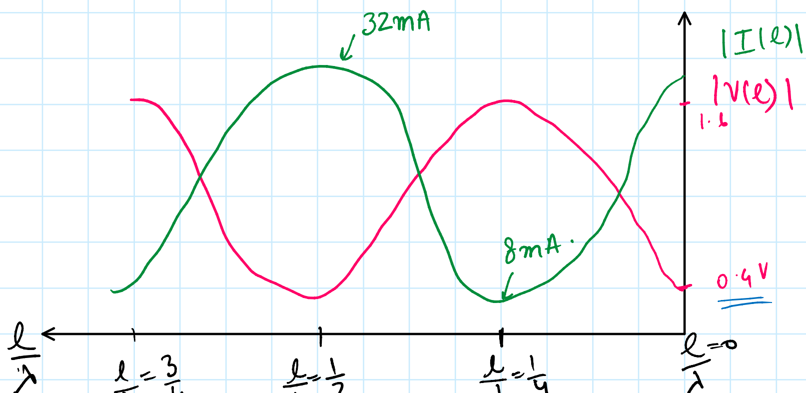
$$|V(\ell)| = 1 + 0.6\left\{\cos[\pi(1-4\ell/\lambda)] + j\sin[\pi(1-4\ell/\lambda)]\right\}$$

$\frac{\ell}{\lambda} = 0 \Rightarrow |V(\ell)| = 0.4V$ Vmin

$\frac{\ell}{\lambda} = \frac{1}{4} \rightarrow \ell = \frac{\lambda}{4} \Rightarrow |V(\ell)| = 1.6V$ Vmax

$\frac{\ell}{\lambda} = \frac{1}{2} \Rightarrow |V(\ell)| = 0.4V$ Vmin

$\frac{\ell}{\lambda} = \frac{3}{4} \Rightarrow |V(\ell)| = 1.6V$ Vmax



Example 2: Transmission Line as Circuit Component.

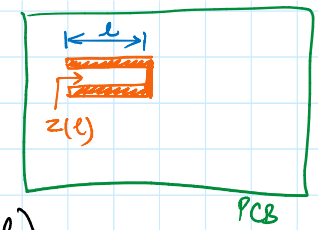
$v_p = 2.07 \times 10^8 m/s$

$Z_0 = 50\Omega$

(a) • We want 15 nH of inductance with minimum length for a short circuited line @ 3GHz.

$Z_{sc} = Z(\ell) = jZ_0 \tan(\beta \ell)$

$\downarrow$ 15nH



$$\cancel{j}\, 2\pi \cdot 3\times10^{9} \times 15\times10^{-9} = \cancel{j}\, 50 \tan\left(\frac{2\pi}{\lambda}\cdot \ell\right)$$

ℓ ← unknown

λ → v, f, λ

$$\frac{2.07\times10^{8}}{3\times10^{9}} = \frac{v}{f} = \lambda$$

$$\ell = \frac{\lambda}{2\pi} \tan^{-1}\left(\frac{2\pi \times 3\times10^{9} \times 15\times10^{-9}}{50}\right)$$

$\boxed{\ell = 1.53\text{ cm}}$

(b) What happens to the impedance at $f = 4$GHz for transmission line trace of $\ell = 1.53$cm ?

$$Z_{sc} = jZ_0 \tan(\beta \ell)$$

Z_0 → 50; ℓ → 1.53 cm

$$\beta = \frac{2\pi}{\lambda}, \quad \lambda \rightarrow \frac{2.07\times10^{8}}{4\times10^{9}} = 5.175\text{ cm.}$$

–ve sign

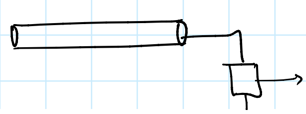
$$Z_{sc} = -j167.4\Omega = \frac{1}{j\omega C}$$

$\omega \rightarrow 2\pi f$, $f \rightarrow 4\times10^{9}$

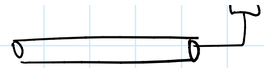
$\boxed{C = 0.238\text{pF}}$

Transmission line acts like a capacitor @ 4GHz.

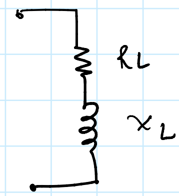
Voltage & Current Standing Wave on a Complex Load.



$Z_L = R_L \pm jX_L$



1. Case I   **Inductive Load**



$$\Gamma_L = \frac{R_L - Z_0 + jX_L}{R_L + Z_0 + jX_L}$$

$R_L = 0$; $X_L = Z_0$

$|\Gamma_L| = ?$ $= 1$

$\angle \Gamma_L = ?$ $=$