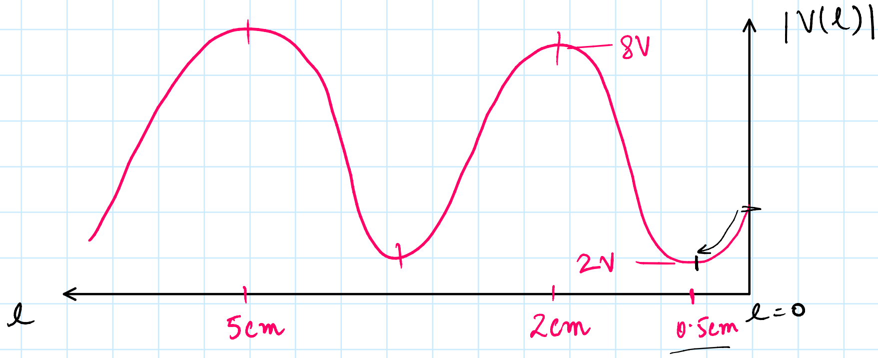
Lec-20

Friday, May 15, 2020 8:31 AM

Example: $Z_0 = 50\Omega$

Z_L = Unknown.



Q_1: What is the VSWR?

$$\rho = VSWR = \frac{|V|_{max}}{|V|_{min}} = \frac{8V}{2V} = 4$$

Q_2: Wavelength on this transmission line

Distance between two voltage maximas or two voltage minima $= \frac{\lambda}{2}$

$$\frac{\lambda}{2} = 3cm$$

$$\lambda = 6cm.$$

Q_3: Calculate V^+

$$V_{max} = V^+(1 + |\Gamma_L|) = 8V$$
$$V_{min} = V^+(1 - |\Gamma_L|) = 2V$$

$V^+ = ?$ $\quad V^+ = \frac{V_{max} + V_{min}}{} = \frac{8+2}{} = 5V$

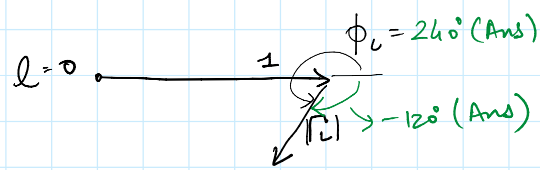
$$V^+ = ? \qquad V^+ = \frac{V_{max} + V_{min}}{2} = \frac{8+2}{2} = 5V$$

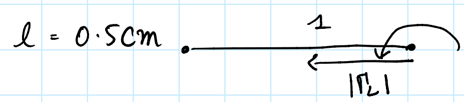
Q4: Calculate the magnitude & phase of the reflection coefficient

$$VSWR = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \rho$$

$$\frac{VSWR - 1}{VSWR + 1} = |\Gamma_L| = \frac{4-1}{4+1} = 0.6$$

Phase calculation :





$$\boxed{\phi_L - 2\beta l = \pi \quad @ \quad l = 0.5\,cm}$$

$$\phi_L - 2\beta l = \pi$$

$$\phi_L - 2\,\frac{2\pi}{\lambda}\cdot l = \pi$$

$$\phi_L - \cancel{2}\cdot\frac{2\pi}{\cancel{6}\,3}\times 0.5 = \pi$$

$$\phi_L = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$\phi_L \quad 180^\circ + 60^\circ = 240^\circ = -120^\circ$$

Q5: Calculate I_{max}, I_{min}.

$$I_{max} = \frac{V_{max}}{Z_0} = \frac{8V}{50\Omega} = 160mA$$

$$I_{min} = \frac{V_{min}}{Z_0} = \frac{2}{50} = 40mA.$$

Q6 What type of load is this?

$\longrightarrow$ Because $\phi_L = -120^\circ$

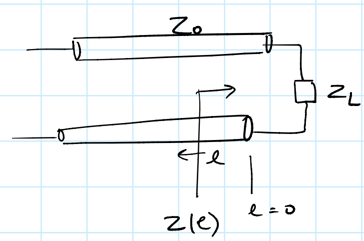
$\longrightarrow$ Because $|\Gamma_L| < 1$

Impedance Transformation Relation & Quater Wavelength Matching

$$V(\ell) = V^+ e^{\gamma\ell} + V^- e^{-\gamma\ell}$$

$$I(\ell) = \frac{V^+}{Z_0} e^{\gamma\ell} - \frac{V^-}{Z_0} e^{-\gamma\ell}$$

$$Z(\ell) = \frac{V(\ell)}{I(\ell)} = Z_0 \left\{ \frac{1 + \frac{V^-}{V^+} e^{-2\gamma\ell}}{1 - \frac{V^-}{V^+} e^{-2\gamma\ell}} \right\}$$



$$Z(\ell) = Z_0 \left\{ \frac{1 + \Gamma_L e^{-2\gamma\ell}}{1 - \Gamma_L e^{-2\gamma\ell}} \right\}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z(\ell) = Z_0 \left\{ \frac{1 + \frac{Z_L - Z_0}{Z_L + Z_0} e^{-2\gamma\ell}}{1 - \frac{Z_L - Z_0}{Z_L + Z_0} e^{-2\gamma\ell}} \right\}$$

$$\left\{ \begin{aligned} \cosh\gamma\ell &= \frac{e^{\gamma\ell} + e^{-\gamma\ell}}{2} \\ \sinh\gamma\ell &= \frac{e^{\gamma\ell} - e^{-\gamma\ell}}{2} \end{aligned} \right.$$

$$Z(\ell) = Z_0 \left\{ \frac{Z_L \cosh\gamma\ell + Z_0 \sinh\gamma\ell}{Z_0 \cosh\gamma\ell + Z_L \sinh\gamma\ell} \right\}$$

Normalized Impedance

$$\bar{Z}(\ell) = \frac{Z(\ell)}{Z_0}$$

$$\bar{Z}(\ell) = \frac{\bar{Z}_L \cosh\gamma\ell + \sinh\gamma\ell}{\cosh\gamma\ell + \bar{Z}_L \sinh\gamma\ell}.$$

For a lossless line $\gamma = j\beta$ $(\alpha = 0)$

$$\cosh(j\beta\ell) = \cos\beta\ell$$

$$\sinh(j\beta\ell) = j\sin\beta\ell.$$

$$\boxed{\bar{Z}(\ell) = \frac{\bar{Z}_L \cos\beta\ell + j\sin\beta\ell}{\cos\beta\ell + j\bar{Z}_L \sin\beta\ell}}$$