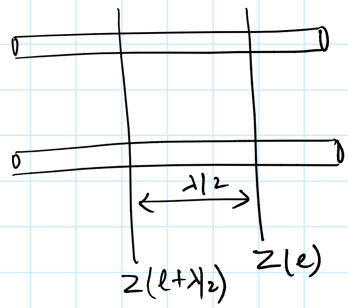
Lec-21

Monday, May 18, 2020 8:26 AM

1. Characteristics of Impedance on a Loss Less Line.

Observation.

* Impedance value repeats after every $\lambda/2$ distance on the transmission line.



$$Z(l+\lambda/2) = Z(l).$$

Let at a location l, the impedance is $Z(l)$

$$\bar{Z}(l) = \frac{\bar{Z}_L \cos\beta l + j\sin\beta l}{\cos\beta l + j\bar{Z}_L \sin\beta l}$$

$$\bar{Z}(l+\lambda/2) = \frac{\bar{Z}_L \cos\beta(l+\lambda/2) + j\sin\beta(l+\lambda/2)}{\cos\beta(l+\lambda/2) + j\bar{Z}_L \sin\beta(l+\lambda/2)}$$

$$\beta(l+\lambda/2) = \frac{2\pi}{\lambda}(l+\lambda/2) = \beta l + \pi$$

$$\cos\beta(l+\lambda/2) = \cos(\beta l+\pi) = -\cos\beta l$$
$$\sin\beta(l+\lambda/2) = \sin(\beta l+\pi) = -\sin\beta l$$

$$\boxed{\bar{Z}(l+\lambda/2) = \frac{\cancel{-}\bar{Z}_L \cos\beta l \cancel{-} j\sin\beta l}{\cancel{-}\cos\beta l \cancel{-} j\bar{Z}_L \sin\beta l} = \bar{Z}(l)}$$

2. Impedance at a distance of $\lambda/4$

Observation

* Normalized Impedance inverts itself at every $\lambda/4$.

$$\bar{Z}(l+\lambda/4) = \frac{1}{\bar{Z}(l)}$$

$$\beta(l+\lambda/4) = \beta l + \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \beta l + \frac{\pi}{2}$$

$$\cos\left(\beta l + \frac{\pi}{2}\right) = -\sin\beta l$$

$$\sin\left(\beta l + \frac{\pi}{2}\right) = \cos\beta l$$

$$\bar{Z}(l+\lambda/4) = \frac{-\bar{Z}_L \sin\beta l + j\cos\beta l}{-\sin\beta l + j\bar{Z}_L\cos\beta l}$$

take j common from N^r & D^r

$$= \frac{\cancel{j}}{\cancel{j}}\left\{\frac{\cos\beta l + j\bar{Z}_L\sin\beta l}{\bar{Z}_L\cos\beta l + j\sin\beta l}\right\}$$

$$\boxed{\bar{Z}(l+\lambda/4) = \frac{1}{\bar{Z}(l)}}$$

3. Matched transmission line.

$$Z_L = Z_0 \Rightarrow \bar{Z}_L = \frac{Z_L}{Z_0} = 1$$

$$\bar{Z}(l) = \left\{\frac{\cancel{\cos\beta l + j\sin\beta l}}{\cancel{\cos\beta l + j\sin\beta l}}\right\} = 1$$

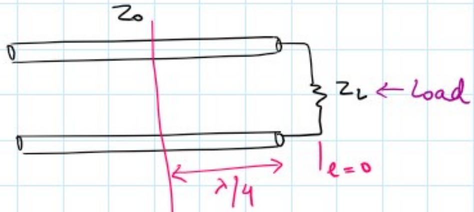
Observation :

If the line is terminated with the characteristic impedance, then impedance seen at any point on the transmission line is equal to the characteristic impedance.

$$\bar{Z}(l) = 1$$

$$\frac{Z(\ell)}{Z_0} = 1 \qquad Z(\ell) = Z_0.$$

Quater Wave Transformer Matching.



$$Z(0) = Z_L$$
$$Z(\ell) = Z_L \quad @ \ \ell = 0$$

$\bar{Z}$ = normalize

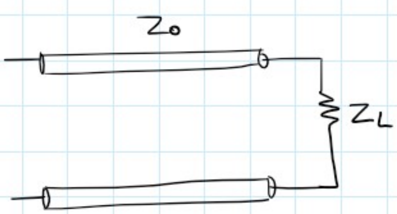
$$\bar{Z}(\ell) = \frac{1}{\bar{Z}_L}$$

$$\frac{Z(\ell)}{Z_0} = \frac{Z_0}{Z_L}$$

$$\boxed{\ell = \lambda/4 \qquad Z(\lambda/4) = \frac{Z_0^2}{Z_L}} \leftarrow$$

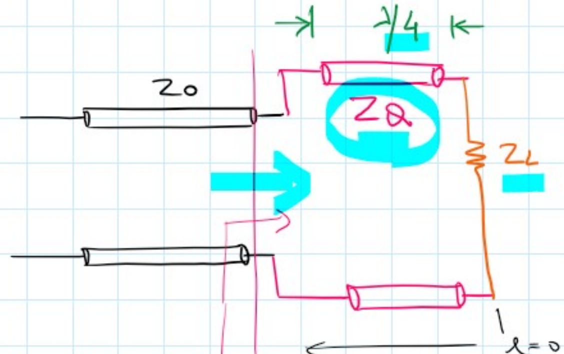
Q: What is $Z\left(\frac{3\lambda}{4}\right)$? $= \frac{Z_0^2}{Z_L}$

Q: What is $Z(\lambda/2)$? $= Z_L$



$Z_L \neq Z_0$

$\longrightarrow$



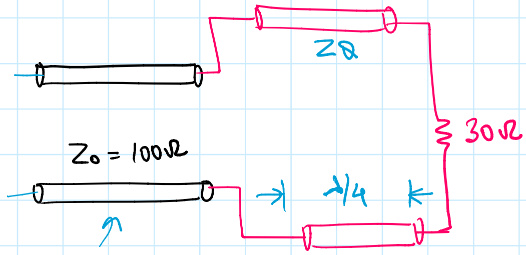
$Z(\ell) = Z_0$
$\downarrow$
$\ell = \lambda/4$

$$Z(\lambda/4) = Z_0 = \frac{Z_Q^2}{Z_L}$$

Goal $\rightarrow$ To estimate Z_Q to achieve quater wave transformer.

$$\boxed{Z_Q = \sqrt{Z_0 \cdot Z_L}}$$

Example 1 Resistive Load.



$Z_Q = ?$

$Z_Q = \sqrt{30 \times 100} = 54.77\,\Omega.$

Example 2 Complex Load.

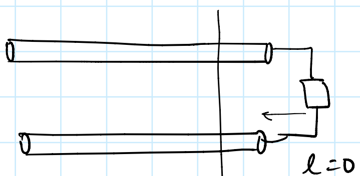
$Z_L = 73 + j42.5\,\Omega$

$Z_0 = 100\,\Omega$

Q: Where to insert the quater wave Transformer ?

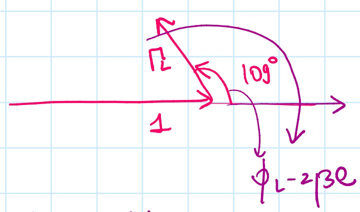
Q: What will be the Z_Q ?

Find a length / point on a line away from your load such that $Z(\ell)$ is real at that point



$Z(\ell) =$ real $= \dfrac{V(\ell)}{I(\ell)}$ when $\angle\Gamma_L = 0^\circ$ or 180°

$$\Gamma_L = \frac{73 + j42.5 - 100}{73 + j42.5 + 100} = 0.283\, e^{j109^\circ}$$



Estimate ℓ for maxima in $|V(\ell)|$ & minima in $I(\ell)$ or vice versa

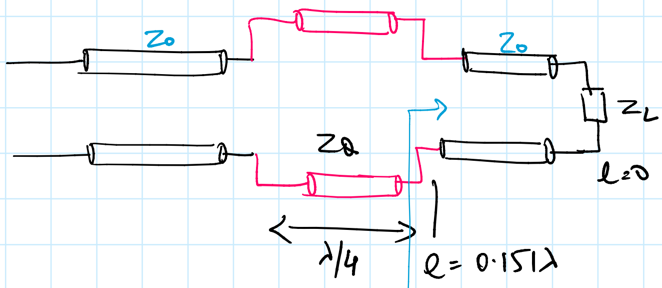
$$\phi_L - 2\beta\ell = 0$$

109°

$$109 \times \frac{2\pi}{360} - 2\frac{2\pi}{\lambda}\cdot \ell = 0$$

$$\underbrace{\ell = 0.151\lambda}$$

Insert the quater wave Transformer here



$Z(l)$ @ $l = 0.151\lambda$

$$\bar{Z}(l) = \frac{\bar{Z}_L + j\tan(\beta l)}{1 + j\bar{Z}_L \tan(\beta l)}$$

$$\bar{Z}(l) = 1.79$$

$$Z(l) = 179\Omega. \quad @\ l = 0.151\lambda$$

$$\boxed{Z_Q = \sqrt{179\Omega \times 100} = 133.7\Omega}$$