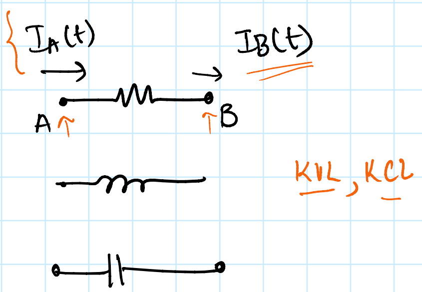
Lec-3

Friday, April 3, 2020 8:52 AM

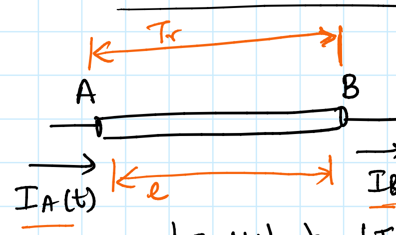
Lumped Circuit



KVL, KCL

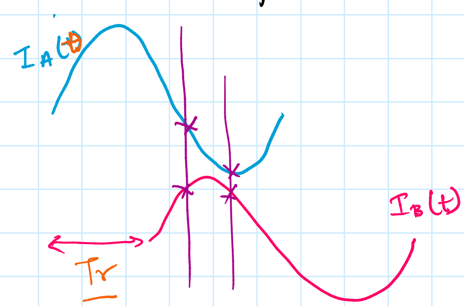
$I_A(t) = I_B(t)$
at all time t

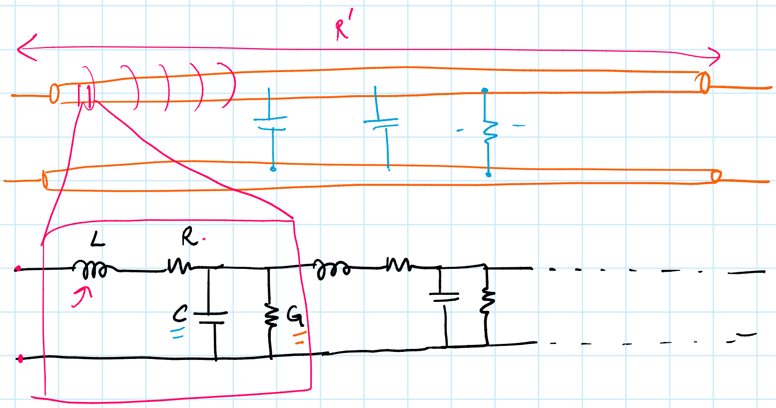
Distributed Circuit.



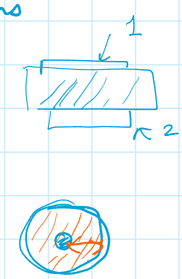
Transmission Line Concepts.

$|I_A(t)| \neq |I_B(t)|$
for all time t.





- FR4
- Rogers



R = Resistance / unit length Ω/m

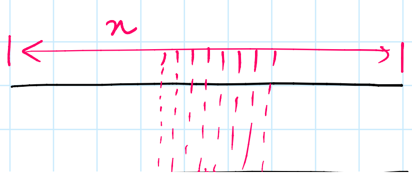
L = Inductance / unit length H/m

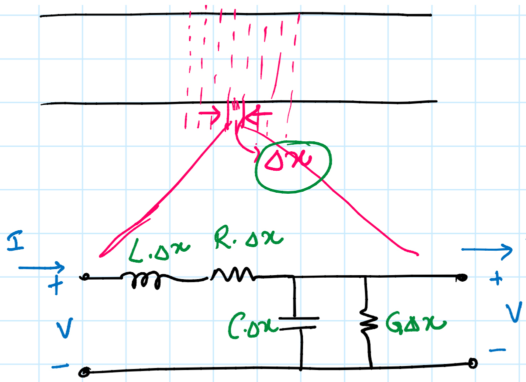
C = Capacitance / unit length F/m

G = Conductance / unit length ℧/m

Primary Parameters of a Transmission Lines.

Analysis of Transmission Line





→ current has changed because some of it has gone through C & G

↳ voltage has changed because of voltage drop in L & R.

Assume the frequency of operation to be f Hz

ω rad/s

$$\Delta V = -(R\Delta x + j\omega L\Delta x) I$$

$$\Delta I = -(G\Delta x + j\omega C\Delta x) V$$

Assuming ΔV & ΔI are very small

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = -(R + j\omega L) I \quad ; \quad \lim_{\Delta x \to 0} \frac{\Delta I}{\Delta x} = -(G + j\omega C) V$$

$$\boxed{\frac{dV}{dx} = -(R + j\omega L) I \quad ; \quad \frac{dI}{dx} = -(G + j\omega C) V}$$

→ These equations are coupled.

Take second derivative to uncouple the equations

$$\frac{d^2V}{dx^2} = -(R + j\omega L)\frac{dI}{dx} \quad ; \quad \frac{d^2I}{dx^2} = -(G + j\omega C)\cdot\frac{dV}{dx}.$$

$$\frac{d^2V}{dx^2} = \underbrace{(R + j\omega L)(G + j\omega C)}_{\gamma^2} V \quad ; \quad \frac{d^2I}{dx^2} = \underbrace{(R + j\omega L)(G + j\omega C)}_{\gamma^2}\cdot I$$

$\gamma^2 \rightarrow$ Propagation Constant

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

→ Represents the characteristic quantity of the transmission line

$$\boxed{\frac{d^2V}{dx^2} = \gamma^2 V} \quad ; \quad \frac{d^2I}{dx^2} = \gamma^2 I$$

$$L = e^{ax}$$

$$\frac{dL}{dx} = a e^{ax}$$

$$\boxed{\frac{d^2L}{dx^2} = a^2 e^{ax} = a^2 L}$$

Solution

$$V(x) = \underbrace{V^+ e^{-\gamma x}}_{\text{Forward travelling signal}} + \underbrace{V^- e^{\gamma x}}_{\text{Backward travelling signal}}$$

$V(x)$ → Voltage as a function of space or distance

$$V(x,t) = V^+ e^{-\gamma x} e^{j\omega t} + V^- e^{\gamma x} e^{j\omega t}$$

$V(x,t)$ → Voltage as a function of space & time

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

γ → It is a complex quantity

$$\boxed{\gamma = \alpha + j\beta}$$

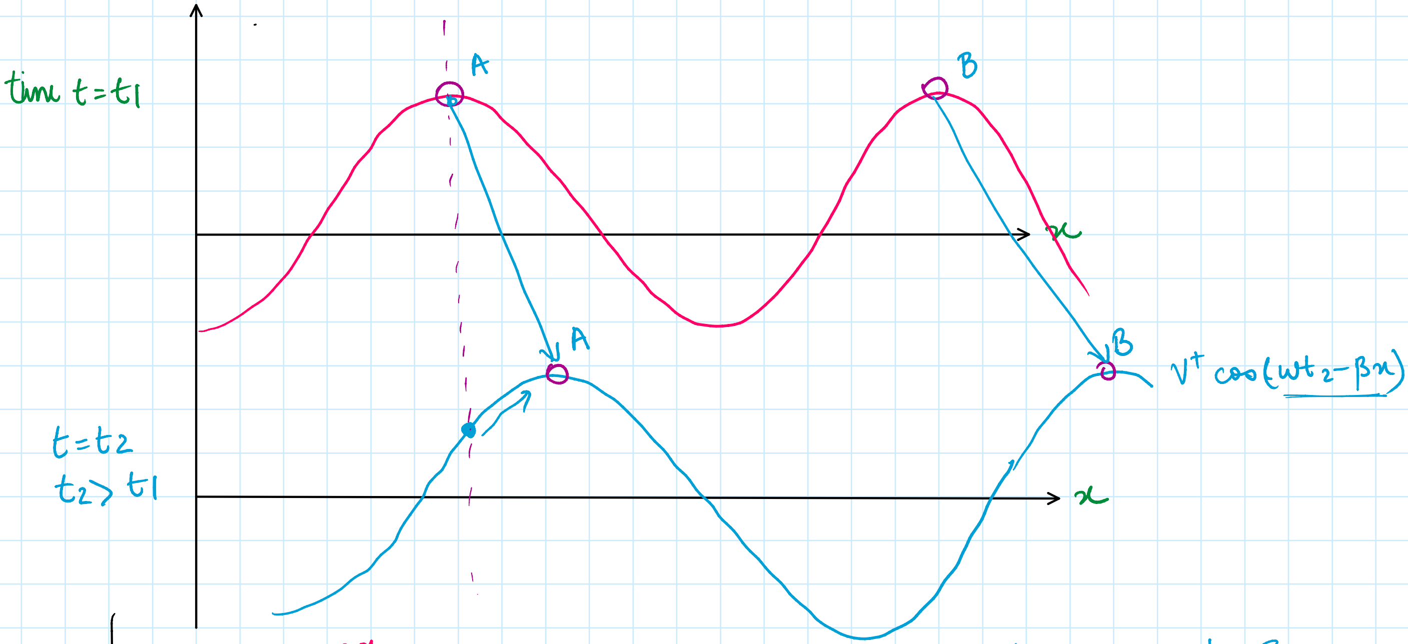
$$\boxed{V(x,t) = V^+ e^{-\alpha x} e^{j(\omega t - \beta x)} + V^- e^{\alpha x} e^{j(\omega t + \beta x)}}$$

Forward Travelling Signal : $\underbrace{V^+ e^{-\alpha x}}_{\text{Amplitude variation with space}} \underbrace{e^{j(\omega t - \beta x)}}_{\text{phase shift as function space \& time}}$

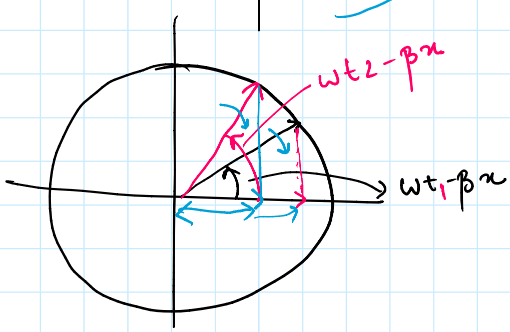
Let us assume $\alpha = 0$

$$Re\{V^+ e^0 e^{j(\omega t - \beta x)}\}$$

$V^+ \cos(\omega t - \beta x)$



$\omega t_2 - \beta x > \omega t_1 - \beta x$



$\phi = \omega t - \beta x$

$x \uparrow \quad \phi \downarrow$