Lec-4

Monday, April 6, 2020 8:18 AM

HW-1 will be posted today.

Reverse Travelling Wave

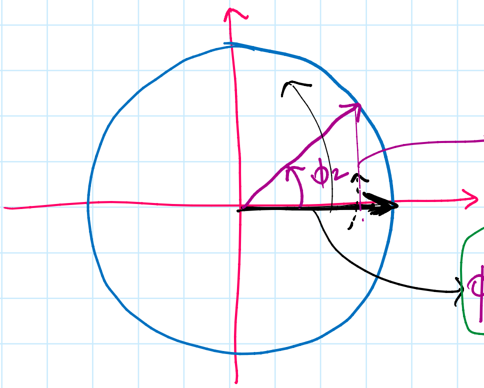
$$V(t) = V^- e^{\alpha x} e^{j(\omega t + \beta x)}$$

assume $\alpha = 0$

$$\mathrm{Re}\left\{ V^- e^{0} e^{j(\omega t + \beta x)} \right\}$$

$$V^- \cos(\underbrace{\omega t + \beta x}) \longrightarrow \phi = \omega t + \beta x$$

as $x \uparrow$ $\phi \uparrow$



$t = t_1$; $x = 0$

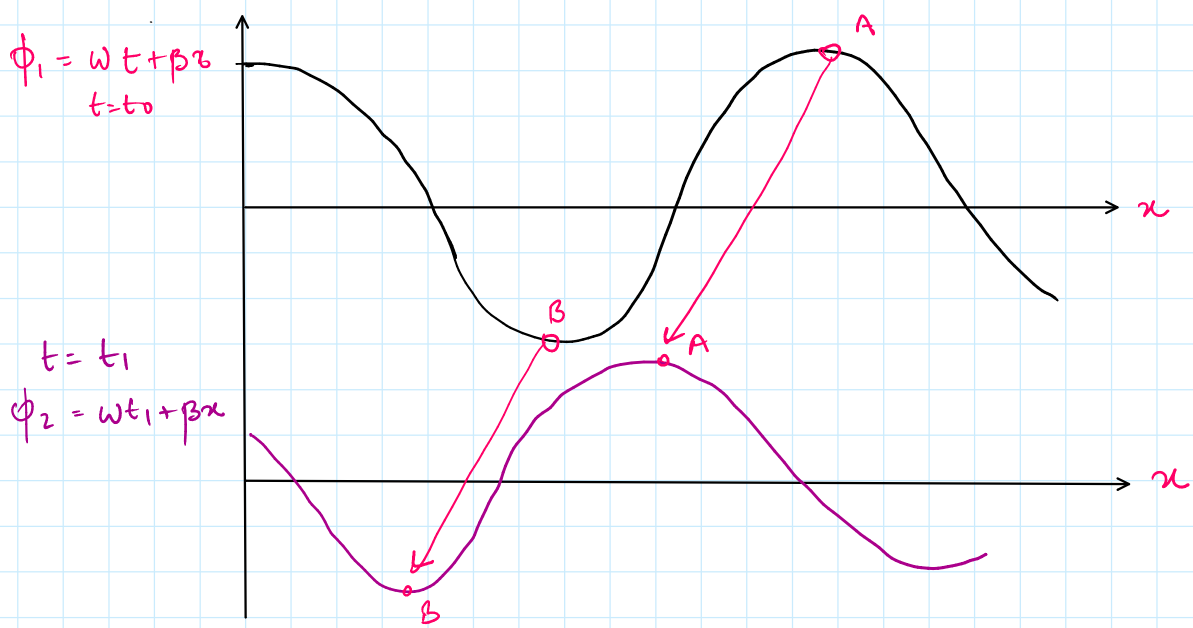
$\phi_2 = \omega t_1 + \beta x = \omega t_1$

$t = 0$; $x = 0$

$\phi_1 = \omega \cdot t + \beta x$

$|\cos(\phi_2)| < |\cos(\phi_1)|$

$x \uparrow$ $\phi \uparrow$



Propagation Constant : $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \underline{\alpha} + j\beta$

Forward Travelling Wave: $V^+ e^{-\gamma x}$

$$\underbrace{V^+ e^{-\alpha x}}_{\text{Magnitude}} \cdot \underbrace{e^{-j\beta x}}_{\text{Phase}}$$

- Real part of γ controls the magnitude of the signal
- Imaginary part of γ controls the phase of the signal

$e^{-j\beta x} \xrightarrow{\text{Phase}} \underbrace{\beta x} \rightarrow$ Phase shift of a signal by 'βx' in space

β = Phase change per unit length.

= Units → rads/m

Wavelength of a signal is λ

Phase shift in one wavelength → 2π

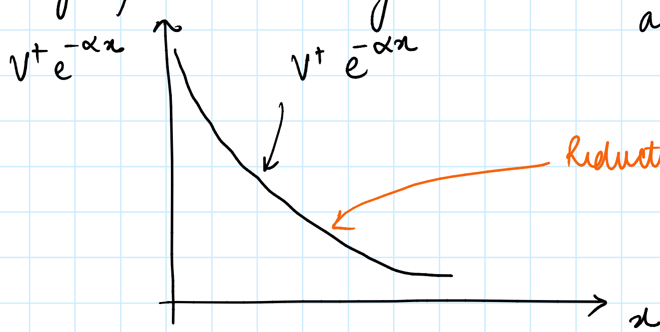
Phase change in covering a distance of $\lambda = 2\pi$

Phase change per unit length $= \dfrac{2\pi}{\lambda} = \beta$

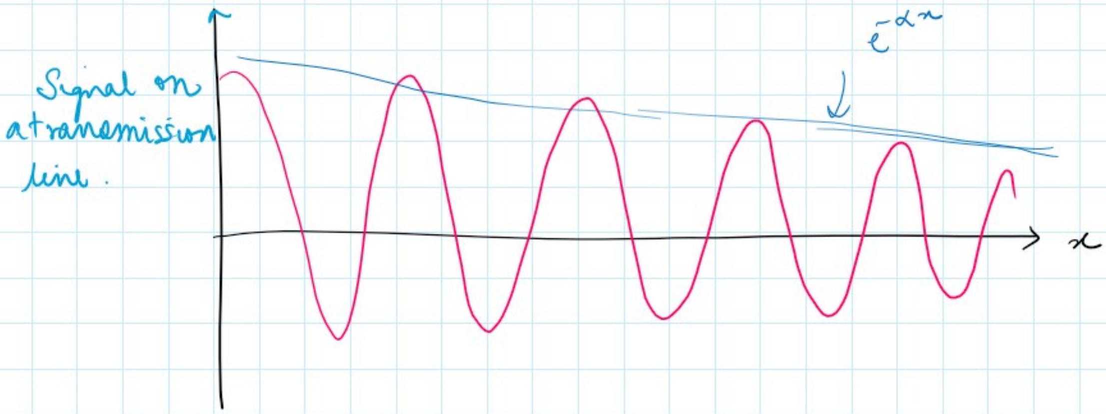
$$\boxed{\beta = \frac{2\pi}{\lambda}}$$

Magnitude of forward travelling wave

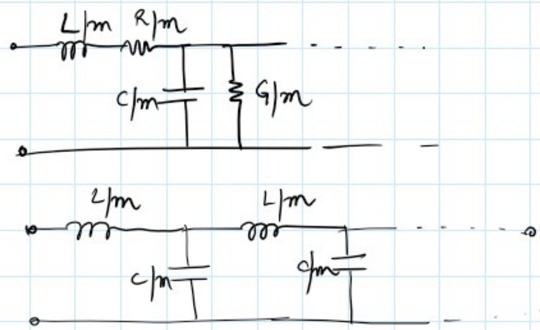
assuming $\alpha > 0$



Reduction in magnitude as it travels the distance x.



$$\boxed{\alpha = \text{Attenuation constant : Nepers/m}}$$



$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

Assume → $R=0$; $G=0$

$$\gamma = \sqrt{-(\omega^2)LC}$$

$$\gamma = j\omega\sqrt{LC} = \alpha + j\beta$$

$$\left.\begin{aligned} \alpha &= 0 \\ \beta &= \omega\sqrt{LC} \end{aligned}\right\} \text{Lossless Transmission Line.}$$

Relationship between dBs & Nepers

Consider forward travelling wave: $V^+ e^{-\alpha x} e^{-j\beta x}$

$x = 0$ $\qquad$ $x = 1\,m$

$|V^+|$ $\qquad$ → $V^+ e^{-\alpha x}$ → Power loss happens because $\alpha > 0$

$$\text{Power loss (dB)} = -10 \log\left(\frac{(V^+ e^{-\alpha x})^2}{(V^+)^2}\right)$$

$$= -20\log\left(e^{-\alpha x}\right)$$

$$\boxed{\text{Power loss (dB) @ } x = 1m = -20\log\left(e^{-\alpha}\right)}$$

Lets put $\alpha = 1$ neper/m

$$-20\log\left(e^{-1}\right) = \underline{-8.68\ dB} = \alpha = 1\ \text{neper/m}$$

$$\boxed{\alpha = 1\ \text{neper/m} = 8.68\ \text{dB of Power loss}}$$

Example 1:

$R = 0.5\Omega/m$
$L = 0.2\ \mu H/m$
$C = 100 pF/m$
$G = 0.1 \mho/m$

@ Freq = 1GHz.

Estimate γ, α, β

$$\gamma = \sqrt{\left(0.5 + j\,2\pi \times 1 \times 10^{9} \cdot 0.2 \times 10^{-6}\right)\left(0.1 + j\,2\pi \times 10^{9} \times 100 \times 10^{-12}\right)}$$

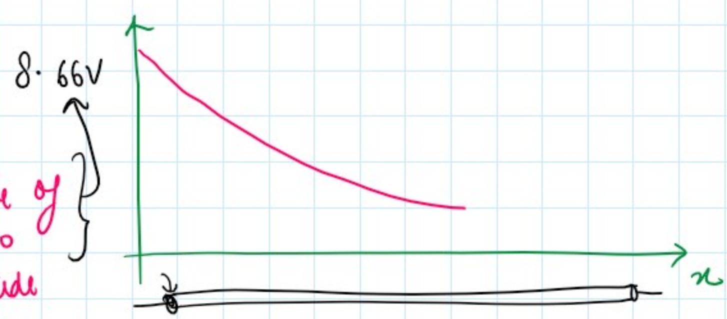
$$\gamma = \underset{\alpha}{2.23} + j\,\underset{\beta}{28.2}$$

pico = 10^{-12}
$\mu = 10^{-6}$
nano = 10^{-9}

Example 2:

- Given the amplitude of wave at $t=0$ & $x=0$
- Estimate the amplitude at $t = 100ns$ $x = 1m$

Use Ex 1 α & β.
Initial phase of the signal is 30°



freq = 1GHz

$$V(t,x) = \mathrm{Re}\left\{V^{+} e^{-\alpha x} e^{-j\beta x + j\omega t}\right\}$$

$8.66V \rightarrow V(t,x)\quad |V^{+}| \cos(\phi + \omega t - \beta x)\, e^{-\alpha x}$

$8.66V \rightarrow V(t,x) \quad \underbrace{|V^+|}\cos(\phi_0 + \omega t - \beta x)\, e^{-\alpha x}$

$\phi_0 \downarrow$ Initial phase

Step 1: Estimate $|V^+|$

$x=0, \; t=0$

$V(t,x) = |V^+| \cos(\phi_0)$