Lec-5

Wednesday, April 8, 2020 8:55 AM

$$\text{Power loss (dBs)} = -20\log\left(e^{-\alpha x}\right)$$

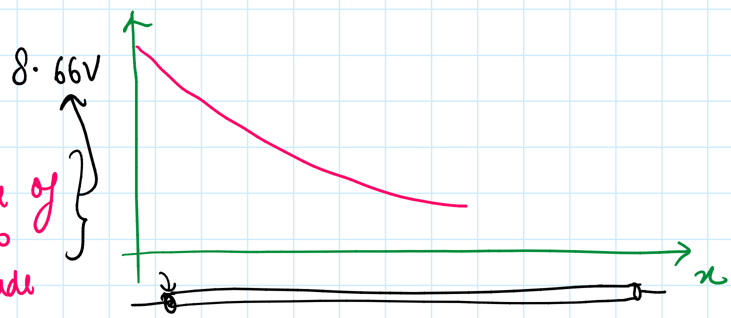
$$\text{Power loss (dBs)} = +20\,\alpha x \log(e)$$

$$x = 1 \; ; \; \alpha = 1$$

$$= \underbrace{8.68\text{ dB}}\; \Big\} \rightarrow \alpha = 1\text{ neper/m}$$

Example $\underbrace{17.36\text{ dB}} \rightarrow \underbrace{\alpha = 2\text{ neper/m}}$

Example 2:



- Given the amplitude of wave at $t=0$ & $x=0$
- Estimate the amplitude at $t = \underline{100\text{ns}}$ $\underline{x = 1\text{ m}}$

Use Ex 1 α & β.

freq = 1 GHz

- Initial phase of the signal is 30°

$$V(t,x) = \mathrm{Re}\left\{ V^+ e^{-\alpha x} e^{-j\beta x + j\omega t} \right\}$$

$$\underline{8.66V} \rightarrow V(t,x)\;\; \underbrace{|V^+|} \cos\left(\phi_0 + \omega t - \beta x\right) e^{-\alpha x} \Big\} \quad t=0 \; ; \; x=0$$

$\phi_0 \downarrow$ Initial phase

$$8.66 = |V^+| \cos(\phi_0)$$

$\phi_0 \downarrow 30^\circ$

$$|V^+| = \frac{8.66}{\cos(30)} = \frac{8.66 \times 2}{\sqrt{3}} = 10\text{ V}$$

$$v(x,t) = 10\cos(\phi_0 + \omega t - \beta x)\, e^{-\alpha x}$$

$\alpha = 2.23$ nepers/m
$\beta = 28.2$ rad/m
$\omega = 2\pi \times 1\times 10^{9}$
$t = 100$ ns
$x = 1$ m

$$\boxed{V(x,t) = -0.88\,V}$$

- Continue with the derivation of transmission line voltage and current.

$$\frac{dV}{dx} = -(R+j\omega L)I$$

Substitute the solution of V & I

$$\frac{d}{dx}\left(V^+ e^{-\gamma x} + V^- e^{\gamma x}\right) = -(R+j\omega L)\left\{I^+ e^{-\gamma x} + I^- e^{\gamma x}\right\}$$

$$-\gamma V^+ e^{-\gamma x} + \gamma V^- e^{\gamma x} = -(R+j\omega L)\left(I^+ e^{-\gamma x} + I^- e^{\gamma x}\right)$$

Seperate the forward and reverse travelling waves.

FTW
Forward Travelling Wave : $\cancel{-}\gamma V^+ \cancel{e^{-\gamma x}} = \cancel{-}(R+j\omega L) I^+ \cancel{e^{-\gamma x}}$

RTW : $\gamma V^- \cancel{e^{\gamma x}} = -(R+j\omega L) I^- \cancel{e^{\gamma x}}$

$$\frac{V^+}{I^+} = \frac{R+j\omega L}{\gamma} \quad ; \quad \frac{V^-}{I^-} = -\frac{(R+j\omega L)}{\gamma}$$

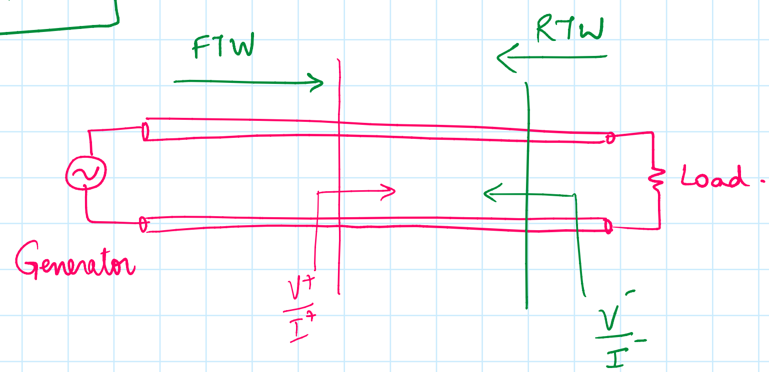
$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\boxed{\frac{V^+}{I^+} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad ; \quad \frac{V^-}{I^-} = -\sqrt{\frac{R+j\omega L}{G+j\omega C}}}$$

Negative Sign

Characteristic Impedance

$$\boxed{Z_0 = \sqrt{\left(\frac{R+j\omega L}{G+j\omega C}\right)}}$$



Energy source is on the left; Load is on the right

RTW goes from right to left → I see that the energy is increasing.

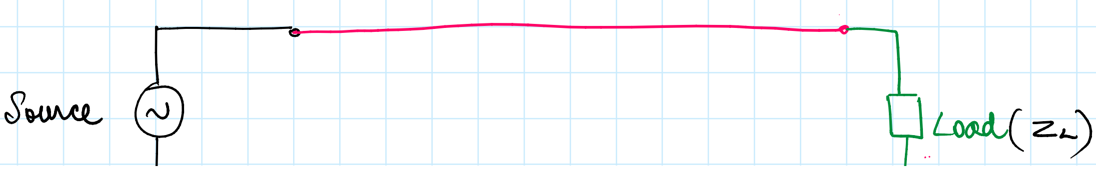
$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

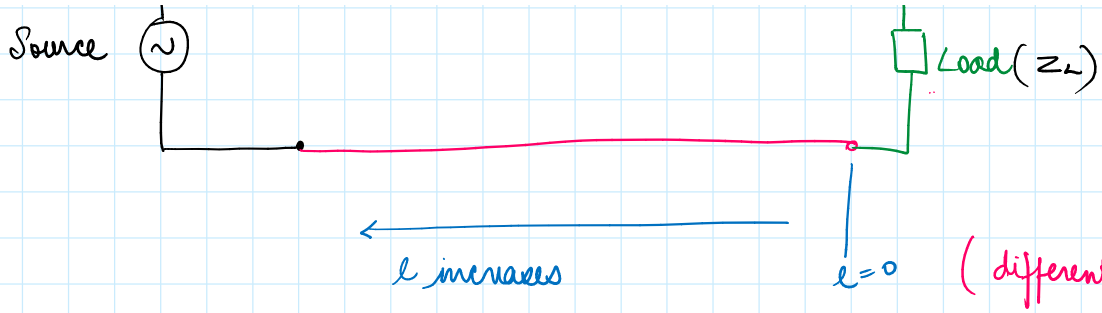
Observations

1. Z_0 = Real for a loss less line
2. Z_0 = Imaginary for a lossy line.

Loss Less → $R = 0$; $G = 0$ $\quad Z_0 = \sqrt{\frac{L}{C}}$

Boundary conditions on a Transmission Line.





(different than text book)

$$V = V^+ e^{\gamma l} + V^- e^{-\gamma l} \quad -(1)$$

$$I = I^+ e^{\gamma l} + I^- e^{-\gamma l} \quad -(2)$$

$$I = \frac{V^+}{Z_0} e^{\gamma l} - \frac{V^-}{Z_0} e^{-\gamma l} \quad -(3)$$

l is made negative because we have assumed that l increases from right to left.

Boundary conditions : $l=0$; $Z = Z_L$ (load impedance)

Take the ratio of eq (1) & (3)

$$Z = \frac{V}{I} = Z_0 \left(\frac{V^+ e^{\gamma l} + V^- e^{-\gamma l}}{V^+ e^{\gamma l} - V^- e^{-\gamma l}} \right)$$

$\longrightarrow$ Apply the boundary conditions.

$l=0$; $Z = Z_L$

$$Z_L = Z_0 \left(\frac{V^+ + V^-}{V^+ - V^-} \right)$$

$$\boxed{\text{Reflection Coefficient} : \Gamma(l) = \frac{V^- e^{-\gamma l}}{V^+ e^{\gamma l}}}$$

at $l=0$ $\Gamma(0) = \frac{V^-}{V^+}$

$$Z_L = Z_0 \left(\frac{1+\Gamma(0)}{1-\Gamma(0)} \right)$$

$$\frac{Z_L}{Z_0} = \frac{1+\Gamma(0)}{1-\Gamma(0)}$$

$$\boxed{\Gamma(0) = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$Z_0 \rightarrow$ Char Impedance

$Z_L \rightarrow$ Load Impedance.

It is a measure of how much energy is reflected from the load end of the transmission line.

Ideal $\Gamma(0) = 0$

Typically $-1 \leq \Gamma \leq 1$

$Z_L = ?$ $\longrightarrow$ $\Gamma(0) = 0$

$Z_L = Z_0$ $\longrightarrow$ $\Gamma(0) = 0$ $\longrightarrow$ Matched Condition

Transmission line is terminated by the Characteristic Impedance.