# ECE 391 Transmission Lines, Winter 2020

**Test Date: 02/05/2020**

**Problems: 4**

**Total Pages: 6**

**Name:** SOLUTION KEY

1. (10 points) ________________
2. (10 points) ________________
3. (20 points) ________________
4. (20 points) ________________

**Total (60 points)** ______________

**Good Luck**

**Problem 1: (10 points)** A surface mount resistor is 2mm in length. Propagation velocity of the signal through this resistor is 0.6c (where c=3x10$^8$m/s). Up to what frequency can you consider this component as lumped parameter?

Note: Transit time effect can be safely ignored if the length of the transmission line is less than 1% of the signal wavelength.

Frequency = 0.9 GHz

To be considered a lumped element,

$$l \le 1\% \text{ of } \lambda \Rightarrow 100\,l \le \lambda$$

$$\Rightarrow 100\,l \le \frac{v_p}{f} \Rightarrow f \le \frac{v_p}{100\,l}$$

$$\Rightarrow f \le \frac{0.6c}{100\,l} \Rightarrow f \le \frac{0.6 \times 3 \times 10^8}{100 \times 2 \times 10^{-3}} \text{ Hz} \Rightarrow f \le 0.9 \text{ GHz}$$

**Problem 2: (10 points)** Parameters of a transmission lines are: R = 0.5Ω/m; L=0.237μH; G = 6.21x10$^{-4}$ S/m; C=106pF; Calculate the characteristic impedance of the transmission line at 1MHz and 10GHz?

Zo (1MHz) = 40.59 + 8.79j

Zo (10GHz) = 47.28 + 0.0014j

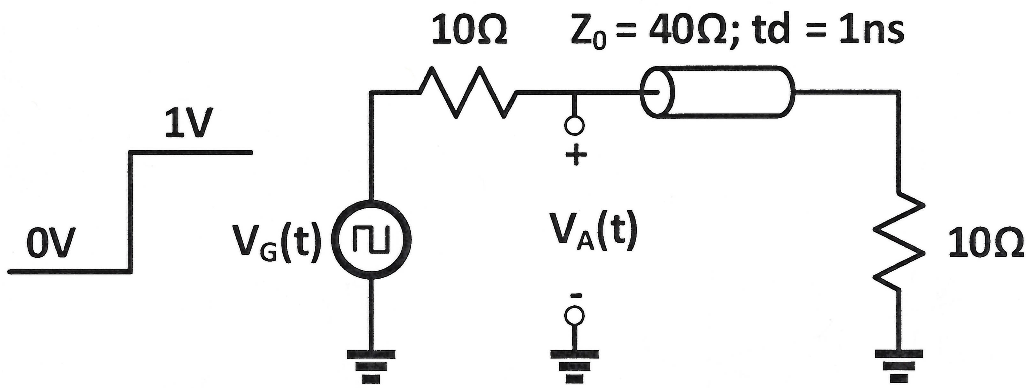
$f_1 = 1\text{MHz}, \quad \omega_1 = 2\pi f_1$

$$Z_{01} = \sqrt{\frac{R + j\omega_1 L}{G + j\omega_1 C}} = 40.59 + 8.79j$$

$f_2 = 10\text{GHz}, \quad \omega_2 = 2\pi f_2$

$$Z_{02} = \sqrt{\frac{R + j\omega_2 L}{G + j\omega_2 C}} = 47.28 + 0.0014j$$

**Problem 3: (20 points)** Given the following transmission line:




(a) **(5 points)** Calculate the reflection coefficient at the generator side and load side:

Reflection Coefficient on Generator ($\Gamma_G$) = $-0.6$

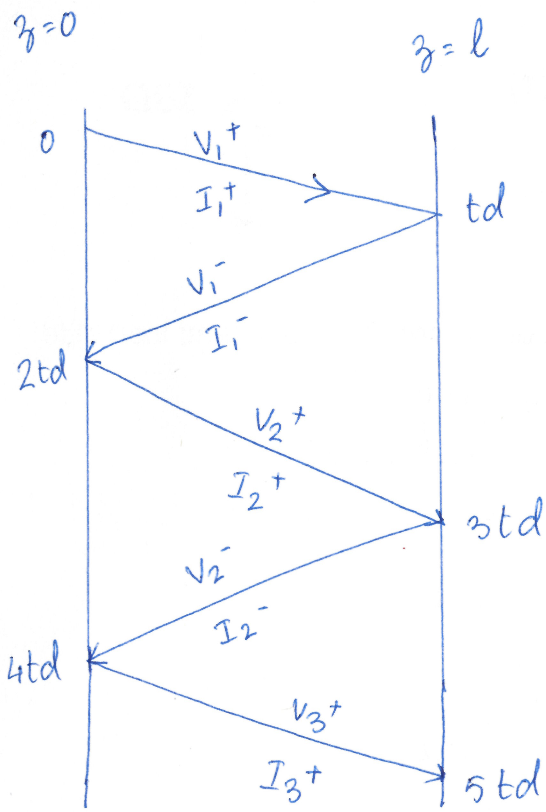
Reflection Coefficient on Load ($\Gamma_L$) = $-0.6$

$$\Gamma_G = \frac{R_G - Z_0}{R_G + Z_0} = \frac{10-40}{10+40} = -0.6$$

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = \frac{10-40}{10+40} = -0.6$$

(b) **(10 points)** Draw lattice diagram for up to 5ns.

$t_d = 1\,ns$




$\Gamma_L = -0.6$

$\Gamma_G = -0.6$

$V_1^+ = V_{inc} = \frac{V_G \cdot Z_0}{Z_0 + R_G} = 0.8\,V$

$V_1^- = \Gamma_L V_1^+ = -0.48\,V$

$V_2^+ = \Gamma_S V_1^- = \Gamma_S \Gamma_L V_1^+ = 0.288\,V$

$V_2^- = \Gamma_L V_2^+ = \Gamma_L^2 \Gamma_S V_1^+ = -0.1728\,V$

$V_3^+ = \Gamma_S V_2^- = \Gamma_L^2 \Gamma_S^2 V_1^+ = 0.10368\,V$

$I_1^+ = V_1^+/Z_0 = 0.02A$

$I_1^- = -\Gamma_L I_1^+ = +0.012A = -V_1^-/Z_0$

$I_2^+ = \Gamma_S I_1^- = 0.0072A = V_2^+/Z_0$

$I_2^- = -\Gamma_L I_2^+ = +0.00432A = -V_2^-/Z_0$

$I_3^+ = \Gamma_S I_2^- = 0.0002592A = V_3^+/Z_0$

$= 0.002592A$

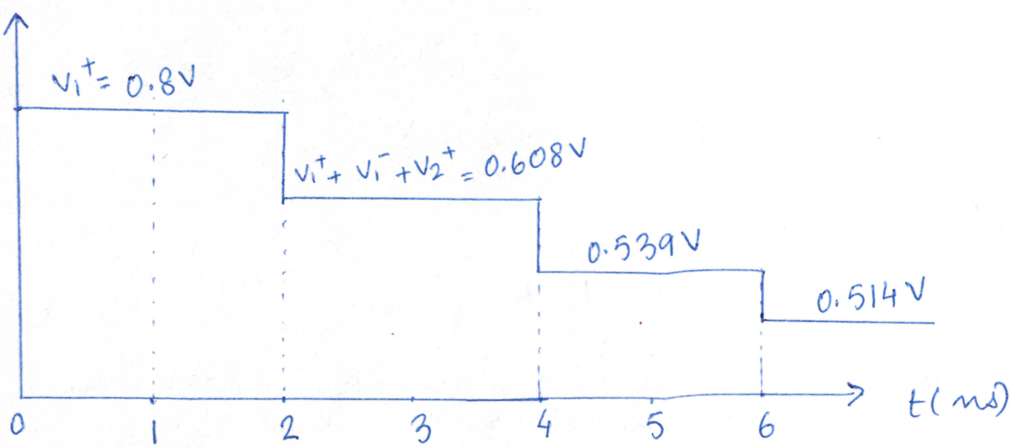
(c) **(10 points)** Plot $V_A(t)$ up to 6ns.

$V_A(t)$ at $t = 0^+ = V_1^+ = 0.8V$

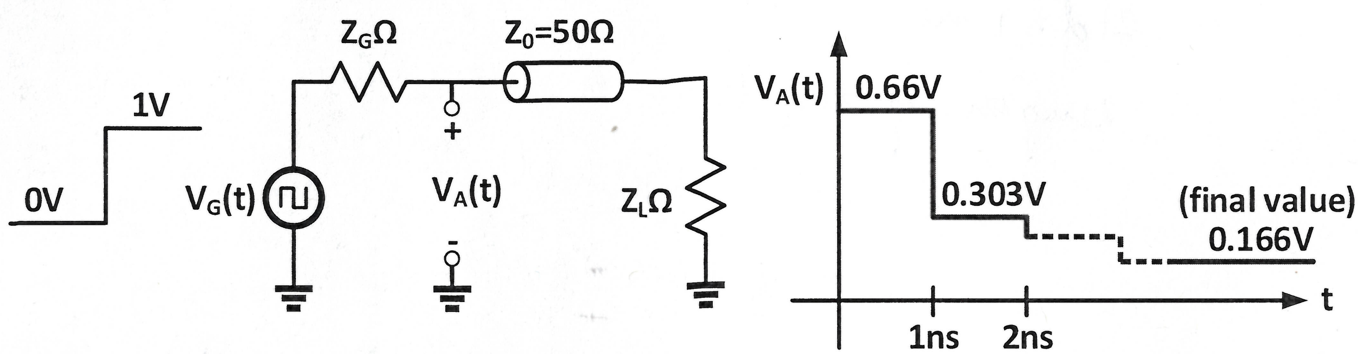
at $t = 2td^+ = V_1^+ + V_1^- + V_2^+ = 0.608V$

at $t = 4td^+ = V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ = 0.539V$

at $t = 6td^+ = V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- + V_4^+ = 0.514V$




**Problem 4: (20 points)** Given the following transmission line and measurement of voltage near generator $V_A(t)$:



(a) **(14 points)** Calculate the impedance $Z_G$ and $Z_L$.

$Z_G$ = 25 Ω

$Z_L$ = 5 Ω

$V_A(t)$ at $t=0^+ = V_1^+ = \frac{V_G \cdot Z_0}{Z_0 + Z_G} = 0.666\ V$

$\Rightarrow Z_G = 25\ \Omega$

$V_A(t)$ at $t \to \infty = \frac{V_G \cdot Z_L}{Z_L + Z_G} = 0.166\ V \Rightarrow Z_L = 5\ \Omega$

Alternate approach, $V_A(t)$ at $t = 2td^+ = V_1^+ + V_1^- + V_2^+ = 0.303V$

$\Rightarrow V_1^+ (1 + \Gamma_L + \Gamma_L \Gamma_G) = 0.303V$

$\Rightarrow \Gamma_L = (Z_L - Z_0)/(Z_L + Z_0) = -0.818 \Rightarrow Z_L = 5\ \Omega$

(b) **(6 points)** Given the propagation velocity of the signal through this transmission line is 0.5c (where c=3x10$^8$m/s), calculate the length of the transmission line.

Length = 75 mm

$v_p = 0.5\ c$

$2td = 1\ ns$

∴ Length of the transmission line

$= \frac{1}{2} \cdot (v_p) \cdot (2td)$

$= \frac{1}{2} \cdot (0.5c) \cdot (2td)$

$= \frac{1}{2} \cdot (0.5 \times 3 \times 10^8)(1 \times 10^{-9})\ m$

$= 75\ mm$