Name: Solutions
(Last name, first name)

Student ID: ____________________

ECE 391

TRANSMISSION LINES

Spring Term 2017

Midterm II

Exam is closed book, closed notes; **one** sheet (2 pages) of notes and formulas allowed; 50 minutes. Show all work on the pages provided. No extra pages (use back if necessary). **Read each question very carefully.**

Box your final answer and include units where appropriate. Number of points for each problem is given in parenthesis (40 points total).

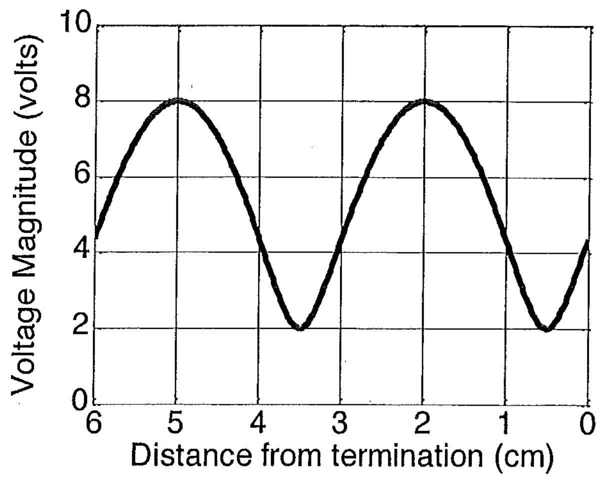
Problem 1 (18 pts.) ____________________

Problem 2 (12 pts.) ____________________

Problem 3 (10 pts.) ____________________

Total (40 pts.) ____________________

1. (18 pts.) A transmission line of characteristic impedance $Z_0 = 50\Omega$ is terminated in an **unknown** load impedance Z_L. The voltage standing-wave pattern along the transmission line as function of distance from the termination is shown below.



(a) What is the standing-wave ratio on the line terminated in the unknown load impedance Z_L?

$$VSWR = \frac{8V}{2V} = \boxed{4}$$

(b) Determine the wavelength on the line.

$$\frac{\lambda}{2} = 5cm - 2cm = 3cm \Rightarrow \boxed{\lambda = 6cm}$$

(c) Determine the voltage magnitude of the outgoing wave, $|V_0^+|$.

$$|V_0^+| = \frac{V_{max} + V_{min}}{2} = \frac{8V + 2V}{2} = \boxed{5V}$$

(d) Determine the reflection coefficient at the termination in **magnitude** and **phase.**

$$|\Gamma| = \frac{VSWR-1}{VSWR+1} = \frac{4-1}{4+1} = \boxed{\frac{3}{5} = 0.6}$$

distance to $V_{max} = 2cm \hat{=} \frac{2}{6}\lambda = \frac{\lambda}{3} \hat{=} 120^0$

$$\Rightarrow \frac{\theta_L}{2} = 120^\circ \Rightarrow \theta_L = 240^0 = \boxed{-120^\circ}$$

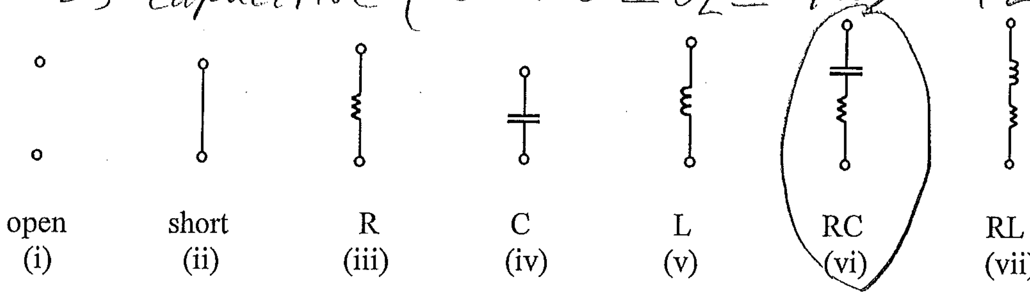
<u>or</u> distance to $V_{min} = 0.5cm = \frac{0.5}{6}\lambda = \frac{\lambda}{12} \hat{=} 30^0$

$$90^0 + \frac{\theta_L}{2} = 30^0 \Rightarrow \frac{\theta_L}{2} = -60^0 \Rightarrow \boxed{\theta_L = -120^0}$$

(e). Indicate the type of termination from the list shown below that produces the standing-wave pattern shown above.

Moving towards the source, V_{min} is seen first

$\Rightarrow$ capacitive (also: $0 \geq \theta_L \geq -180°$) $+ |\Gamma_L| < 1 \Rightarrow$ resistive

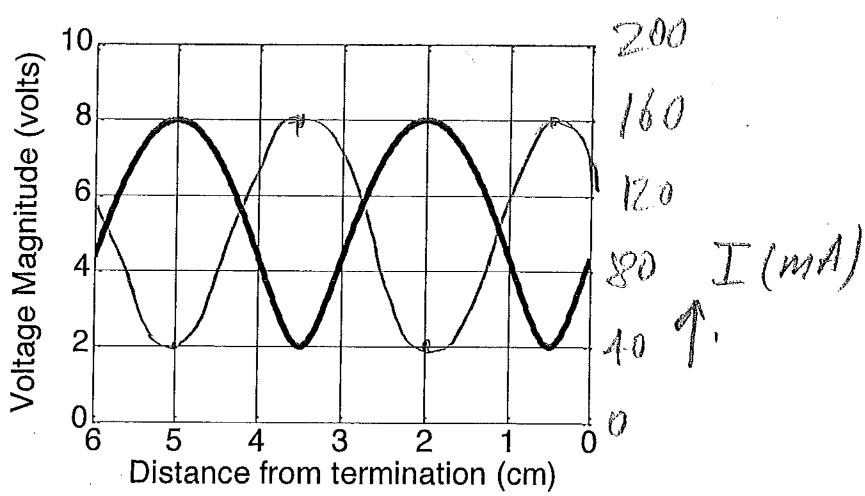


(f) Determine $|I|_{max}$ and $|I|_{min}$ on the line.

$$|I_{max}| = \frac{|V_{max}|}{Z_0} = \frac{8V}{50\Omega} = \boxed{160mA}$$

$$|I_{min}| = \frac{|V_{min}|}{Z_0} = \frac{2V}{50\Omega} = \boxed{40mA}$$

(g) Sketch the corresponding current standing-wave plot in the graph below.



2. (12 pts.) A low-loss 50Ω transmission line of 100m length is found to attenuate a sinusoidal wave traveling from one end to the other by 6dB. It is known that dielectric loss is negligible.

(a) What is the voltage magnitude across a matched load if the magnitude of the voltage across the input terminals of the line is 10V?

$$20\log_{10}\frac{V_2}{V_1} = -6dB \Rightarrow \frac{V_2}{V_1} = 10^{-\frac{6}{20}} = 0.5$$

$$\boxed{V_2 = V_{FE} = 5V}$$

(b) Determine the attenuation constant in Np/m.

$$\alpha_{dB} = \frac{6dB}{100m} = 0.06\frac{dB}{m} \Rightarrow \alpha_{Np} = \alpha_{dB}\cdot\frac{1}{8.686\frac{dB}{Np}} =$$

$$= \frac{0.06}{8.686}\frac{Np}{m} = \boxed{0.0069\frac{Np}{m}}$$

(c) Applying the low-loss approximation, determine the resistance per-unit-length, R, of the transmission line in ohms/meter.

$$\alpha_{Np} \cong \frac{R}{2Z_0} \Rightarrow R = \alpha_{Np}2Z_0 = 0.0069\frac{1}{m}2\cdot 50\Omega$$

$$= \boxed{0.69\frac{\Omega}{m}}$$

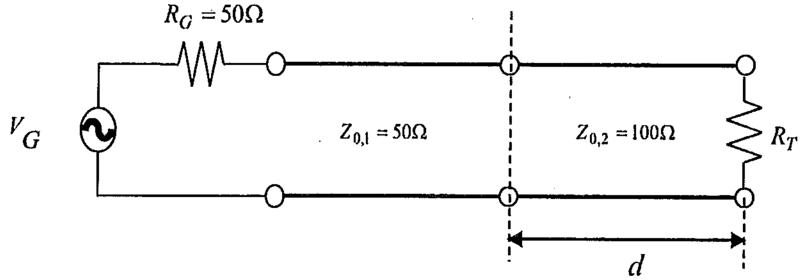
(d) Applying the low-loss approximation, determine the remaining per-unit-length parameters L, C, and G (don't forget to specify the proper units). Assume a phase velocity of $v_p = 20$ cm/ns.

$$Z_0 \cong \sqrt{\frac{L}{C}};\ V_p \cong \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{Z_0}{V_p} = \frac{50\Omega}{0.2\,m/ns} = \boxed{250\frac{nH}{m}}$$

$$C = \frac{1}{Z_0 V_p} = \frac{1}{50\Omega \times 0.2\frac{m}{ns}} = \frac{1}{10}10^{-9}\frac{F}{m} = \boxed{100\frac{pF}{m}}$$

$G \cong 0$ (dielectric loss is negligible)

3. (10 pts.) An unknown resistive load R_T is connected through a transmission line section of length d and characteristic impedance $Z_{0,2} = 100\Omega$ to a $Z_{0,1} = 50\Omega$ transmission line, as shown below. At frequency f_0, line length d corresponds to a quarter-wavelength ($d = \lambda/4$).



(a) What is the physical length of the 100Ω line if $f = f_0 = 200\,\text{MHz}$ and the effective dielectric constant of the transmission line is $\varepsilon_{r,eff} = 4$?

$$\text{at } f=f_0:\ d = \lambda/4 \quad \lambda = \frac{v_p}{f} = \frac{3\times10^8 \frac{m}{s}}{\sqrt{4}\ 200\times10^6 \frac{1}{s}} = \frac{3}{4}\,m = 75\,cm$$

$$\Rightarrow d = \frac{\lambda}{4} = \boxed{18.75\,cm}$$

(b) Determine R_T if at $f = f_0$ the voltage standing-wave ratio on the 50Ω line is $VSWR = 1$.

line 1 sees a match

$$\Rightarrow Z_{in} = \frac{Z_{0,2}^2}{R_T} = Z_{0,1} = 50\Omega \Rightarrow R_T = \frac{Z_{0,2}^2}{Z_{0,1}} = \frac{100^2}{50}\Omega = \boxed{200\Omega}$$

(c) What is the voltage standing-wave ratio on the 50Ω line (with R_T from part b) if the frequency is doubled ($f = 2f_0$)?

$$\text{if } f = 2f_0 \Rightarrow d = \frac{\lambda}{2} \Rightarrow Z_{in} = R_T$$

$$\Gamma = \frac{R_T - Z_0}{R_T + Z_0} = \frac{200-50}{200+50} = \frac{150}{250} = \frac{3}{5} = 0.6 \Rightarrow SWR = \frac{1+0.6}{1-0.6} \boxed{= 4}$$

$$\text{or: } SWR = \frac{R_T}{Z_0} = \frac{200}{50} = 4$$

(d) What is the voltage standing-wave ratio on the 50Ω line if the load resistor R_T is replaced with an inductor $L = 4\,\text{nH}$ and the frequency is $f = f_0 = 200\,\text{MHz}$?

Termination is purely reactive

$$\Rightarrow |\Gamma_L| = 1 \Rightarrow VSWR \rightarrow \infty$$