ECE 391 Transmission Lines, Winter 2020

Test Date: 03/04/2020

Problems: 4

Total Pages: 7

Name: Midterm-2 Solutions

1. (10 points) ______________
2. (10 points) ______________
3. (15 points) ______________
4. (15 points) ______________

Total (50 points) ____________

Good Luck

Problem 1: (10 points) The following data are specified at f=1MHz for a given transmission line. $Z_0=(99.85 - j3.008)\Omega$; $\alpha=4.345$dB/m; $\beta= 16.328\times10^{-3}$rad/m. Determine the per-unit-length R, L, G, C transmission line parameters.

R = 50Ω/m

L = 20nH/m

G = 5mS/m

C = 50pF/m

$$\gamma = \underset{\text{np/m}}{\alpha} + j\underset{\text{rad/m}}{\beta} = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$$R = Re\{Z_0 \cdot \gamma\}$$

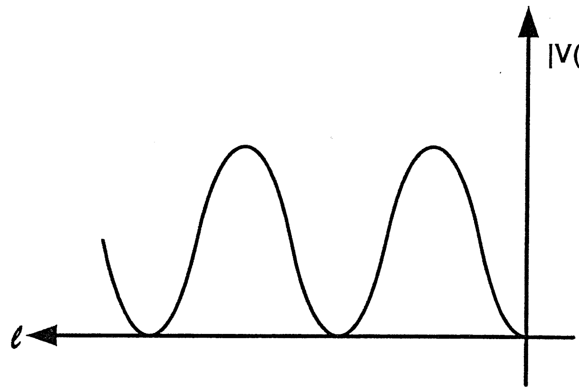
$$\omega L = Img\{Z_0 \gamma\}$$

$$G = Re\{\gamma / Z_0\}$$

$$\omega C = Img\{\gamma / Z_0\}$$

Problem 2: (10 points) Circle the termination load on the transmission line from the voltage standing wave and give reason for your answer.

(a)



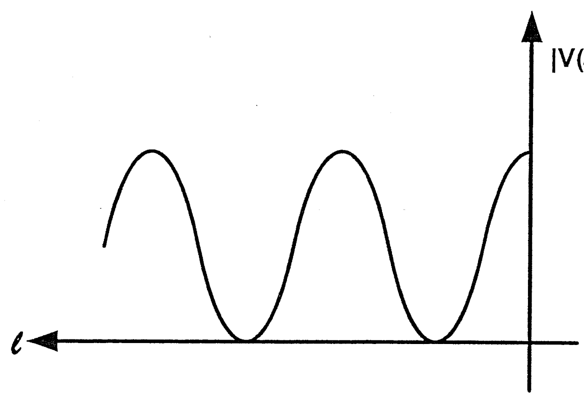
$Z_L=R;\ R>Z_0$	$Z_L=j\omega L$
$Z_L=R;\ R<Z_0$	$Z_L=1/j\omega C$
$Z_L=0$	$Z_L=R+1/j\omega C$
$Z_L=\infty$	$Z_L=R+j\omega L$

Reason:

$\Gamma_L = -1$ for a short

$V(\ell)\big|_{\ell=0} = V^+ (1 + \Gamma_L)$ $\quad V(\ell) = 0$ at the load end

(b)



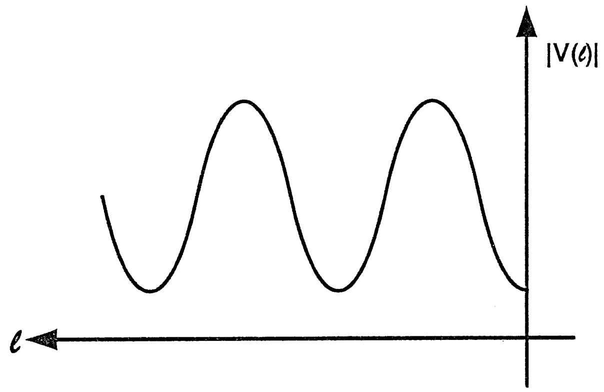
$Z_L=R;\ R>Z_0$	$Z_L=j\omega L$
$Z_L=R;\ R<Z_0$	$Z_L=1/j\omega C$
$Z_L=0$	$Z_L=R+1/j\omega C$
$Z_L=\infty$	$Z_L=R+j\omega L$

Reason:

$\Gamma_L = 1$ for open

$V(\ell)\big|_{\ell=0} = V^+ (1 + \Gamma_L)$ $\quad |V(\ell)|$ is maximum at the load end

(c)



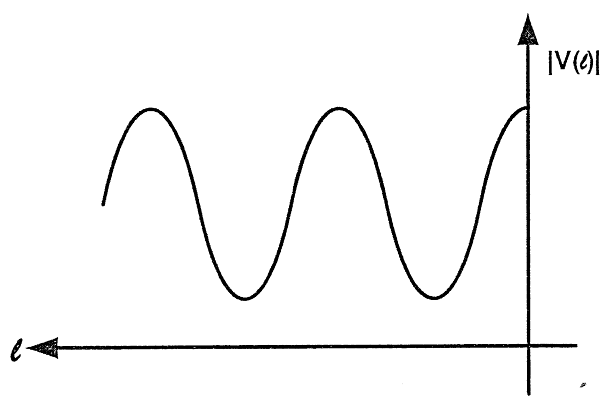
$Z_L=R;\ R>Z_0$	$Z_L=j\omega L$
$Z_L=R;\ R<Z_0$	$Z_L=1/j\omega C$
$Z_L=0$	$Z_L=R+1/j\omega C$
$Z_L=\infty$	$Z_L=R+j\omega L$

Reason:

$|V(\ell)|_{\ell=0} = V^+(1+\Gamma_L)$ $\quad \Gamma_L < 0$ for $R < Z_0$

$|V(\ell)|_{\ell=0} \rightarrow$ minimum of $|V(\ell)|$

(d)



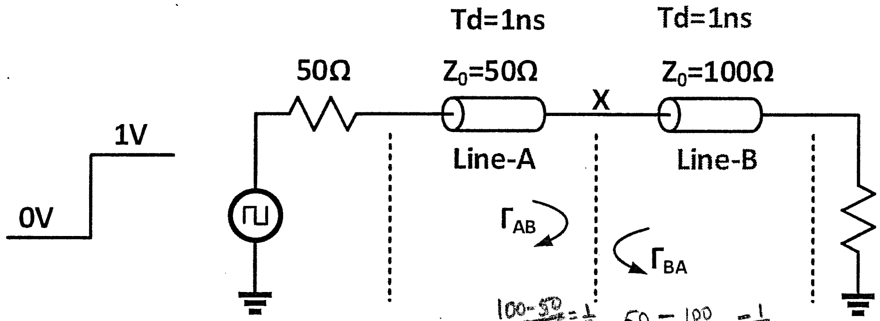
$Z_L=R;\ R>Z_0$	$Z_L=j\omega L$
$Z_L=R;\ R<Z_0$	$Z_L=1/j\omega C$
$Z_L=0$	$Z_L=R+1/j\omega C$
$Z_L=\infty$	$Z_L=R+j\omega L$

Reason:

$|V(\ell)|_{\ell=0} = V^+(1+\Gamma_L)$ $\quad \Gamma_L > 0$ for $R > Z_0$

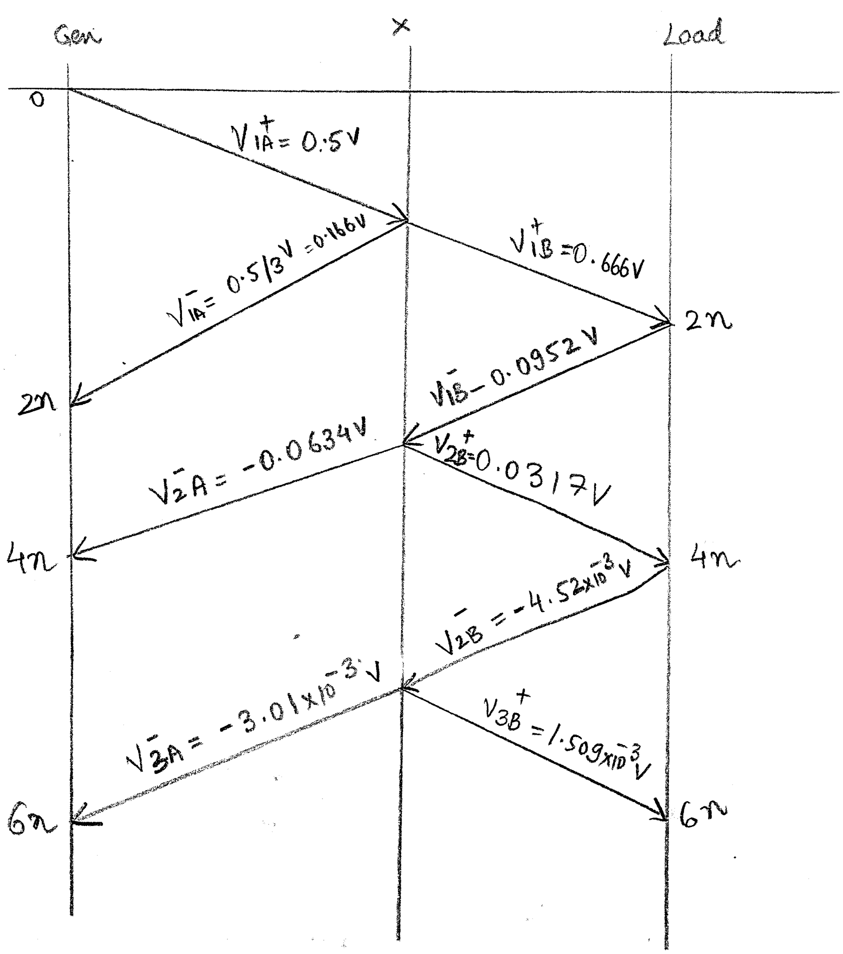
$|V(\ell)|_{\ell=0} \rightarrow$ maximum of $|V(\ell)|$

Problem 3: (15 points) Given the following transmission line:

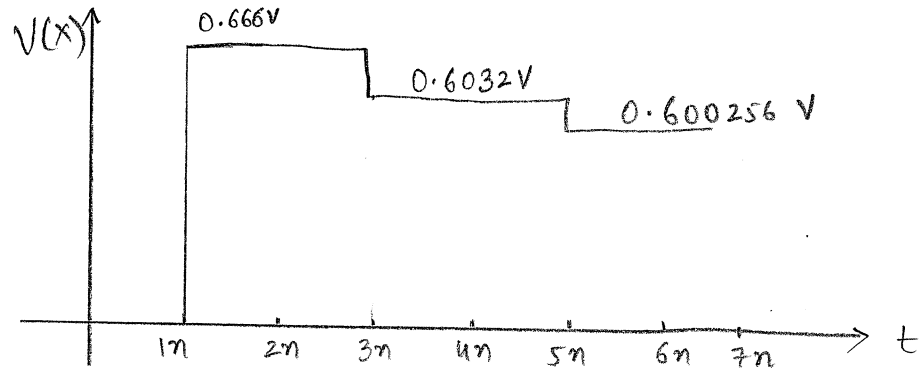


$\Gamma_G = 0$ $\quad \frac{100-50}{150} = \frac{1}{3}$ $\quad \frac{50-100}{150} = -\frac{1}{3}$ $\quad \Gamma_L = \frac{75-100}{75+100} = \frac{-25}{175} = -\frac{1}{7}$

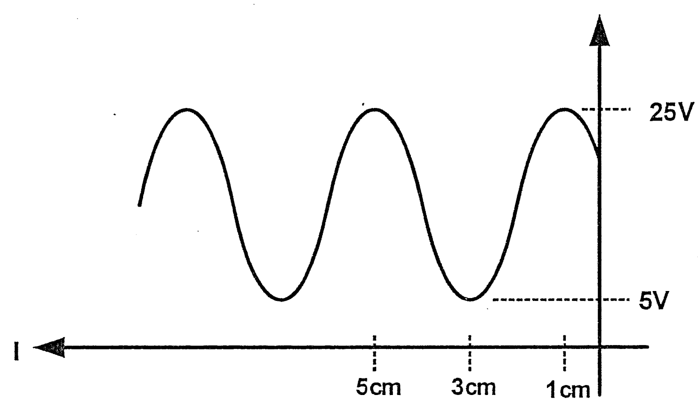
(a) **(10 points)** Draw lattice diagram for up to 6ns.



(b) **(5 points)** Draw waveform of voltage on node X versus time for up to 6ns.



Problem 4: (15 points) A transmission line with characteristic impedance of Z_0=50Ω is terminated with an un-known load impedance Z_L. The voltage standing wave pattern along the transmission line as a function of distance is shown below.



(a) **(10 points)** Calculate VSWR, wavelength on the line, magnitude of outgoing wave $|V^+|$, magnitude of maximum and minimum current.

VSWR = $25/5 = 5$

λ = $\lambda/2 = 4\text{ cm} \Rightarrow \lambda = 8\text{ cm}$

$|V^+|$ = $V^+ = \frac{V_{max} + V_{min}}{2} = 15$

$|I_{max}|$ = $25/50 = 0.5\text{ A}$

$|I_{min}|$ = $5/50 = 0.1\text{ A}$

(b) **(5 points)** Calculate the phase of the reflection coefficient at the load-end and circle the load Z_L at the end of the transmission line.

Phase = $\phi_L = \pi/2.$

Voltage Maxima @ 1 cm

$$\phi_L - 2\beta l = 0$$

$l = 1\text{cm}$

$$\phi_L - 2 \cdot \frac{2\pi}{\lambda} \cdot 1\text{cm} = 0$$

$$\phi_L = \frac{4\pi}{8} = \frac{\pi}{2}$$

$Z_L=R;\ R>Z_0$	$Z_L=j\omega L$
$Z_L=R;\ R<Z_0$	$Z_L=1/j\omega C$
$Z_L=0$	$Z_L=R+1/j\omega C$
$Z_L=\infty$	$Z_L=R+j\omega L$

Voltage Maxima comes first

or

$0 < \phi_L < \pi$