Name: Solutions

(Last name, first name)

Student ID: ____________________

ECE 391

TRANSMISSION LINES

Spring Term 2017

Final Exam

Exam is closed book, closed notes; **three** sheet (6 pages) of notes and formulas allowed; 1 hour 50 minutes. Show all work on the pages provided. No extra pages (use back if necessary). **Read each question very carefully**.

Box your final answer and include units where appropriate. Number of points for each problem is given in parenthesis (50 points total).

Problem 1 (12 pts.) ____________________

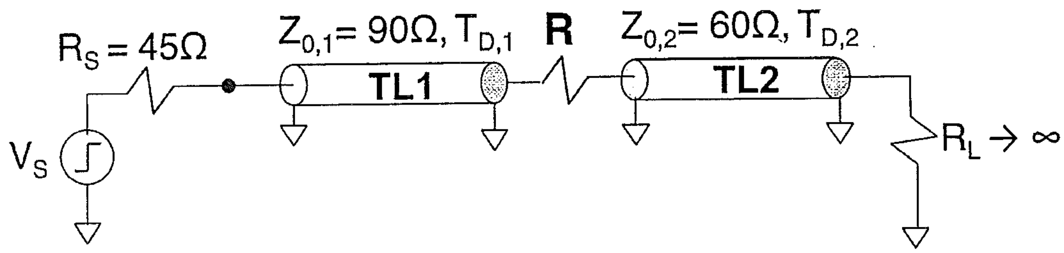
Problem 2 (11 pts.) ____________________

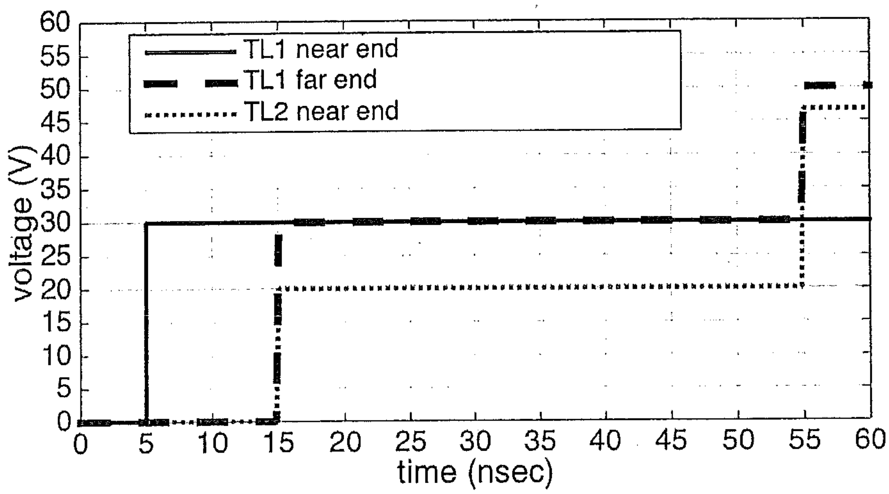
Problem 3 (11 pts.) ____________________

Problem 4 (16 pts.) ____________________

Total (50 pts.) ____________________

1. [12 pts.] Two PCB traces of different widths are connected via a resistor R, as shown in the circuit diagram below. The DC source voltage is turned on **at time t = 5ns** and the resulting voltage waveforms at the near end and far end of transmission line 1 (TL1) and at the near end of transmission line 2 (TL2) are recorded and plotted below.





(a) Determine the delay times TD1 and TD2 of the two PCB traces TL1 and TL2, respectively.

$$TD1 = (15-5)\,ns = \boxed{10ns}$$

$$TD2 = \frac{55-15}{2}\,ns = \boxed{20ns}$$

(b) Determine the reflection coefficient at the source side (near end of TL1) and at the termination of trace 2 (far end of TL2).

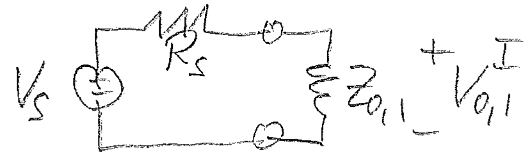
$$\rho_s = \frac{45-90}{45+90} = \frac{-45}{135} = \boxed{-\frac{1}{3}}$$

$$\rho_L = +1 \text{ (open circuit)}$$

(c) Determine the voltage of the first **outgoing** wave **on trace 1 (TL1).**

$V_{0,1}^{I} = \boxed{30V}$ (from plot)

(d) Determine the source voltage, V_S.



$$V_{0,1}^{I} = V_S \frac{Z_{0,1}}{Z_{0,1} + R_S}$$

$$\Rightarrow V_S = V_{0,1}^{I} \frac{Z_{0,1} + R_S}{Z_{0,1}} = 30V \frac{90+45}{90} = \boxed{45V}$$

(e) Determine the voltage of the first **outgoing** wave **on trace 2 (TL2).**

from plot: $V_{0,1}^{II} = \boxed{20V}$

(f) Determine the transmission coefficient, ρ_{21} at the junction between TL1 and TL2.

$$\rho_{21} = \frac{V_{0,1}^{II}}{V_{0,1}^{I}} = \frac{20V}{30V} = \boxed{\frac{2}{3}}$$

(g) Determine the reflection coefficient, ρ_{11}, at the far end of TL1. / near-end of TL1

from plot: no change in voltage at near-end of TL1 although $\rho_S \neq 0$

$\Rightarrow \boxed{\rho_{11} = 0}$

(h) Determine the value of resistor R connecting TL1 and TL2.

$$\rho_{11} = \frac{R + Z_{0,2} - Z_{0,1}}{R + Z_{0,2} + Z_{0,1}} = 0 \Rightarrow R = Z_{0,1} - Z_{0,2} = \boxed{30\Omega}$$

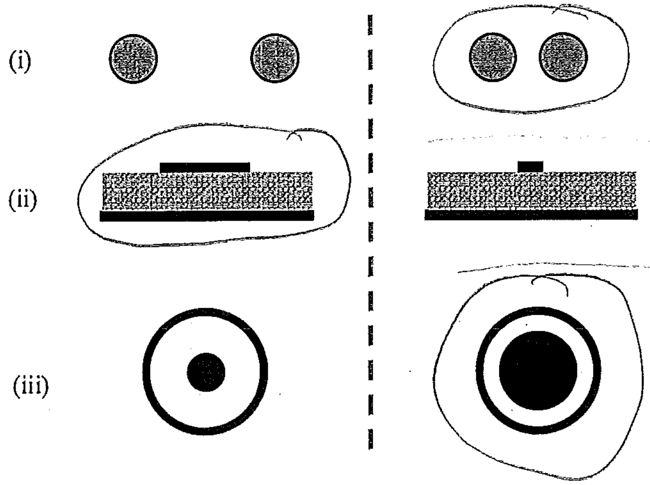
(i) Determine the steady-state voltage across series resistor, R, and across load resistor, $R_L \to \infty$.

no current in steady-state

$\Rightarrow V_R = \boxed{0}$

$V_{R_L} = V_S = \boxed{45V}$

2. (a) [3 pts.] For each pair of transmission lines (shown by the cross-sections), **explain** and clearly indicate which of the two has the **lower** characteristic impedance.

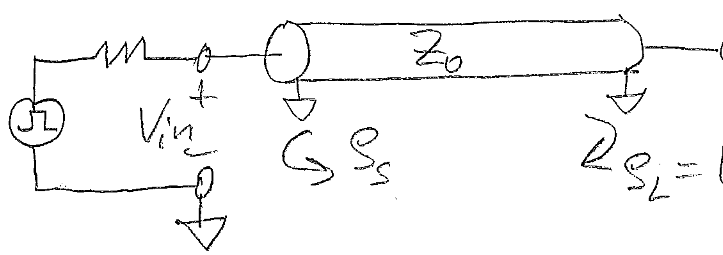


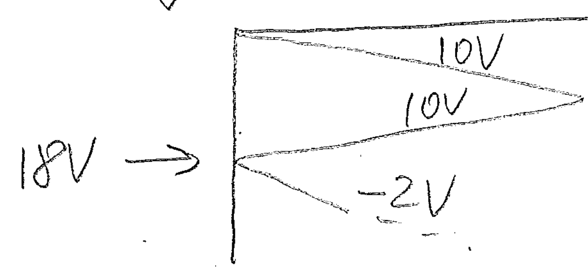
(i) smaller spacing, $\Rightarrow$ increased C, decreased L $\Rightarrow Z_0 = \sqrt{\frac{L}{C}}$ decreased

(ii) larger width: $\Rightarrow$ increased C, decreased L $\Rightarrow$ decreased Z_0

(iii) smaller spacing $\Rightarrow$ increased C, decreased L $\Rightarrow$ smaller Z_0

(b) [4 pts.] You would like to determine the characteristic impedance of a coaxial cable of length 50 meters. In your lab, you connect one end of the cable to a **50Ω** pulse generator and leave the other end open, and then monitor the voltage at the input of the cable on your oscilloscope. The pulse duration is set to 10 milliseconds. The initial voltage you measure is **10V**, and after **500 nsec** you measure **18V** at the input. (i) Determine the characteristic impedance of the coaxial cable assuming a lossless line. Hint: Draw a circuit diagram and a lattice diagram. (ii) If, instead, the cable is lossy and has characteristic impedance $Z_0 = 50\Omega$, what is the attenuation constant of the cable?



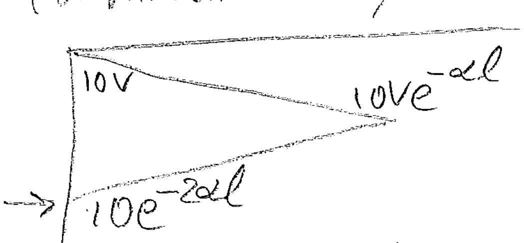


$\Rightarrow \rho_s = \frac{-2V}{10V} = -0.2$

$R_s = Z_0 \frac{1+\rho_s}{1-\rho_s}$ or $Z_0 = R_s \frac{1-\rho_s}{1+\rho_s}$

$Z_0 = \frac{1.2}{0.8} R_s = 1.5 R_s = \boxed{75\Omega}$

For matched lossy line:

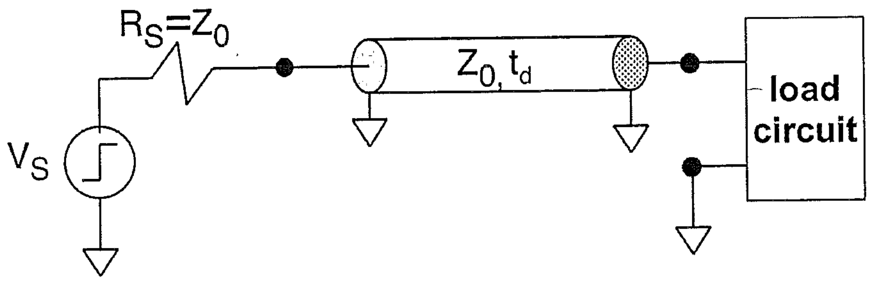


$\Rightarrow 8V = 10Ve^{-2\alpha\ell}$

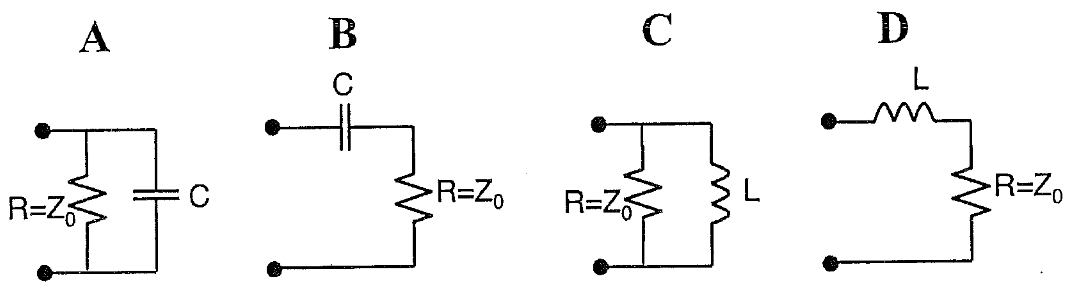
$\alpha = \frac{\ln 1.25}{2\ell} = \boxed{0.0022 \frac{Np}{m}}$

$\triangleq 0.0194 \frac{dB}{m}$

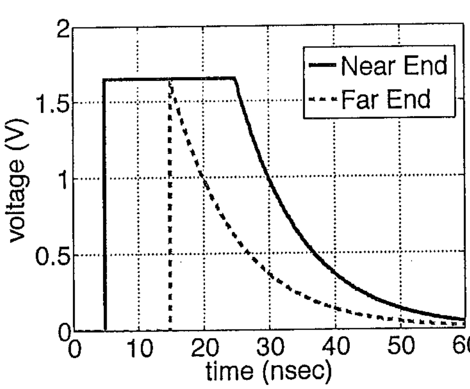
(c) [4 pts.] A series terminated digital driver is connected via a PCB trace to a set of two different loads. A schematic diagram of the transmission line circuit and the two different load circuits are shown below. The near-end and far-end responses for the two loads are also shown. Indicate in the box next to each response plot the corresponding load circuit (A-D).

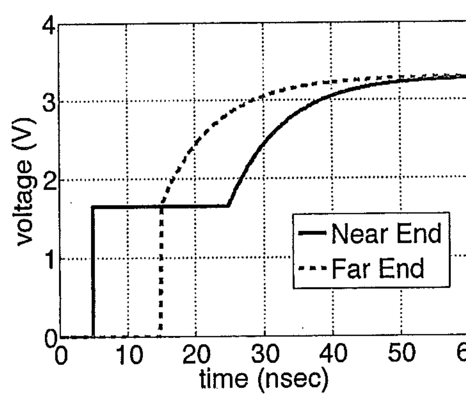


Load circuits:



Step responses:





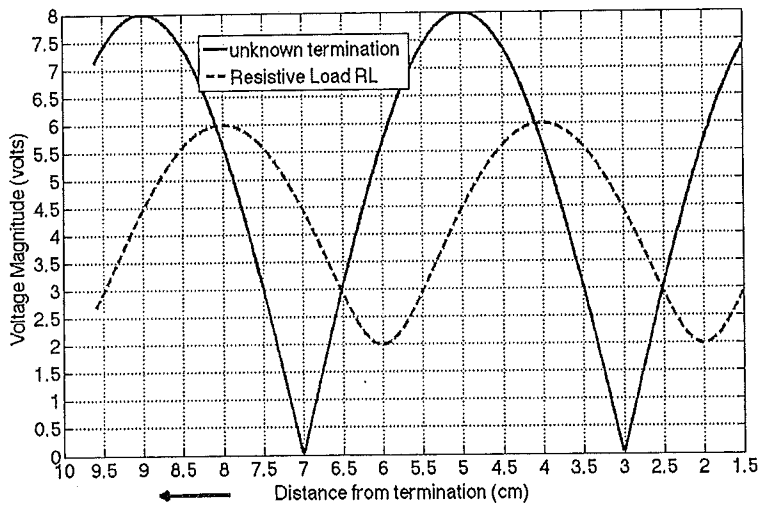
$\boxed{C}$

initially matched
⇒ C in series
or parallel L
finaly short: ⇒ parallel L

$\boxed{B}$

initially: matched:
⇒ series C or parallel L
finally: open circuit
⇒ series C

3. [11 pts.] A lossless transmission line with characteristic impedance $Z_0 = 50\Omega$ is terminated in a resistive load $\mathbf{R_L} > \mathbf{Z_0}$. A portion of the voltage standing-wave plot is shown below (dashed curve). The solid curve, also shown below, corresponds to an unknown termination and will be discussed later. An optional Smith chart is included after this problem.



(a) What is the standing-wave ratio on the line for the resistive termination (dashed curve)?

$$SWR = \frac{V_{max}}{V_{min}} = \frac{6V}{2V} = \boxed{3}$$

(b) Determine R_L (note: $R_L > Z_0$).

$$R_L = Z_{max} = Z_0 \cdot SWR = 3Z_0 = \boxed{150\Omega}$$

(c) Determine the voltage magnitude of the incident wave, $|V_0^+|$?

$$|V_0^+| = \frac{V_{max} + V_{min}}{2} = \frac{6V + 2V}{2} = \boxed{4V}$$

$$\text{or } \frac{8V - 0}{2} = 4V$$

(d) Determine the wavelength on the line.

$$\lambda = 2 \times (7-3)\,cm = \boxed{8cm}$$

Now, the resistive load is replaced with unknown load impedance Z_T and a new standing-wave pattern is obtained (solid curve above).

(e) Determine the standing-wave ratio on the line terminated in unknown load impedance $Z_T = R_T + jX_T$ (solid curve).

$V_{min} = 0 \Rightarrow VSWR = \frac{8V}{0} = \boxed{\infty}$

(f) Determine the unknown terminating impedance $Z_T = R_T + jX_T$. Show all your steps.

$R_T = 0$ since $V_{min} = 0 \Rightarrow |\Gamma_L| = 1$

$\frac{\theta_L}{2} \triangleq$ shift of V_{max} from open circuit case or $R_L > Z_0$ case

here: $\frac{\theta_L}{2} = \frac{(5-4)\,cm}{8\,cm} 360^\circ = 45^\circ$

$\Rightarrow \theta_L = 90^\circ \Rightarrow \Gamma_L = 1 \cdot e^{j90^\circ} = j$

$Z_T = Z_0 \frac{1+\Gamma_L}{1-\Gamma_L} = Z_0 \frac{1+j}{1-j} = Z_0 \frac{(1+j)^2}{1+1} = \frac{2j}{2} Z_0 = jZ_0$

$= \boxed{j50\Omega}$

or from $\frac{X_T}{Z_0} = \tan \frac{\theta_L}{2} = \tan 45^\circ = 1 \Rightarrow X_T = Z_0$

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from Smith chart:

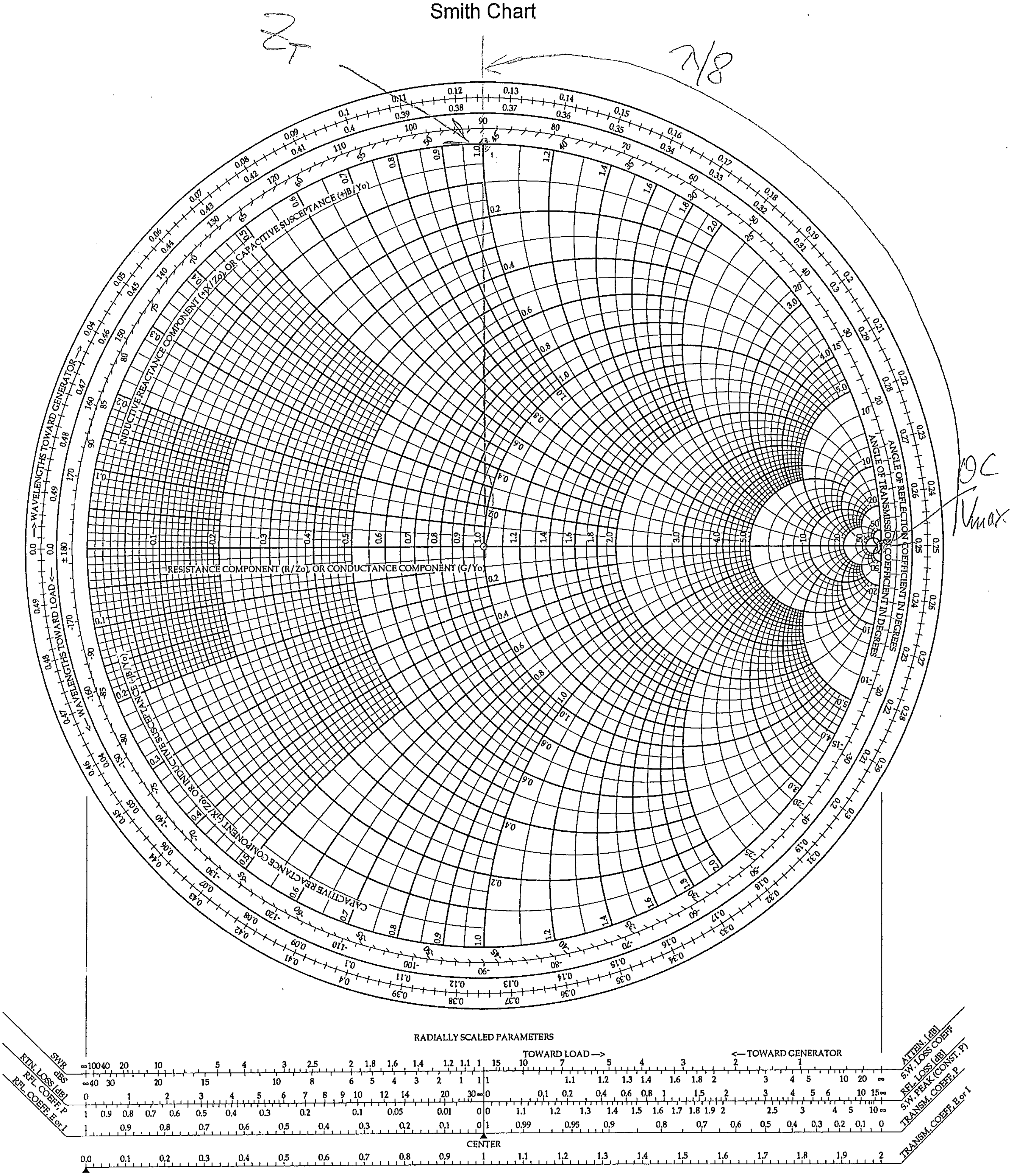
enter at V_{max} → rotate by $\frac{\lambda}{8} = 0.125\lambda$ in counter-clockwise direction

→ read off $Z_T/Z_0 = j \cdot 1$

$\Rightarrow Z_T = jZ_0 = j50\Omega$

Optional Smith Chart for Problem 3

Smith Chart



4. [16 pts.] A lossless transmission line ($Z_0 = 100\Omega$) is terminated in a complex load impedance Z_L. The normalized load impedance Z_L/Z_0 is shown on the attached Smith Chart. Use the Smith Chart to answer the following questions. **Clearly mark your answers on the attached Smith Chart.**

(a) Specify the **normalized** load impedance and unnormalized load impedance Z_L **in Ohms**.

$$\frac{Z_L}{Z_0} = \boxed{0.8 - j2.0} \Rightarrow Z_L = \boxed{(80 - j200)\,\Omega}$$

(b) Using the Smith Chart, determine the reflection coefficient at the termination (specify both magnitude and phase in degrees).

$$\boxed{\begin{array}{l} |\Gamma_L| \cong 0.75 \\ \theta_L \cong -48^\circ \end{array}}$$

(analytical solution:
$|\Gamma_L| = 0.7470$
$\theta_L = -47.698^\circ$)

(c) Using the Smith Chart, determine the voltage standing-wave ratio on the line.

$$\boxed{SWR \cong 6.9}$$

(analytical solution: $SWR = 6.9052$)

(d) Using the Smith Chart, determine the **normalized** load admittance. Specify the load admittance in **Siemens**.

$y_L \cong 0.175 + j0.43$

$$\Rightarrow Y_L = y_L \frac{1}{Z_0} = \boxed{(1.75 + j4.3)\,mS}$$

analytical solution:
$y_L = 0.1724 + j0.4310$

(e) Using the Smith Chart, determine the electrical distance $z'_{i,min}/\lambda$ from the termination to the nearest current minimum.

current minimum $\cong$ voltage maximum at phase ϕ, 2π

$$\frac{z_{i,min}}{\lambda} = (0.5 - 0.316) + 0.25 = \boxed{0.434}$$

(going in clock-wise direction starting at $\phi = 0$)

Now, a lossless open-circuited stub of length $l = \lambda/8$ and characteristic impedance $Z_0 = 100\Omega$ is connected in parallel (shunt) to the transmission line at distance $d = \lambda/4$ from the termination.

(f) Using the Smith Chart, determine the normalized input admittance of the open-circuited stub.

$\frac{l}{\lambda} = \frac{1}{8} = 0.125 \Rightarrow$ go from open circuit (in admittances) to $\frac{z'}{\lambda} = 0.125$

$\Rightarrow y_{stub} = +j\,1.0$

(g) Using the Smith Chart, determine the voltage-standing wave ratio on the source side of the transmission line **with** the stub connected.

$d = \frac{\lambda}{4} \Rightarrow$ rotate from Y_L by $180° \Rightarrow Y_{in,l} = 0.8 - j2.0$

add parallel stub:

$y_{in} = y_{in,l} + Y_{stub} = 0.8 - j2 + j1 = \boxed{0.8 - j}$

$\rightarrow$ draw circle and read off $\boxed{VSWR \doteq 3}$

(analytical solution: $SWR = 2.9624$)

(h) How does the voltage standing-wave ratio found in part (g) change if the operating frequency is doubled?

f doubled $\Rightarrow \lambda$ halved

$\Rightarrow \frac{d}{\lambda} = \frac{1}{2}$ and $\frac{l}{\lambda} = \frac{1}{4}$

Open-circuited stub of length $l = \frac{\lambda}{4} \Rightarrow Y_{stub} = \infty$
($Z_{stub} = 0$)

$\Rightarrow$ main line is short-circuited

$\Rightarrow \boxed{VSWR = \infty}$

Smith Chart for Problem 4