Lec-13

Wednesday, February 12, 2020 8:45 AM

Analysis of Transmission Line with Analog signal

$\hookrightarrow$ RF

- Assume Loss-Less Transmission Line : $R=0; G=0$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \underbrace{\alpha}_{0} + j\beta$$

Voltage : $V(l) = V^{+} e^{j\beta l} + V^{-} e^{-j\beta l}$

$$V(l) = V^{+} e^{j\beta l}\left[1 + \frac{V^{-}}{V^{+}} e^{-j2\beta l}\right]$$

Standing Voltage Equation

$$\boxed{V(l) = V^{+} e^{j\beta l}\left[1 + \Gamma_L e^{-j2\beta l}\right]}$$

$$\Gamma_L = \frac{V^{-}}{V^{+}} \rightarrow \frac{Z_L - Z_0}{Z_L + Z_0}$$

Complex quantity

$$\Gamma_L = |\Gamma_L| e^{j\phi_L}$$

Current : $I(l) = \frac{V^{+}}{Z_0} e^{j\beta l} - \frac{V^{-}}{Z_0} e^{-j\beta l}$

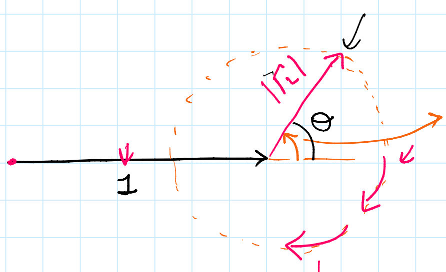
$$\boxed{I(l) = \frac{V^{+}}{Z_0} e^{j\beta l}\left[1 - \Gamma_L e^{-j2\beta l}\right]}$$

Standing Current Equation.

Phasor representation

$$V(l) = \left|V^{+} e^{j\beta l}\right|\left[1 + |\Gamma_L| e^{j(\phi_L - 2\beta l)}\right]$$

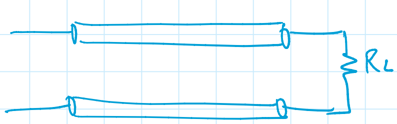
$$I(l) = \frac{V^{+}}{Z_0} e^{j\beta l}\left[1 - |\Gamma_L| e^{j(\phi_L - 2\beta l)}\right]$$



As $l \uparrow$

$\theta = (\phi_L - 2\beta l) \downarrow$

Moving towards generator



$\xleftarrow{\ell \text{ increases}} \quad \ell = 0$

$V(\ell)$ maximum $= V^+ e^{j\beta\ell}\left[1 + |\Gamma_L|\right]$; $I(\ell)$ minimum $= \frac{V^+}{Z_0} e^{j\beta\ell}\left[1 - |\Gamma_L|\right]$

$V(\ell)$ minimum $= V^+ e^{j\beta\ell}\left[1 - |\Gamma_L|\right]$; $I(\ell)$ maximum $= \frac{V^+}{Z_0} e^{j\beta\ell}\left[1 + |\Gamma_L|\right]$

Observation :

1. Point of maximum voltage corresponds to minimum current
2. Point of minimum voltage corresponds to maximum current.

VSWR : Voltage Standing Wave Ratio.

$$\boxed{\rho = \frac{|V|_{max}}{|V|_{min}}} = \frac{\cancel{V^+ e^{j\beta\ell}}\left[1 + |\Gamma_L|\right]}{\cancel{V^+ e^{j\beta\ell}}\left[1 - |\Gamma_L|\right]} = \boxed{\frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \rho}$$

The goal should be to have $\rho = 1$

$$\boxed{1 \leqslant \rho < \infty}$$

$$\boxed{-1 \leqslant \Gamma_L \leqslant 1}$$

Case 1: Short Circuit Transmission Line.

Standing Wave Pattern

$$\left.\begin{aligned} V(\ell) &= V^+ e^{j\beta\ell} + V^- e^{-j\beta\ell} \\ I(\ell) &= \frac{V^+}{Z_0} e^{j\beta\ell} - \frac{V^-}{Z_0} e^{-j\beta\ell} \end{aligned}\right\} \quad \begin{aligned} \Gamma_L &= -1 \\ V^- &= -V^+ \end{aligned}$$

$$V(\ell) = V^+ e^{j\beta\ell} - V^+ e^{-j\beta\ell} = V^+\left[e^{j\beta\ell} - e^{-j\beta\ell}\right]$$

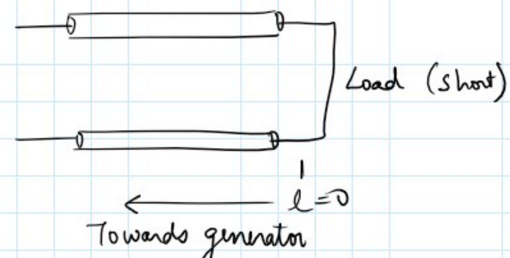
$$I(\ell) = \frac{V^+}{Z_0} e^{j\beta\ell} + \frac{V^+}{Z_0} e^{-j\beta\ell} = \frac{V^+}{Z_0}\left[e^{j\beta\ell} + e^{-j\beta\ell}\right]$$

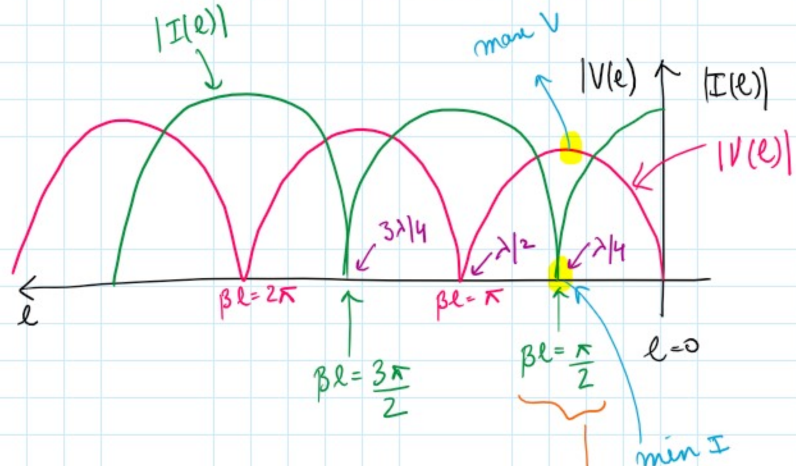
$$I(\ell) = \frac{V^+}{Z_0} e^{j\beta\ell} + \frac{V^+}{Z_0} e^{-j\beta\ell} = \frac{V^+}{Z_0}\left[e^{j\beta\ell} + e^{-j\beta\ell}\right]$$

$$V(\ell) = j\, V^+ \cdot 2 \sin(\beta\ell) \leftarrow$$

$$I(\ell) = \frac{V^+}{Z_0} \cdot 2 \cos(\beta\ell) \leftarrow$$

$$\boxed{Z(\ell) = \frac{V(\ell)}{I(\ell)} = jZ_0 \tan(\beta\ell)}$$





$$\frac{2\cancel{\pi}}{\lambda} \cdot \ell = \frac{\cancel{\pi}}{2}$$

$$\boxed{\ell = \frac{\lambda}{4}}$$