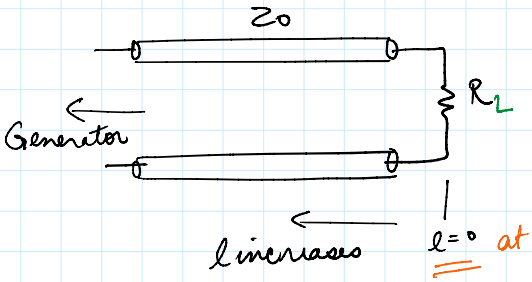
Lec-15

Friday, February 21, 2020 8:40 AM

HW-4 Posted Online.

VSWR with a resistive load.



Loss Less Line → Z_0 is real

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

$R = 0$, $G = 0$

$$V(\ell) = V^+ e^{j\beta\ell}\left(1 + |\Gamma_L| e^{j(\phi_L - 2\beta\ell)}\right)$$

$$I(\ell) = \frac{V^+}{Z_0} e^{j\beta\ell}\left(1 - |\Gamma_L| e^{j(\phi_L - 2\beta\ell)}\right)$$

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$

Real quantity with angle 0

$$\Gamma_L = |\Gamma_L| e^{j\phi_L}, \quad \phi_L = 0$$

$$V(\ell) = V^+ e^{j\beta\ell}\left(1 + \Gamma_L e^{-j2\beta\ell}\right)$$

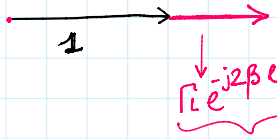
$$I(\ell) = \frac{V^+}{Z_0} e^{j\beta\ell}\left(1 - \Gamma_L e^{-j2\beta\ell}\right)$$

@ $\ell = 0$ (at Load) =

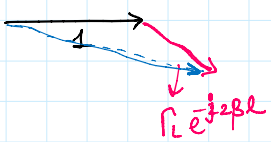
$$V(0) = V^+ \cdot 1\left(1 + \Gamma_L\right)$$

$$I(0) = \frac{V^+}{Z_0}\left(1 - \Gamma_L\right)$$

Phasor for Voltage at $\ell = 0$



$\ell = 1$ cm

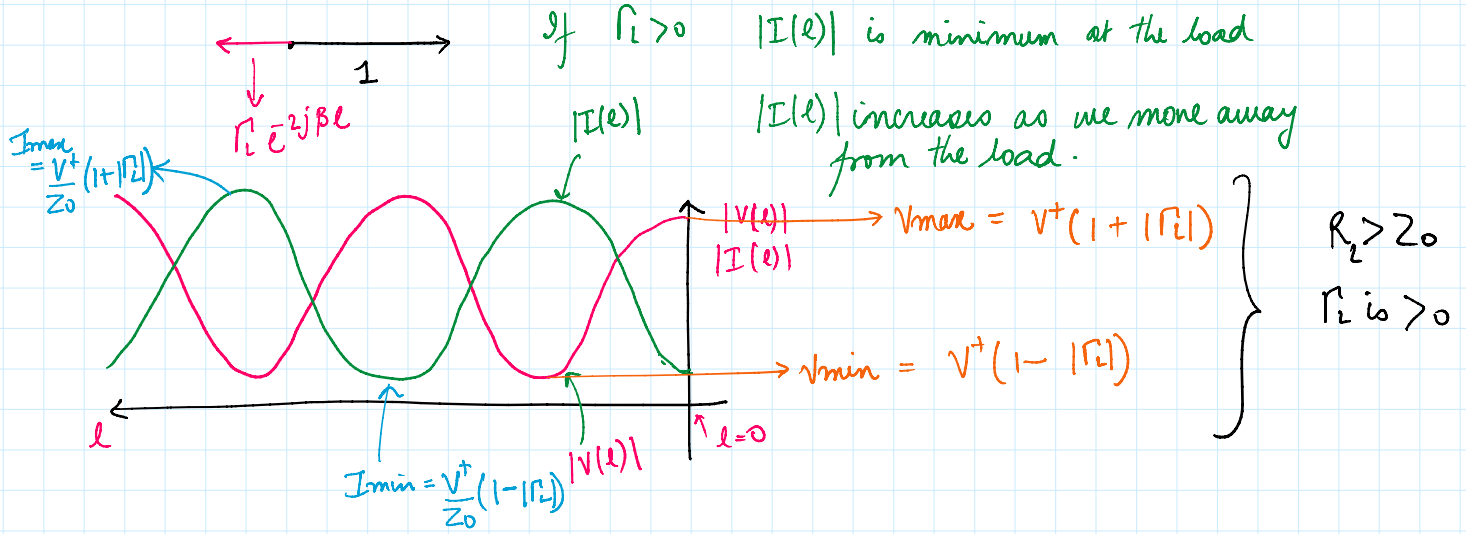


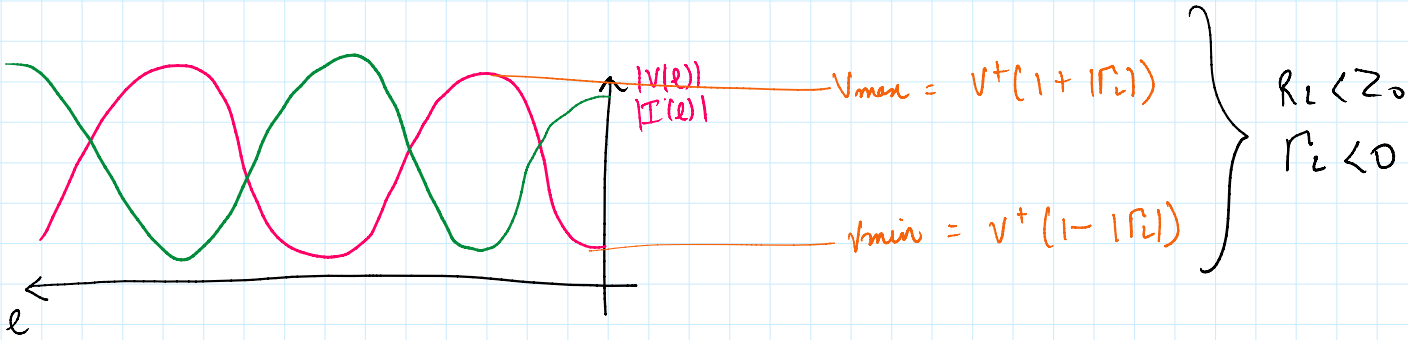
Observation: If $\Gamma_L > 0$, Γ_L is positive, $R_L > Z_0$

$|V(\ell)|$ is maximum at the load

$|V(\ell)|$ reduces as we move away from the load.

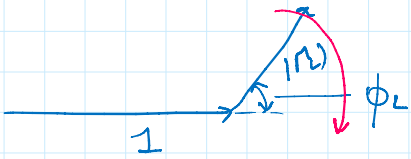
Phasor for Current



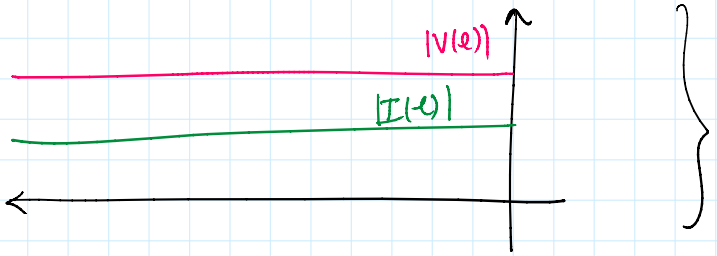


$$V(\ell) = V^+ e^{j\beta\ell}\left(1 + |\Gamma_L| e^{j(\phi_L - 2\beta\ell)}\right)$$

@ $\ell = 0$



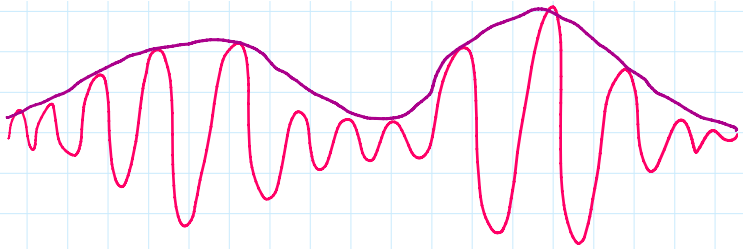
@ $\ell = 0$



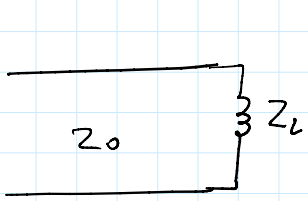
$R_L = Z_0$
$\Gamma_L = 0$

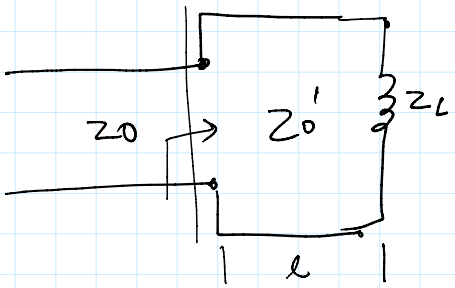
$R_L = Z_0$

$V(\ell)$ =

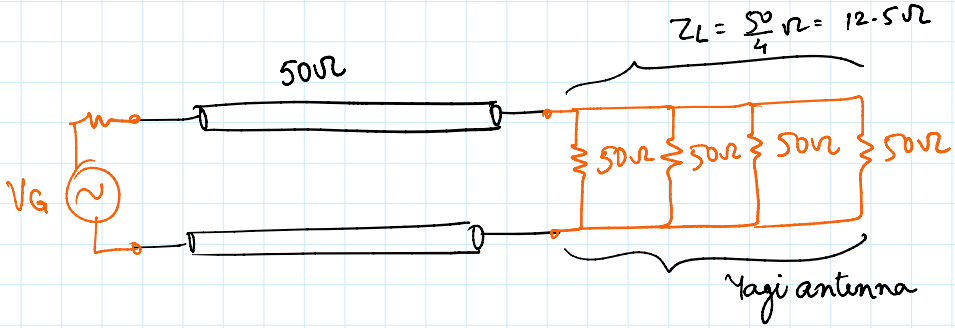


$R_L \neq Z_0$





Example: 1



a) $\Gamma_L = \frac{12.5 - 50}{12.5 + 50} = -0.6 = 0.6\, e^{j\pi}$

b) Calculate V_{max}, V_{min}, I_{max}, I_{min}

$V^+ = 1V$

$V_{max} = 1.6V \rightarrow V^+(1 + |\Gamma_L|)$

$1 + 0.6$

$V_{min} = 0.4 \rightarrow V^+(1 - |\Gamma_L|)$

$I_{max} = \frac{V^+}{Z_0}(1 + |\Gamma_L|) = \frac{1.6}{50} = 32mA$

$I_{min} = \frac{V^+}{Z_0}(1 - |\Gamma_L|) = \frac{0.4}{50} = 8mA$

c) Sketch $|V(\ell)|$ & $|I(\ell)|$ as a function of ℓ/λ starting at $\ell = 0$

$$V(\ell) = V^+ e^{j\beta\ell}\left(1 + 0.6\, e^{j\pi - 2j\beta\ell}\right) \qquad \beta = \frac{2\pi}{\lambda}$$

$$V(\ell) = \underbrace{V^+}_{1}\, e^{\frac{j2\pi}{\lambda}\ell}\left(1 + 0.6\, e^{j\pi(1-4\ell/\lambda)}\right)$$

↓ take absolute magnitude

$$\rightarrow |V(\ell)| = \left|\left(1 + 0.6\cos[\pi(1-4\ell/\lambda)] + j0.6\sin[\pi(1-4\ell/\lambda)]\right)\right|$$

$\frac{\ell}{\lambda} = 0$	$\lvert V(\ell)\rvert = 0.4V$
$\frac{\ell}{\lambda} = \frac{1}{4}$	$= 1.6V$
$\frac{\ell}{\lambda} = \frac{1}{2}$	$= 0.4V$
$\frac{\ell}{\lambda} = \frac{3}{4}$	$= 1.6V$
$\frac{\ell}{\lambda} = 1$	$= 0.4V$