Lec-18

Friday, February 28, 2020   8:40 AM

HW-4 Due Today
Midterm-2   Next Week

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

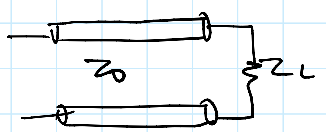
$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

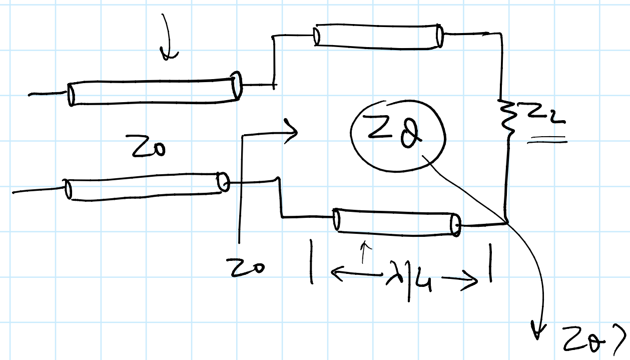
$$\gamma \cdot Z_0 = R+j\omega L$$

$$\frac{\gamma}{Z_0} = G+j\omega C$$

Impedance Transformation → Quater wavelength matching.

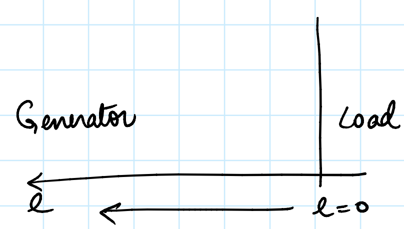
$Z_0 \neq Z_L$





$$V(l) = V^+ e^{\gamma l} + V^- e^{-\gamma l}$$

$$I(l) = \frac{V^+}{Z_0} e^{\gamma l} - \frac{V^-}{Z_0} e^{-\gamma l}$$



$$\frac{V(l)}{I(l)} = Z(l) = Z_0 \left\{ \frac{1+\Gamma_L e^{-2\gamma l}}{1-\Gamma_L e^{-2\gamma l}} \right\}$$

$$\Gamma(l) = \Gamma_L e^{-2\gamma l}$$

$$\Gamma_L \rightarrow \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z(l) = Z_0 \left\{ \frac{1 + \frac{Z_L - Z_0}{Z_L + Z_0} e^{-2\gamma l}}{1 - \frac{Z_L - Z_0}{Z_L + Z_0} e^{-2\gamma l}} \right\}$$

$$Z(l) = Z_0 \left\{ \frac{Z_L \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_L \sinh \gamma l} \right\}$$

$$\cosh \gamma l = \frac{e^{\gamma l} + e^{-\gamma l}}{2}$$

$$\sinh \gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{2}$$

Normalized Impedance

$\frac{Z(l)}{Z_0} = \bar{Z}(l) \rightarrow$ normalize $Z(l)$

$\frac{Z_L}{Z_0} = \bar{Z}_L \rightarrow$ normalized load

For a loss less line $\alpha = 0$

$$\gamma = \alpha + j\beta$$
$$\downarrow 0$$
$$\gamma = j\beta$$

$$Z(l) = Z_0 \left\{ \frac{Z_L \cosh(j\beta l) + Z_0 \sinh(j\beta l)}{Z_0 \cosh(j\beta l) + Z_L \sinh(j\beta l)} \right\}$$

$$\boxed{\bar{Z}(l) = \frac{\bar{Z}_L \cos \beta l + j \sin \beta l}{\cos \beta l + j \bar{Z}_L \sin \beta l}}$$

1. Normalized impedance $\bar{Z}(l)$ repeats itself every $\lambda/2$

$$\bar{Z}(l) = \bar{Z}(l + \lambda/2)$$

$$\beta(l + \lambda/2) = \frac{2\pi}{\lambda} . l + \cancel{\frac{2\pi}{\lambda}} \cancel{\frac{\lambda}{2}}$$

$$\beta(l + \lambda/2) = \beta l + \pi$$

$$\beta(l+\lambda/2) = \frac{2\pi}{\lambda}\cdot l + \frac{2\pi}{\cancel{\lambda}}\cdot\frac{\cancel{\lambda}}{\cancel{2}}$$

$$\beta(l+\lambda/2) = \beta l + \pi$$

$$\bar{Z}(l+\lambda/2) = \frac{\bar{Z}_L \cos(\beta l+\pi) + j\sin(\beta l+\pi)}{\cos(\beta l+\pi) + j\bar{Z}_L \sin(\beta l+\pi)}$$

$$= \frac{-\bar{Z}_L \cos(\beta l) - j\sin(\beta l)}{-\cos(\beta l) - j\bar{Z}_L \sin(\beta l)}$$

$$= \frac{\bar{Z}_L \cos\beta l + j\sin\beta l}{\cos\beta l + j\bar{Z}_L \sin\beta l} = \bar{Z}(l)$$

2. Normalized impedance INVERTS at every $l = \lambda/4$

$\bar{Z}(l)$

$$\bar{Z}(l+\lambda/4) = \frac{1}{\bar{Z}(l)}$$

βl

$$\beta(l+\lambda/4) \rightarrow \beta l + \frac{2\pi}{\cancel{\lambda}}\cdot\frac{\cancel{\lambda}}{\cancel{4}2}$$

$$= \beta l + \pi/2$$

$$\bar{Z}(l+\tfrac{\lambda}{4}) = \frac{\bar{Z}_L \cos(\beta l+\pi/2) + j\sin(\beta l+\pi/2)}{\cos(\beta l+\pi/2) + j\bar{Z}_L \sin(\beta l+\pi/2)}$$

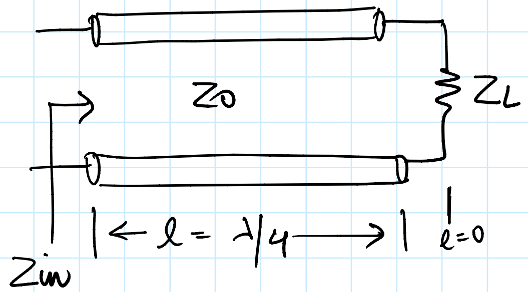
$$\bar{Z}(l+\lambda/4) \quad \frac{-\bar{Z}_L \sin\beta l + j\cos\beta l}{-\sin\beta l + j\bar{Z}_L \cos\beta l}$$

multiply & divide by j

$$= \frac{-\cos\beta l - j\bar{Z}_L \sin\beta l}{\bar{Z}_L \cos\beta l - j\sin\beta l}$$

$$= \frac{-\cos\beta l - j\bar{Z}_L \sin\beta l}{-\bar{Z}_L \cos\beta l - j \sin\beta l}$$

$$\boxed{\bar{Z}(l+\lambda/4) = \frac{\cos\beta l + j\bar{Z}_L \sin\beta l}{\bar{Z}_L \cos\beta l + j \sin\beta l} = \frac{1}{\bar{Z}(l)}}$$



$$Z(l)\Big|_{\text{at } l=0} = Z_L$$

$$Z_{in} = Z(l=\lambda/4)$$

$$\bar{Z}(l=\lambda/4) = \frac{1}{\bar{Z}_L}$$

$$\frac{Z_{in}}{Z_0} = \frac{Z_0}{Z_L}$$

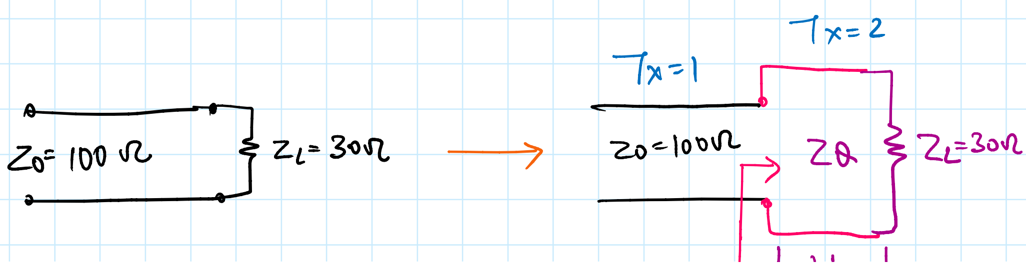
$$Z_0^2 = Z_{in} \cdot Z_L$$

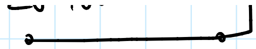
$$\boxed{Z_0 = \sqrt{Z_{in} \cdot Z_L}}$$

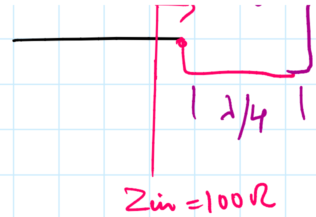
works out for a resistive load in hand calculations.

Assume Z_L is resistive

Example:







No VSW on Tx 1

$Z_Q = \sqrt{100 \times 30}$

$Z_Q = \sqrt{3000} \approx 54.8\Omega$

↓

real

(lossless) $\Rightarrow$ Terms inside sqr root must be real $\Rightarrow$ Z_L must be real