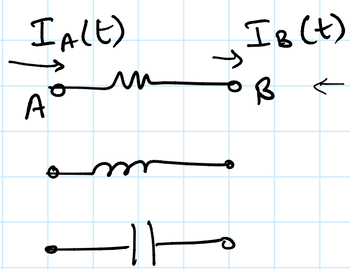
Lec-2

Wednesday, January 8, 2020 8:43 AM

No class on Friday 01/10/2020

Lumped Circuit

322



$I_A(t) = I_B(t)$

for all time t

Distributed Circuit.

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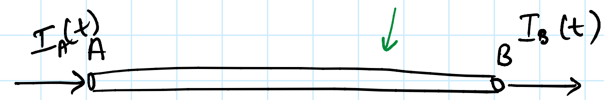
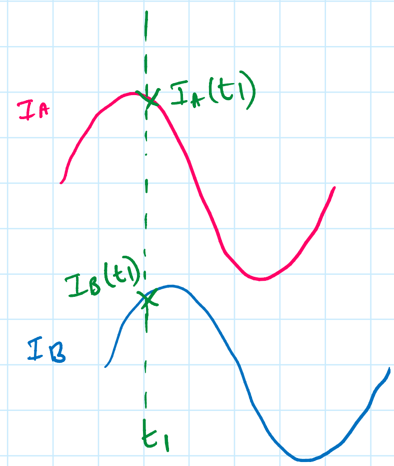
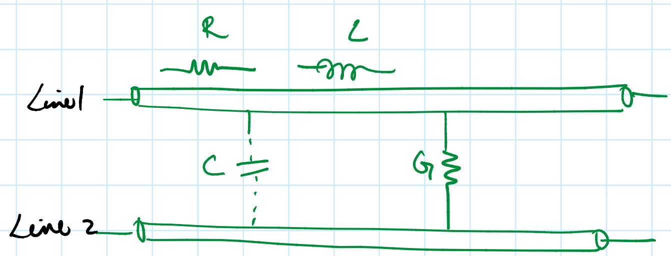


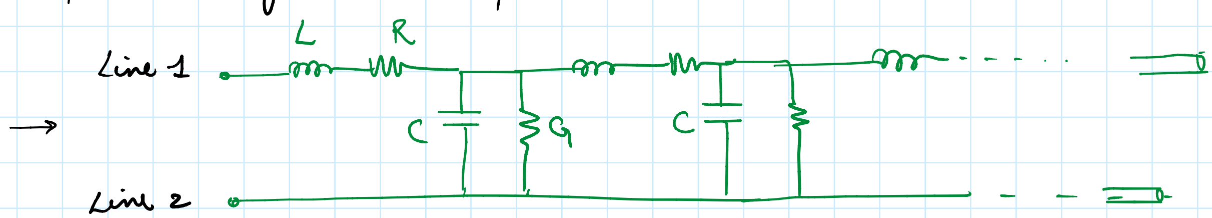
$|I_A(t)| \neq |I_B(t)|$

for all time t

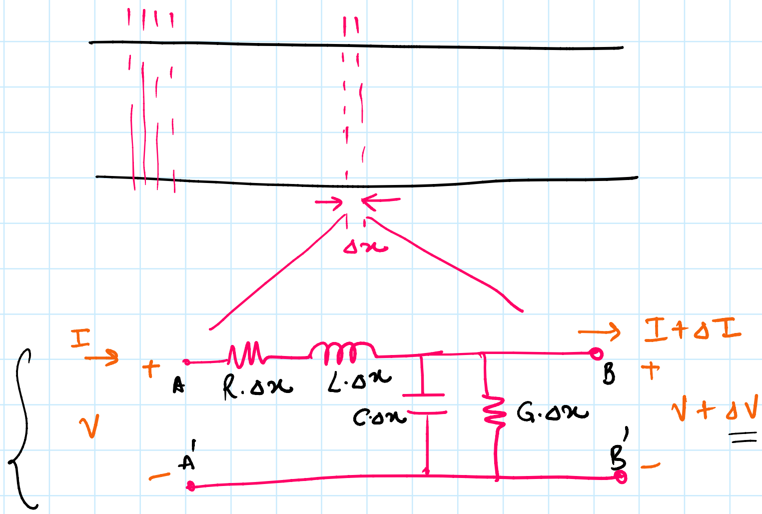




Representation of distributed parameters in a transmission line.



- L : Inductance / unit length H/m
- C : Capacitance / unit length F/m
- R : Resistance / unit length Ω/m
- G : Conductance / unit length ℧/m



Let us assume frequency of signal = f Hz $\qquad \omega$ rad/sec.

Voltage drop b/w AA' & BB'

$$\Delta V = -(R\Delta x + j\omega L\Delta x)\,I$$

$$\Delta I = -(G\Delta x + j\omega C\Delta x)\,V$$

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = -(R + j\omega L)\,I$$

$$\lim_{\Delta x \to 0} \frac{\Delta I}{\Delta x} = -(G + j\omega C)\,V$$

$$\left.\begin{aligned} \frac{dV}{dx} &= -(R + j\omega L)\,I \\ \frac{dI}{dx} &= -(G + j\omega C)\,V \end{aligned}\right\} \text{Coupled equations}$$

Take a second derivative to uncouple the equations.

$$\frac{d^2V}{dx^2} = -(R + j\omega L)\frac{dI}{dx} \qquad \text{— (1)}$$

$$\frac{d^2I}{dx^2} = -(G + j\omega C)\frac{dV}{dx}. \qquad \text{— (2)}$$

$$\frac{d^2I}{dx^2} = -(G+j\omega C)\frac{dV}{dx} \qquad -(2)$$

Subtitute $\frac{dI}{dx}$ in (1) & $\frac{dV}{dx}$ in (2)

$$\frac{d^2V}{dx^2} = +(R+j\omega L)(G+j\omega C).V$$

$$\frac{d^2I}{dx^2} = +\underbrace{(R+j\omega L)(G+j\omega C)}_{\gamma^2} I$$

↳ Characteristic of a transmission line

↓

Constant for a given transmission line

$$\gamma^2 = (R+j\omega L)(G+j\omega C) = \text{Propagation Constant}$$

$$\boxed{\begin{aligned} \frac{d^2V}{dx^2} &= \gamma^2 V \\ \frac{d^2I}{dx^2} &= \gamma^2 I \end{aligned}}$$

Solution of the differential equations

$$V(x) = V^+ e^{-\gamma x} + V^- e^{\gamma x}$$

$V^- e^{\gamma x}$ ↳ Backward travelling signal.

$V^+ e^{-\gamma x}$ ↓ Forward travelling signal

↳ This voltage solution is wrt distance $- x$

Multiply the above equation by $e^{j\omega t}$ to get the instantaneous value of voltage.

$$V(x,t) = V^+ e^{-\gamma x} e^{j\omega t} + V^- e^{\gamma x} e^{j\omega t}$$

$\underbrace{V(x,t)}$ → Voltage in space & time.

Propagation constant (Complex quantity) $\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$

- Only consider forward travelling wave

$$V(x,t) = V^+ e^{-\gamma x} e^{j\omega t}$$

Amp — Phase

$$\mathrm{Re}\{V(x,t)\} = \mathrm{Re}\{\boxed{V^+ e^{-\alpha x}}\ \boxed{e^{-j\beta x} e^{j\omega t}}\}$$

$$V(x,t) = \underbrace{V^+ e^{-\alpha x}} \cos(\underbrace{\omega t - \beta x}_{\phi})$$