

Mapping b/w Impedance Plane & Complex Γ plane.

$$\bar{Z} = r + ja = \frac{1 + \Gamma}{1 - \Gamma}$$

$$r + ja = \frac{1 + u + jv}{1 - u - jv}$$

$$= \frac{(1 + u + jv)(1 - u + jv)}{(1 - u)^2 + v^2}$$

$$r + ja = \frac{(1 + u)(1 - u) + jv(1 + u) + jv(1 - u) - v^2}{(1 - u)^2 + v^2}$$

Separating real & imaginary terms

$$r = \frac{(1 + u)(1 - u) - v^2}{(1 - u)^2 + v^2} \quad - (1)$$

$$x = \frac{jv(1 + u) + jv(1 - u)}{(1 - u)^2 + v^2} \quad - (2)$$

$$= \frac{jv + jvu + jv - jv}{(1 - u)^2 + v^2}$$

After rearrangement of terms.

$$u^2 - 2\left(\frac{r}{1+r}\right)u + v^2 + \frac{r-1}{r+1} = 0$$

Constant Resistance Circle

After rearrangement

$$u^2 + v^2 - 2u - \left(\frac{2}{x}\right)v + 1 = 0$$

Constant Reactance circle

Constant Resistance circle. ^(r)

Remember:
(normalized resistance & reactance)

• Locus of all reactance points for a fixed resistance

Circle Centre = $\left(\frac{r}{r+1}, 0\right)$ Radius = $\frac{1}{r+1}$

- 1. $r = 0$ Centre = $(0, 0)$ Radius = $\frac{1}{1} = 1$
- 2. $r = 1$ Centre = $(0.5, 0)$ Radius = $\frac{1}{2} = 0.5$

Observation:

- 1) As r increases, centre shifts to the right
- 2) " " " , radius reduces.
- 3) At $r = \infty$, radius = 0
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 open in the resistance

Constant Reactance Circles ^(x)

Centre : $\left(1, \frac{1}{x}\right)$
 Radius : $\frac{1}{x}$

• Locus of all resistances for a constant reactance.

- a) $x = 0$ Centre = $1, \infty$
 Radius = ∞ } Denotes a straight line

b) x is positive
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Centre $1, \frac{1}{x}$
radius $= \frac{1}{x}$

$x = 1$ Centre $= (1, 1)$
Radius $= (1)$

$x = \infty$, Centre $= 1, 0$
Radius $= 0$

c) x is negative

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$x = -1$ Centre $= (1, -1)$
Radius $= 1$

Smith chart \rightarrow Super position of constant x circle & constant r circle.