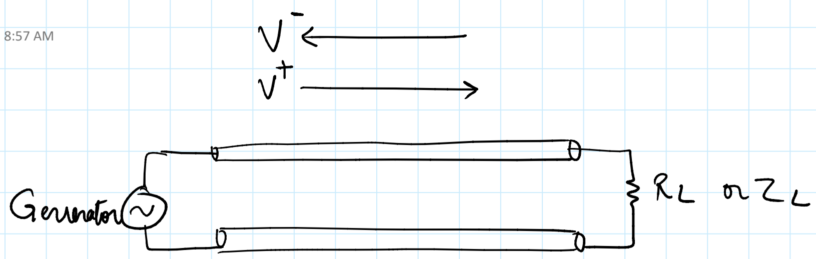


V^+ - Amplitude of forward travelling wave : Generator to Load

V^- - Amplitude of reverse travelling wave : Load to generator.

Example 2 Continued :

$$v(t) = Re\left\{ V^+ e^{-\alpha x} e^{(-j\beta x + \omega t)} \right\}$$

$$= \rightarrow V^+ e^{-\alpha x} \cos(\beta x - \omega t + \phi_0)$$

$x = 0$; $t = 0$
$v(t) = 8.66V$
$\phi_0 = 30^\circ$

Substitute $x = 0$ & $t = 0$

$$8.66 = V^+ . 1 \cos(\phi_0)$$

$$V^+ = \frac{8.66}{\cos 30^\circ} = \frac{8.66}{\sqrt{3}} 2$$

$$\boxed{V^+ = 10V}$$

Step 2: Calculate the voltage at $t = 100nS$, $x = 1m$ $f = 1GHz$

$$v(t) = V^+ e^{-\alpha x} \cos(\omega t - \beta x + \phi_0)$$

$\alpha = 2.23$ nepers/m

$\beta = 28.2$ rad/m

$$v(t) = -0.88V \}$$

- More analysis on the differential equations of Transmission Line

$$\frac{dV}{dx} = -(R+j\omega L)I$$

$$V = V^+e^{-\gamma x} + V^-e^{\gamma x}$$
$$I = I^+e^{-\gamma x} + I^-e^{\gamma x}$$

$$\frac{d}{dx}\left(V^+e^{-\gamma x} + V^-e^{\gamma x}\right) = -(R+j\omega L)\left[I^+e^{-\gamma x} + I^-e^{\gamma x}\right]$$

$$\underline{V^+(-\gamma)}e^{-\gamma x} + V^-(\gamma)e^{\gamma x} = -(R+j\omega L)\left[\underline{I^+e^{-\gamma x}} + I^-e^{\gamma x}\right]$$

Separate out the forward travelling & reverse travelling components.

$$V^+(\cancel{-}\gamma)\cancel{e^{-\gamma x}} = \cancel{-}(R+j\omega L)\, I^+\cancel{e^{-\gamma x}}$$

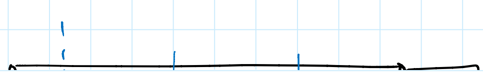
$$\frac{V^+}{I^+} = \frac{R+j\omega L}{\gamma} \qquad\qquad \frac{V^-}{I^-} = -\frac{(R+j\omega L)}{\gamma}$$

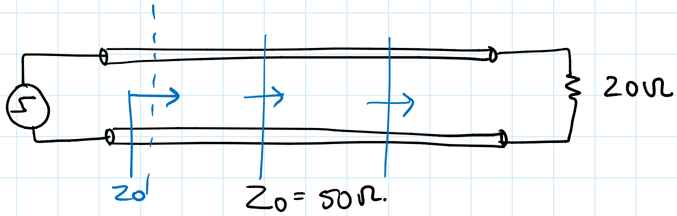
$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\boxed{\frac{V^+}{I^+} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \qquad\qquad \frac{V^-}{I^-} = -\sqrt{\frac{R+j\omega L}{G+j\omega C}}}$$

Zo: Characteristic Impedance

Impedance seen by the forward travelling or reverse travelling wave





Boundary Conditions on a Transmission Lines

$$\left.\begin{aligned} V &= V^{+} e^{+\gamma l} + V^{-} e^{-\gamma l} \quad -(1) \\ I &= I^{+} e^{\gamma l} + I^{-} e^{-\gamma l} \quad -(2) \end{aligned}\right\}$$

l is made negative because we are assuming l increases from right to left.

Boundary Condition : At $l = 0$ $\quad Z = Z_L$ (load)

Take ratio of (1) & (2) & apply the boundary condition.

$$Z_L = \left.\frac{V}{I}\right|_{l=0} = \frac{V^{+} + V^{-}}{I^{+} + I^{-}}$$

$$\left\{ \frac{V^{+}}{I^{+}} = Z_0 \; ; \; \frac{V^{-}}{I^{-}} = -Z_0 \right\}$$

Characteristic Impedance

$$I^{+} = \frac{V^{+}}{Z_0} ; I^{-} = \frac{V^{-}}{-Z_0}$$

$$Z_L = \left(\frac{V^{+} + V^{-}}{V^{+} - V^{-}}\right) Z_0$$

Reflection Coefficient :

$$\boxed{\Gamma(l) = \frac{V^{-} e^{-\gamma l}}{V^{+} e^{\gamma l}}}$$

$$\underset{l=0}{\Gamma(0)} \text{ at load} = \frac{V^{-}}{V^{+}}$$

Re write

Re write

$$\frac{Z_L}{Z_0} = \frac{1 + \frac{V^-}{V^+}}{1 - \frac{V^-}{V^+}} = \left(\frac{1+\Gamma(0)}{1-\Gamma(0)}\right)$$

$$\frac{Z_L}{Z_0} = \frac{1+\Gamma(0)}{1-\Gamma(0)}$$

$$\frac{Z_L - Z_0}{Z_L + Z_0} = \Gamma(0)$$

$$\left\{ \begin{array}{c} \frac{a}{b} = \frac{c}{d} \\ \frac{a-b}{a+b} = \frac{c-d}{c+d} \end{array} \right\}$$

$$\boxed{\Gamma(0) = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$$\Gamma(0) = \frac{V^-}{V^+}$$

Ideally we don't want anything to reflect from the load.

→ V^- should ideally be 0

↓ The requires

$$Z_L = Z_0$$

- Terminate the transmission Line with the characteristic impedance.