Name: Solution Key
(**Last name**, first name)

Student ID: ____________________

ECE 391

TRANSMISSION LINES

Spring Term 2017

Midterm I

Exam is closed book, closed notes; **one** sheet (2 pages) of notes and formulas allowed; 50 minutes. Show all work on the pages provided. No extra pages (use back if necessary). **Read each question very carefully**.

Box your final answer and include units where appropriate. Number of points for each problem is given in parenthesis (40 points total).

Problem 1 (4 pts.) ____________________

Problem 2 (6 pts.) ____________________

Problem 3 (20 pts.) ____________________

Problem 4 (10 pts.) ____________________

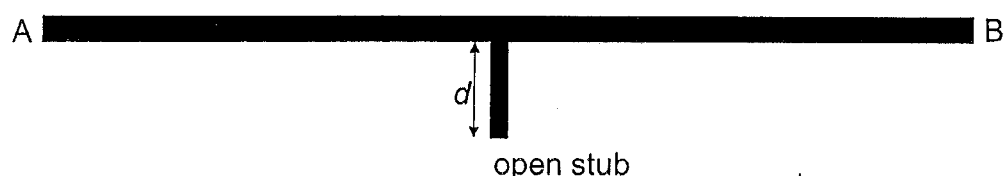
Total (40 pts.) ____________________

1. [4 pts.] One of your colleagues was tasked to design a 60Ω microstrip transmission line of 1 nsec delay time on a PCB. After the PCB was fabricated, it was handed to you to be tested. You discover that the actual characteristic impedance of the fabricated microstrip line is 50Ω instead of the specified 60Ω. After further exploration, you find out that the fabricated microstrip trace has the wrong width. **Explain,** if the actual width of the microstrip is larger or smaller than the width specified in the design.

for a microstrip: increase in width increases C and decreases L ⇒ decreases $Z_0 = \sqrt{\frac{L}{C}}$

⇒ the actual width is larger than the width specified in the design since Z_0 is smaller

2. [6 pts.] A trace is placed on a PCB to route a digital signal from point A to point B, as illustrated in the figure below. At about half way down the trace, a stub of length d = 10mm and characteristic impedance Z_0 = 50Ω has been added for a possible connection to another device in the future. For now, the stub is left open circuit. The digital signal has rise and fall times of about 2nsec. What effect does the open-circuited stub have on the main trace from A to B? Assume a velocity of propagation of 20cm/ns on the PCB traces.



The delay time of the stub is $TD = \frac{d}{V_p} = \frac{1\,cm}{20\frac{cm}{ns}} = 0.05\,ns$

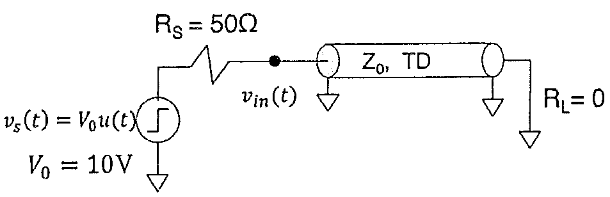
The rise time of the signal is much larger than the delay time of the stub: $t_r = 2ns \gg TD$

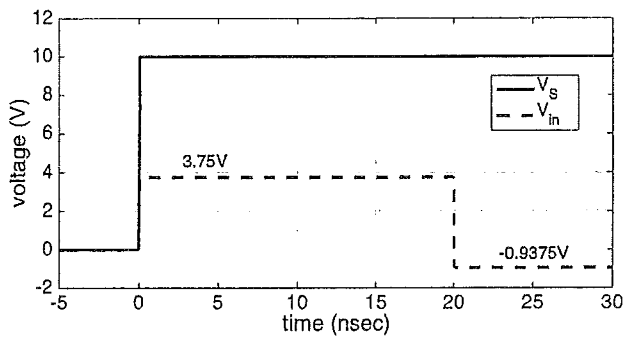
⇒ the stub acts like a lumped element connected in parallel to the main line at the connection point

Since the stub is open-circuited, it represents a capacitance $C_{Stub} = \frac{TD}{Z_0} = \frac{0.05\,ns}{50\Omega} = 1\,pF$

The additional capacitance at the stub junction causes reflections at the stub junction, which will be observed at A and B.

3. [20 pts.] You have found a piece of 2m long piece of coaxial cable of unknown characteristic impedance, Z_0. To characterize the cable, you connect one end of the cable to the TDR instrument in your lab and short-circuit the other end, as illustrated in the figure below. The open circuit voltage of the TDR system is $V_0 = 10\text{V}$ and the output impedance is $R_S = 50\Omega$. The recorded step response $v_{in}(t)$ at the input of the coaxial cable is shown below for $-5 \leq t \leq 30$ nsec (dashed curve).





(a) Determine the delay time (TD) of the coaxial transmission line.

Reflection at short circuit at the far end is seen at the near end at 20ns. ⇒ 2TD = 20ns ⇒ $\boxed{TD = 10ns}$

(b) Determine the propagation velocity on the coaxial cable.

$$V_p = \frac{\text{length}}{TD} = \frac{2m}{10ns} = 2\times 10^8 \frac{m}{s} \quad \left(20 \frac{cm}{ns}\right)$$

(c) Determine the characteristic impedance of the coaxial cable.

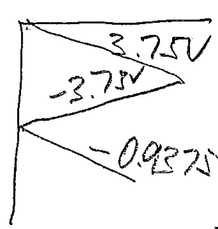
Method 1:

$$V_1^+ = 3.75V = \frac{Z_0}{Z_0 + R_S} 10V$$

$$\Rightarrow 3.75V \cdot Z_0 + 3.75V \cdot R_S = 10V Z_0$$

$$\Rightarrow Z_0 = \frac{3.75}{6.25} R_S = 0.6 R_S = \boxed{30\Omega}$$

Method 2:



$$\Rightarrow \rho_S = \frac{-0.9375}{-3.75} = 0.25$$

$$\Rightarrow Z_0 = R_S \frac{1-\rho_S}{1+\rho_S} = \boxed{30\Omega}$$

(d) Determine the reflection coefficient at the source (ρ_S) and the load (ρ_L).

$$\rho_S = \frac{-0.9375}{-3.75} = \boxed{0.25} \quad \text{or} \quad \rho_S = \frac{R_S - Z_0}{R_S + Z_0} = \frac{50-30}{50+30} = \frac{1}{4}$$

$\boxed{\rho_L = -1}$ (short circuit)

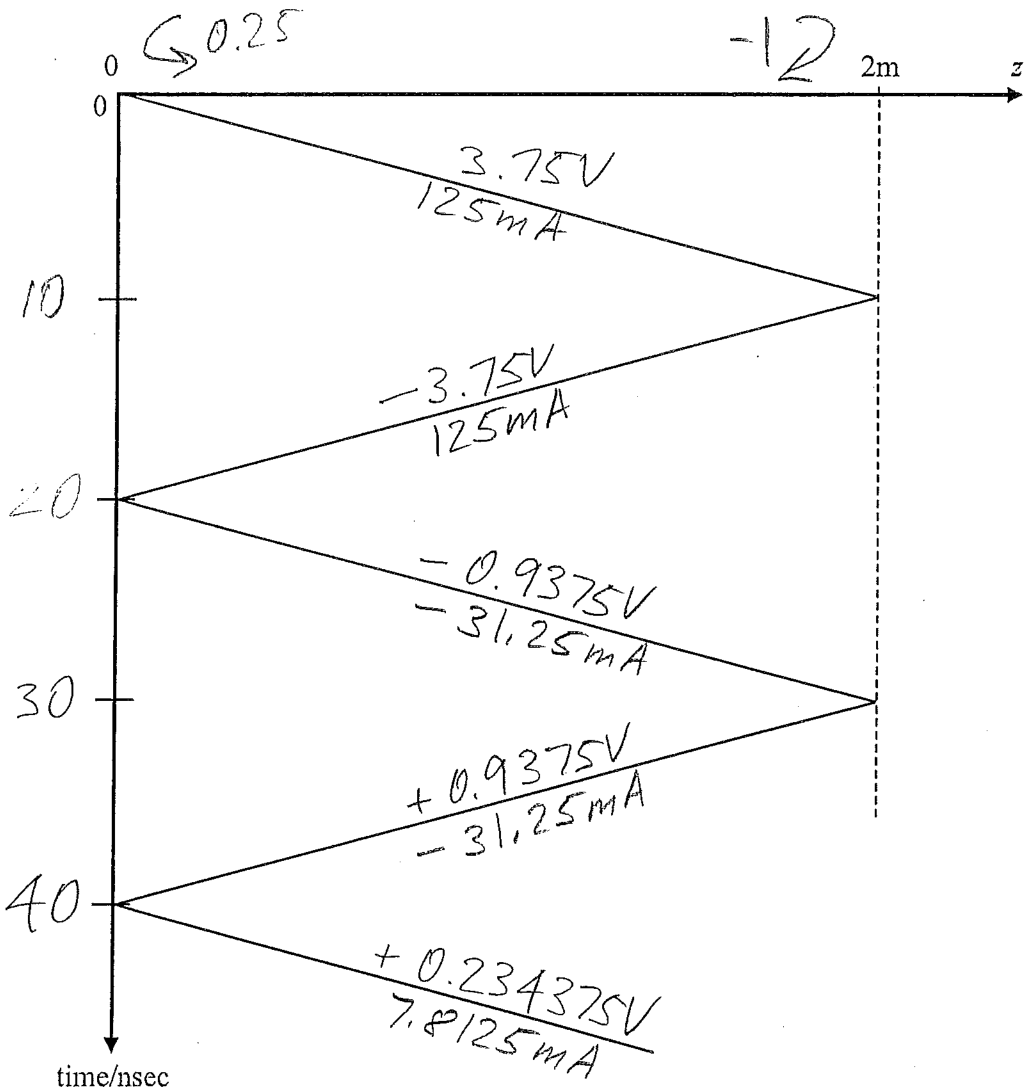
(e) Determine the capacitance per unit length of the coaxial cable.

$$C = \frac{1}{V_p Z_0} = \frac{1}{2\times10^8 \frac{m}{s}\ 30\Omega} = \frac{1}{60} 10^{-8} \frac{F}{m} = \boxed{166.7 \frac{pF}{m}}$$

(f) Determine the inductance per unit length of the coaxial cable.

$$L = \frac{Z_0}{V_p} = \frac{30\Omega}{2\times10^8 \frac{m}{s}} = 15\times10^{-8} \frac{H}{m} = \boxed{150 \frac{nH}{m}}$$

(g) Add time scale and numerical values for voltage and current for the first 5 wave components in the lattice diagram below.



(h) Determine the voltage at the input of the coaxial cable, v_{in}, at time t = 45nsec.

$$V_{in}(t=45ns) = 3.75V + (-3.75V) + (-0.9375V) + 0.9375V + 0.23475V$$

$$= \boxed{0.23475V \approx 0.235V}$$

(i) Determine the current into the coaxial cable (source side) at t = 5nsec and t = 25nsec.

$$i_{in}(t=5ns) = 125mA$$

$$i_{in}(t=25ns) = 125mA + 125mA + (-31.25mA)$$

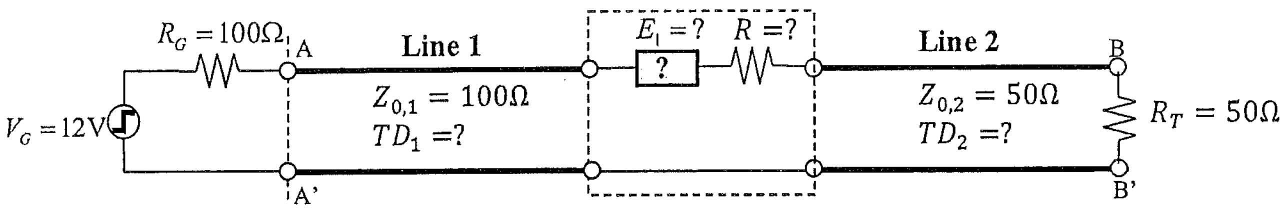
$$= \boxed{218.75mA}$$

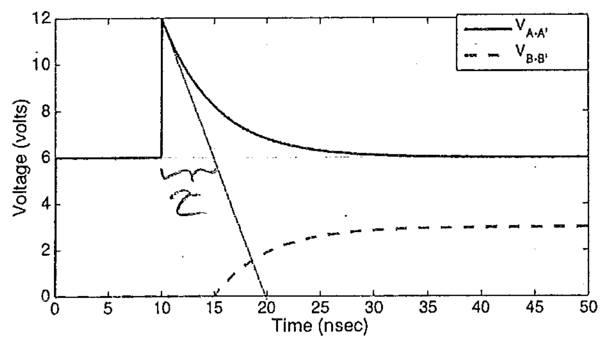
(k) What are the steady-state voltage and current at the input of the coaxial cable?

$$V_{\infty} = 0 \text{ (short circuit)}$$

$$i_{\infty} = \frac{V_0}{R_S} = \frac{10V}{50\Omega} = \boxed{200mA}$$

4. [10 pts.] Two transmission lines with different characteristic impedances are connected via a series combination of a resistor and an unknown lumped element, as shown below. Line 1 is matched at the source (near end) and line 2 is matched at the far end. At time $t = 0$, a 12V step voltage generator connected to line 1 is turned on, and the voltages at the input terminals of line 1 (at A-A') and across the load resistor R_T (at B-B') are observed on an oscilloscope over a finite duration of time (see below).





(a) Determine the delay times of line 1 and line 2, respectively.

$2TD_1 = 10ns \Rightarrow \boxed{TD_1 = 5ns}$

$TD_1 + TD_2 = 15ns \Rightarrow TD_2 = 15ns - 5ns = \boxed{10ns}$

(b) Determine the value of series resistance R connected between the two lines.

for $t \to \infty$ the junction between the 2 lines is matched ($V_{AA'}(t \to \infty) = \frac{1}{2} V_G = 6V$) $\Rightarrow \boxed{R = 50\Omega}$

(c) Specify the **type** of lumped element E_1 (see circuit above) to give the response as shown in the figure above?

initial response is that of an open circuit at the junction (doubling of voltage) $\Rightarrow \boxed{E_1 \hat{=} L}$

(d) Determine the circuit value of the unknown element E_1.

time constant $\tau = \frac{L}{Z_{0,1} + R + Z_{0,2}} = \frac{L}{200\Omega} = 5ns$ (from graph)

$\Rightarrow L = 5ns \cdot 200\Omega = 1000\, nH = \boxed{1mH}$