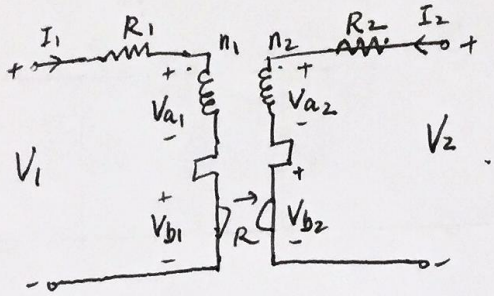
Q1.



$$\begin{cases} Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \\ V_1 = I_1 R_1 + V_{a1} + V_{b1} \\ V_{a1} = \frac{n_1}{n_2} V_{a2} \\ V_{b1} = -RI_2 \\ I_2 = -\frac{n_1}{n_2} I_1 \end{cases}$$

$$\Rightarrow Y_{11} = \frac{n_2^2}{n_1^2 R_2 + n_2^2 R_1}$$

$$\begin{cases} Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \\ V_2 = I_2 R_2 + V_{a2} + V_{b2} \\ V_{a2} = \frac{n_2}{n_1} V_{a1} \\ V_{b2} = RI_1 \\ I_1 = -\frac{n_2}{n_1} I_2 \end{cases}$$

$$\Rightarrow Y_{22} = \frac{n_1^2}{n_1^2 R_2 + n_2^2 R_1}$$

$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{n_2}{n_1} \cdot \left.\frac{I_2}{V_2}\right|_{V_1=0} = -\frac{n_2}{n_1} Y_{22}$$

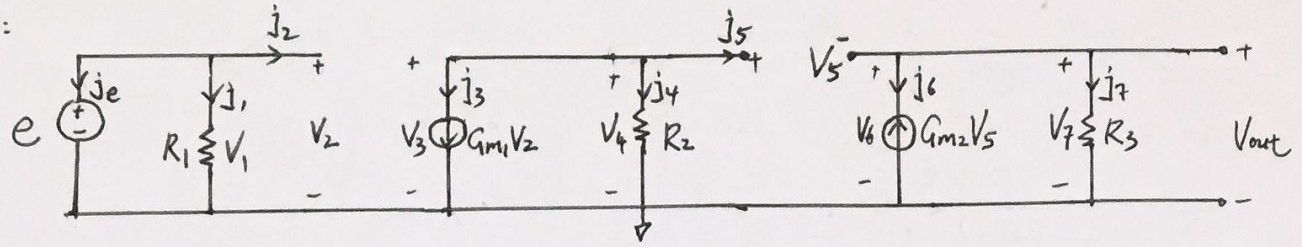
$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = -\frac{n_1}{n_2} Y_{11}$$

$$\Rightarrow Y_a = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{n_1^2 R_2 + n_2^2 R_1} \begin{bmatrix} n_2^2 & -n_1 n_2 \\ -n_1 n_2 & n_1^2 \end{bmatrix}$$

$$\Rightarrow S = 1 - 2Y_{an} = 1 - \begin{bmatrix} \sqrt{R_1} & 0 \\ 0 & \sqrt{R_2} \end{bmatrix} Y_a \begin{bmatrix} \sqrt{R_1} & 0 \\ 0 & \sqrt{R_2} \end{bmatrix}$$

$$= \frac{1}{n_1^2 R_2 + n_2^2 R_1} \begin{bmatrix} n_1^2 R_2 - n_2^2 R_1 & 2n_1 n_2 \sqrt{R_1 R_2} \\ 2n_1 n_2 \sqrt{R_1 R_2} & n_2^2 R_1 - n_1^2 R_2 \end{bmatrix}$$

Q2:

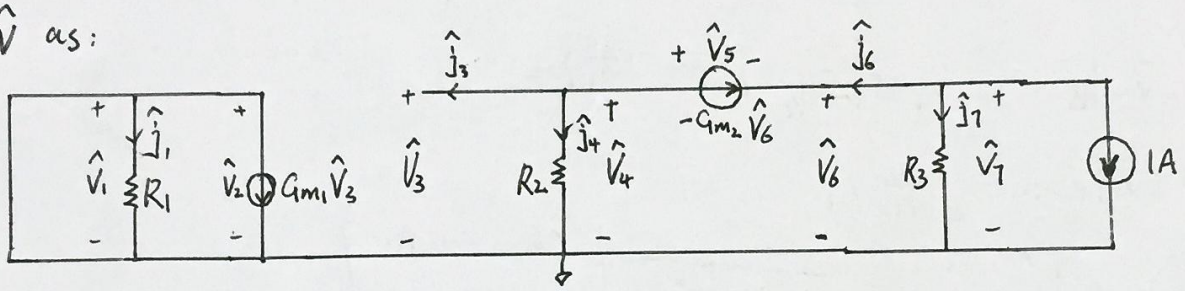


$$[j] = G[V]$$

$$\Rightarrow G = \begin{bmatrix} G_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_{m_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & G_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -G_{m_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & G_3 \end{bmatrix} \quad \text{where } G_i = \frac{1}{R_i}$$

$$\Rightarrow \hat{G} = G^T$$

$\Rightarrow \hat{N}$ as:



In N: $V_1 = V_2 = V_{in}$. $V_3 = V_4 = -G_{m_1} R_2 V_{in}$. $V_5 = -\frac{G_{m_1} R_2 V_{in}}{1 + G_{m_2} R_3}$. $V_6 = V_7 = -\frac{G_{m_1} G_{m_2} R_2 R_3 V_{in}}{1 + G_{m_2} R_3}$

In $\hat{N}$: $\hat{V}_1 = \hat{V}_2 = 0$. $\hat{V}_3 = \hat{V}_4 = -\frac{G_{m_2} R_2 R_3}{1 + G_{m_2} R_3}$. $\hat{V}_6 = \hat{V}_7 = \frac{R_3}{1 + G_{m_2} R_3}$

$$\Rightarrow \begin{cases} \frac{\partial V_o}{\partial R_1} = \frac{\partial V_o}{\partial G_2} \cdot \left(-\frac{1}{R_1^2}\right) = 0. & \frac{\partial V_o}{\partial G_2} = \hat{V}_4 V_4. \quad \frac{\partial V_o}{\partial G_3} = \hat{V}_7 \cdot V_7 \\ \frac{\partial V_o}{\partial R_2} = \frac{\partial V_o}{\partial G_2}\left(-\frac{1}{R_2^2}\right) = \frac{G_{m_1} G_{m_2} R_3}{1 + G_{m_2} R_3} V_{in}, \\ \frac{\partial V_o}{\partial R_3} = \frac{\partial V_o}{\partial G_3}\left(-\frac{1}{R_3^2}\right) = -\frac{G_{m_1} G_{m_2} R_2}{(1 + G_{m_2} R_3)^2} V_{in} \\ \frac{\partial V_o}{\partial G_{m_1}} = V_2 \hat{V}_3 = -\frac{G_{m_2} R_2 R_3}{1 + G_{m_2} R_3} V_{in}. \\ \frac{\partial V_o}{\partial G_{m_2}} = \hat{V}_6 V_5 = -\frac{G_{m_1} R_2 R_3}{(1 + G_{m_2} R_3)^2} V_{in} \end{cases}$$