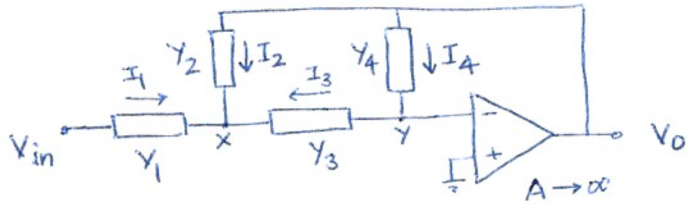
HOMEWORK 3

Q1)



(a) $V_y = 0$

KCL at NODE Y,

$$I_3 = I_4 \qquad \text{——— (1)}$$

KCL at NODE X,

$$I_1 + I_2 + I_3 = 0$$

$$(V_{IN} - V_x) Y_1 + (V_o - V_x) Y_2 - V_x Y_3 = 0$$

$$V_{IN} Y_1 + V_o Y_2 - V_x (Y_1 + Y_2 + Y_3) = 0$$

$$I_4 = V_o Y_4$$

$$V_x = \frac{-I_3}{Y_3}$$

USING (1),

$$V_x = -V_o \frac{Y_4}{Y_3}$$

$$\therefore V_{IN} Y_1 + V_o Y_2 + V_o \frac{Y_4}{Y_3} (Y_1 + Y_2 + Y_3) = 0$$

$$V_{IN} Y_1 = -V_o \left[Y_2 + \frac{Y_4}{Y_3} (Y_1 + Y_2 + Y_3) \right]$$

$$\boxed{\frac{V_o}{V_{IN}} = \frac{-Y_1 Y_3}{Y_2 Y_3 + Y_4 (Y_1 + Y_2 + Y_3)}}$$

(b) FOR A BANDPASS SYSTEM, THE TRANSFER FUNCTION IS OF THE FORM,

$$H(s) = \frac{A_o \left(\frac{\omega_o}{Q} \right) s}{s^2 + \frac{\omega_o}{Q} s + \omega_o^2}$$

THERE ARE 2 POSSIBLE SOLUTIONS,

<u>CASE 1</u> $\quad Y_1 = \frac{1}{R} = G \quad \& \quad Y_3 = sC$

SINCE DENOMINATOR IS SECOND ORDER, ATLEAST TWO OF THE ELEMENTS HAVE TOBE CAPACITORS.

SAY $Y_4 = sC \quad \& \quad Y_2 = \frac{1}{R} = G$

$$H(s) = \frac{-sCG}{sGC + sC(G+G+sC)} = \frac{-sCG}{sC(3G+sC)}$$

THIS CANNOT BE A SOLUTION.

THEREFORE,

$$\boxed{\begin{array}{ll} Y_1 = \frac{1}{R} = G & Y_3 = sC \\ Y_2 = sC & Y_4 = \frac{1}{R} = G \end{array}}$$

$$H(s) = \frac{-sCG}{s^2C^2 + G(2sC+G)}$$

$$\boxed{H(s) = \frac{-s\,G/C}{s^2 + 2s\,G/C + G^2/C^2}}$$

<u>CASE 2</u> $\quad Y_1 = sC \quad Y_3 = \frac{1}{R} = G$

USING THE SAME ARGUMENTS AS IN CASE 1, ONLY POSSIBLE COMBINATION IS,

$$\boxed{\begin{array}{ll} Y_1 = sC & Y_3 = \frac{1}{R} = G \\ Y_2 = \frac{1}{R} = G & Y_4 = sC \end{array}}$$

$$\cancel{H(s) = \frac{-sCG}{G^2 + G(sC+sC+G)}}$$

$$H(s) = \frac{-sCG}{G^2 + sC(sC+G+G)}$$

$$\boxed{H(s) = \frac{-sG/C}{s^2 + 2sG/C + G^2/C^2}}$$

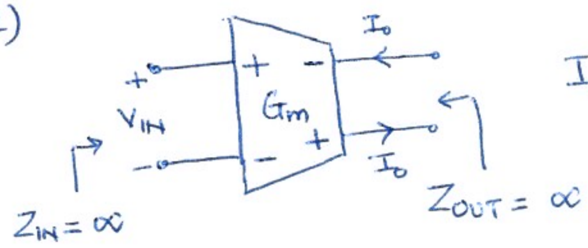
c) $\omega_o^2 = \dfrac{G^2}{C^2}$

$$\therefore \boxed{\omega_o = \frac{G}{C} = \frac{1}{RC}}$$

$$\frac{\omega_o}{Q} = 2\frac{G}{C}$$

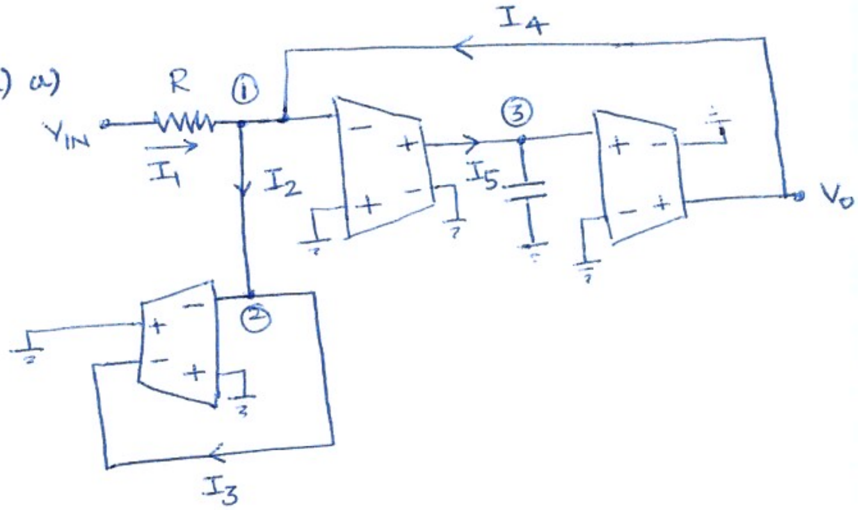
$$\therefore \boxed{Q = \frac{C\omega_o}{2G} = \frac{1}{2}}$$

Q2)



$I_o = G_m V_{IN}$

a) a)



NODE 1 & NODE 2 ARE THESAME (SHORTED)

$V_1 = V_2 = V_o$

$I_3 = I_2 = -G_m V_o$

KCL AT NODE 1

$I_1 + I_4 = I_2$

$$\frac{V_{IN} - V_o}{R} + G_m V_3 = -G_m V_o \quad — (1)$$

KCL AT NODE 3

$I_5 = V_3 sC$

$V_3 = \frac{I_5}{sC}$

BUT $I_5 = -G_m V_o$

$$\therefore V_3 = \frac{-G_m V_o}{sC}$$

SUBSTITUTING V_3 IN EQN. (1)

$$\frac{V_{IN}}{R} = V_o\left[\frac{1}{R} - G_m\right] + \frac{G_m^2 V_o}{sC}$$

$$V_{IN} = V_o\left[1 - G_mR + \frac{G_m^2 R}{sC}\right]$$

$$\therefore \frac{V_o}{V_{IN}} = \frac{sC}{(1-G_mR)sC + G_m^2 R}$$

$$\boxed{\frac{V_o}{V_{IN}} = \frac{sC/G_m^2 R}{1 + sC\,\frac{(1-G_mR)}{G_m^2 R}}}$$

a) b) $$\frac{V_o}{V_{IN}} = \frac{\frac{s}{1-G_mR}}{s + \frac{G_m^2 R}{(1-G_mR)C}}$$

FOR A STABLE FILTER, POLE MUST BE IN LHP,

$$\therefore \quad s = \frac{-G_m^2 R}{(1-G_mR)C} < 0$$

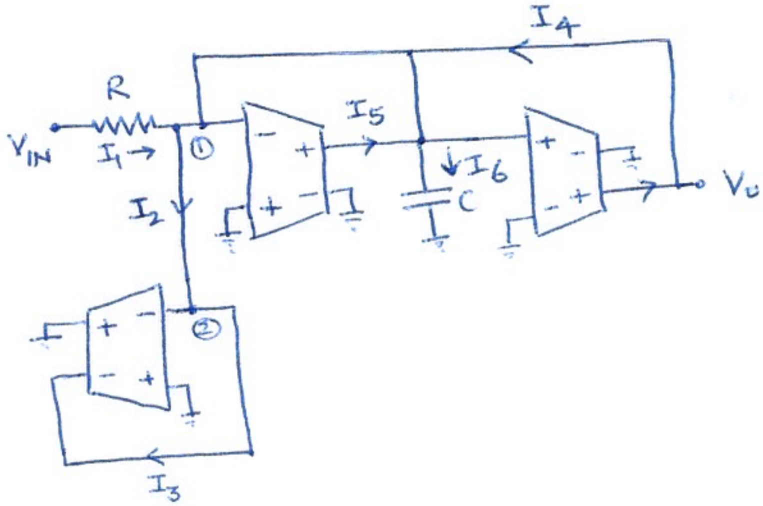
since $G_m > 0$, $R > 0$ & $C > 0$

$$1 - G_mR > 0$$

$$\therefore \quad G_mR < 1$$

$$\boxed{G_m < \frac{1}{R}}$$

b)



KCL AT NODE ①

$$I_1 + I_5 + I_4 = I_2 + I_6$$

$$I_1 = \frac{V_{IN} - V_o}{R}$$

$$I_5 = -G_m V_o$$

$$I_4 = G_m V_o$$

$$I_2 = I_3 = -G_m V_o$$

$$I_6 = V_o s C$$

$$\therefore \frac{V_{IN} - V_o}{R} \cancel{- G_m V_o} \cancel{+ G_m V_o} = -G_m V_o + V_o s C$$

$$\frac{V_{IN}}{R} = V_o \left[\frac{1}{R} - G_m + sC \right]$$

$$V_{IN} = V_o \left[1 - G_m R + sRC \right]$$

$$\boxed{\frac{V_o}{V_{IN}} = \frac{1}{1 - G_m R + sRC}}$$