

THE EFFECT OF STOCHASTIC VOLATILITY IN PREDICTING HIGHWAY BREAKDOWNS

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ABSTRACT

In this paper we pursue the effect of volatility on the probability of highway breakdown. Because daily aggregated flow values exhibit dissimilar levels of variability throughout the day, a stochastic volatility (SV) model was pursued. Under this format, assuming traffic flow volatility follows an autoregressive process of order one, volatility was estimated at each 15-minute time step per day during the collection period of 205 days. A computational Bayesian format was used to fit parameters of 205 SV models to data collected from one bottleneck site along Interstate 93 in Salem, NH. Fitted results show that volatility estimates successfully capture flow variability observed from the traffic data. To evaluate the effect of SV on breakdown, a sampling scheme was created in which sampled demands and capacities were compared. To estimate demand, a signal was first extracted from the flow aggregates using a Functional Data Analysis (FDA) model and then combined with a sampled SV estimate. For stochastic capacity, daily flow maxima were used as either censored or uncensored estimates of capacity based on breakdown occurrence. By extreme value theory, these capacity values are distributed as a Generalized Extreme Value (GEV) distribution. From a GEV model fitted to our data, capacity estimates are sampled and compared to SV-based demand to produce breakdown probabilities by time of day. Finally, our breakdown probabilities are compared to those estimated by the sample data only to illustrate the effect of SV on breakdown.

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INTRODUCTION

Theoretically, highway breakdown will occur when flow q_t , some flow (q) at time (t), equals or exceeds roadway capacity c . However, because breakdowns do not necessarily occur at the same demand levels (Elefteriadou et al., 1995), our breakdown protocol $q_t \geq c$ cannot be confirmed with field data (Brilon et al., 2007; Lorenz and Elefteriadou, 2001). In light of this, breakdown will be treated as a chance event with the following probability model:

$$P(D_t \geq C)$$

where travel demand at time t , D_t , and roadway capacity, C , are both considered random variables. In order to estimate this probability expression, we must identify suitable models for both traffic demand and capacity. A complication arises as flow values, values used to calibrate our models, are typically very noisy. To address this, to mitigate the irregular fluctuations, flow values are typically extracted from coarse aggregates. That is, traffic flow variability (noise) is removed to identify a realistic signal. What if we could estimate the natural variability in demand and re-introduce it into our breakdown prediction? In so doing, how would this volatility in demand affect the performance of the roadway? Will traffic noise increase the chance of breakdown? In this work we aim to answer these questions and quantify the effect of flow volatility on the probable occurrence of breakdown.

APPROACH

A dominant characteristic of freeway traffic flow is the time-varying volatility. That is, a roadway commonly experiences distinct periods of contrasting levels of flow variability (Figure 1). Such behavior is also commonly observed in financial or economic time series. To account for this type of behavior, Taylor (1982) suggests that volatilities evolve in a stochastic nature, an approach commonly referred to as the ‘stochastic volatility’ (SV) model. So, in our expression for the probability of breakdown, $P(D_t \geq C)$, for any day,

$$D_t \sim N(q_t, \sigma_t),$$

or demand at time t is normally distributed about some flow q_t with a time-indexed standard deviation (Note the distinction between demand and flow, where, going forward, demand is a combination of known flow and random variability). The unique feature of the SV model is the time-varying standard deviation, σ_t , which is itself introduced as a random variable.

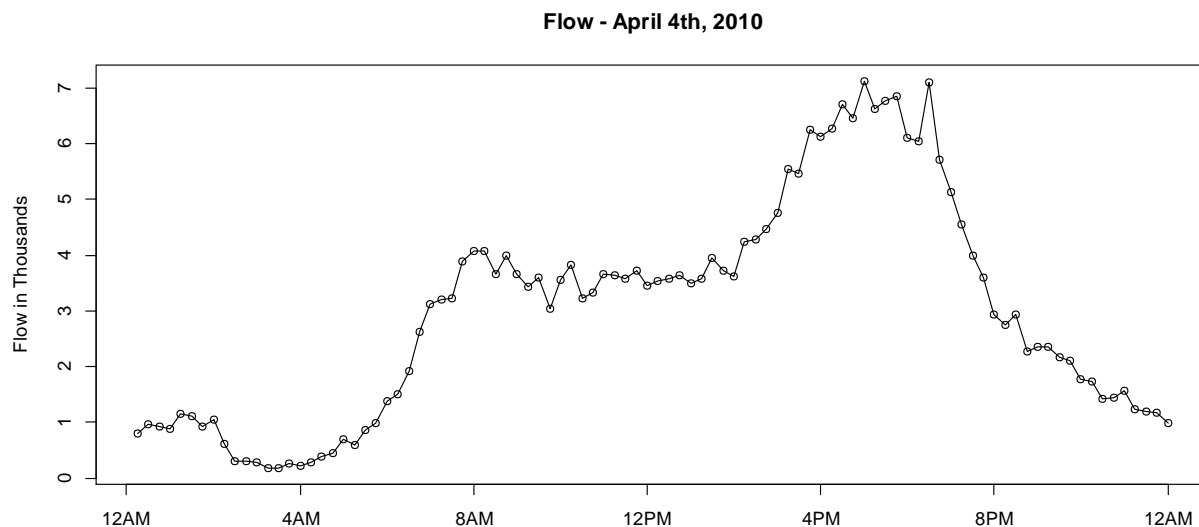


FIGURE 1 Illustration of time-varying volatility in traffic flows for a typical weekday during the collection period. We observe that flow values are more volatile during morning and evening commuting hours.

DATA

Data was collected by the State of New Hampshire (NH) Department of Transportation along northbound lanes of interstate I-93 at one location in Salem, NH that observes both daily breakdowns and heavy demand. Data collection was made possible by a side-fire radar unit located just north of an exit ramp, exit 1, and just south of an entrance ramp to I-93. Just north of the radar device (downstream), I-93 is physically constricted from three to two lanes. The positioning of this device is ideal as data collected here fit perfectly to the theoretical framework of flow-based capacity analyses (Minderhoud et al., 1997). As stated by Brilon et al. (2005), the capacity of the freeway segment is analyzed most precisely at or slightly upstream of a bottleneck.

The radar device was scheduled to intermittently measure traffic at irregular but frequent time periods about 1 minute apart. The data collection period occurred between April 1 and November 30, 2010, a total of 244 days. The observations (raw data) consist of the following measurements: vehicle counts, average speed, occupancy, and speed of individual vehicles observed over the interval. During this collection period, there were several scheduled days where the radar devices were shut down for maintenance. In the end, after omitting these days where no traffic measurements were recorded, a total of 205 days were retained for our analysis.

Aggregation

Because radar data are collected over very short, irregular time intervals, these measurements were aggregated into uniform intervals. The HCM recommends that transportation engineers use aggregates (intervals) no shorter than 15-minutes in order to ensure ‘stable’ traffic flow rates. Longer intervals are especially suitable for macroscopic/speed-flow analyses and intervals shorter than 5-minutes should be avoided (Highway Capacity Manual,

2000). That is, for shorter intervals, it is possible to observe speeds and flows that simply cannot be sustained over longer periods, and these measurements are not accurate representations of traffic conditions. As stated by Greenwood et al. (2007), analyzing capacity is a macroscopic endeavor related to management, policy, design and regional speed/flow comparisons, thus stable, longer interval durations are appropriate. While exact lengths vary among analyses, 15-minute intervals are commonly used for capacity studies (See, for example, Agyemang-Duah and Hall, 1991; Yang and Zhang, 2005; Lorenz and Elefteriadou, 2001; Elefteriadou and Lertworawanich, 2002; Minderhoud et al., 1997). For these reasons, and to produce results that are directly comparable to those put forth by the HCM, observations were binned into 15-minute aggregates.

From each of the raw vehicle counts (total volume, or number of vehicles, observed during a precisely defined interval), the corresponding interval length during which they were observed and a flow rate, or equivalent hourly rate of vehicles passing a point (the 'traffic flow rate,' by definition), was simply calculated. This then yields flow rates that are expressed in the customary scale of vehicles per hour (vph), but based on short, unstable time intervals. Next, these flow rates were binned into non-overlapping, sequential 15-minute intervals based on original observation times. Lastly, for each bin, the component flow rates were averaged to produce aggregated flow (q) rates for each 15-minute interval during the collection period. Thus, we have 96 estimates per day, or $t = 1, \dots, 96$. Addressing traffic speed (u) is a simpler task as precise speed (in mph) and time measurements are recorded for every vehicle observed during the collection period. Speed aggregates are then calculated by first separating all observations into 15-minute bins, and then simply taking their average.

CAPACITY DATA

Daily Maxima as Estimates of Capacity

Modern methods of estimating breakdown probabilities are based on stochastic capacity, or treating capacity as a random variable (See, for example, Brilon et al., 2005; Lorenz and Elefteriadou, 2001). That said, no universally accepted measure of capacity has been established. Breakdown flows, however, those flows measured immediately before the onset of congestion, have been widely adopted as estimators of capacity (See, for example, Elefteriadou and Lertworawanich, 2002; Brilon et al., 2005; Lorenz and Elefteriadou, 2001; Minderhoud et al., 1996). However, one concern when using breakdown flows is that higher flows or daily flow maxima often occur prior to congested conditions. These cases suggest that breakdown flows are actually underestimating capacity since higher flows were observed just prior to breakdown. By instead considering *daily flow maxima*, we would capture these higher flows prior to breakdown and ultimately obtain more representative measures of capacity, the maximum flow that a roadway can sustain. Another concern when using breakdown flows is that they are highly dependent on subjective, and often arbitrary, breakdown identification criteria. Daily flow maxima, of course, have the advantage of being independent of the methods used to identify breakdowns.

Another commonly used estimate of freeway capacity is the maximum pre-breakdown flow (See, for example, Hall and Agyemang-Duah, 1991; Hall et al., 1992), or the maximum sustained flow prior to breakdown. Based on aggregated traffic data, Elefteriadou and Lertworawanich (2002) provide evidence that distributions of breakdown flows and maximum

pre-breakdown flows are statistically similar and, by extension, that maximum pre-breakdown flows are suitable estimates for capacity. Because daily flow maxima typically occur prior to breakdowns, in practice, daily flow maxima closely resemble maximum pre-breakdown flows, although the two are conceptually different. Thus, using daily flow maxima to estimate capacity has many of the same benefits as using pre-breakdown flows. For one, both maxima and pre-breakdown flows account for cases (as described above) where flows decrease prior to congestion, a so-called ‘lingering’ or lagged effect, and are better representations of the true maximum throughput of the roadway. However, while maxima and pre-breakdown flows are somewhat similar, maxima have the advantage of being consistently available. That is, extraction of pre-breakdown flows is completely dependent on breakdowns, while daily maxima, on the other hand, have no such dependence and may be obtained on a regular basis regardless of breakdown occurrence. Using maxima, then, is especially beneficial when breakdowns are rare as such cases would yield relatively few pre-breakdown flows. Laflamme (2013) presents a detailed discussion on the use and advantage of using daily maxima as capacity values.

Censoring

As stated by the HCM, the capacity for a given facility is the flow rate that can be achieved repeatedly for peak periods of sufficient demand (Highway Capacity Manual, 2000). Therefore, not every maximum flow is a suitable estimate of freeway capacity, and only those daily maxima associated with ‘sufficiently high demand’, demand typically resulting in breakdown, should be considered as such. On days where demand is insufficient, when breakdowns are not observed, daily conditions are not adequate (sufficiently extreme) to assess the true capacity of the roadway. In these cases, under a survival analysis premise, the corresponding maxima are deemed censored (right-censored) capacity values as the roadway can surely service higher demands. That is, breakdowns would occur at some higher flow rates, and the resulting capacity, the maximum daily flows, would necessarily be larger than the observed value. Alternatively, we may simply consider these cases as incomplete data records where the true maximum, the capacity, is simply missing. Despite the incompleteness associated with censored values, they still contain valuable information and will therefore be considered in the calibration of our capacity distribution (See, for example, Geistefeldt, 2010). In a comparison of capacity distributions approaches, Geistefeldt and Brilon (2009) found that using censored data achieves significantly more precise estimates, especially at higher quantiles. Lastly, our censoring designation establishes a correspondence between extreme flows and breakdown which we intuitively know exists.

The Generalized Extreme Value Distribution

Our capacity data, daily maxima extracted from the traffic stream measurements, are approximated by the Generalized Extreme Value distribution (GEV). That is, $C \sim GEV(\alpha, \beta, \xi)$, where C represents the capacity random variable. The GEV is three-parameter distribution given by the following form:

$$G(x) = \exp \left\{ - \left[1 + \xi \left(\frac{x-\alpha}{\beta} \right) \right]^{-1/\xi} \right\},$$

defined on $\{x: 1 + \frac{\xi(x-\alpha)}{\beta} > 0\}$ with α and β the respective location and scale parameters. The shape parameter of the GEV, ξ , characterizes the rate of tail decay, where $\xi > 0$, $\xi = 0$, and $\xi < 0$ correspond to data with heavy tails, light tails, and short tails, respectively. For our data, in the expression above, we replace the random variable X with capacity data C as both censored and uncensored (observed) daily flow maxima. By theory, our capacity data, the maximal flows, can be suitably and accurately approximated by the GEV distribution.

Despite violating the model assumption, that a series of independent random variables define the block maxima, the conclusion that the block maxima have a GEV distribution is still reasonable (Coles, 2001). Quantile plots for daily maxima show strong correspondence between observed and predicted values, which illustrates the appropriateness of the model class and justifies the use of the GEV for our data. Additionally, the GEV approach assumes daily maxima be identically distributed which, from observing the 8 months of data, appears to be the case. That is, we observe no true seasonality ('heavy' season), no trend, no oscillations nor systematic patterns in maximal values.

TRAVEL DEMAND

When forecasting $P(D_t \geq C)$, simply specifying demand as a constant flow value, q_t , would naively suggest that the travel demand is known with certainty, and breakdown prediction would be $P(q_t \geq C)$. The noise observed in the daily demand profile, as seen in Figure 1, would not play a role in forecasting breakdown. We simply do not know that this is the case. In our approach, we consider $D_t = q_t + N(0, \sigma_t)$ for each of the 205 days. To investigate whether or not noise in flow has a destabilizing effect on reliability, we will assess the effect that both q_t and σ_t have on our breakdown probability estimates.

Stochastic Volatility

For each of the 205 days and at each of the 96 time periods, stochastic volatility was estimated. Following Kastner and Fruhwirth-Schnatter (2013), the variance in flow measurements (q_t), σ_t , for each day is assumed to have the following properties:

$$\sigma_t | h_t \sim N(0, \exp h_t)$$

$$h_t | h_{t-1}, \mu, \phi, \lambda_\eta \sim N(\mu + \phi(h_{t-1} - \mu), \lambda_\eta^2)$$

$$h_0 | \mu, \phi, \lambda_\eta \sim N(\mu, \lambda_\eta^2 / (1 - \phi^2))$$

where $\exp(h_t)$ is called the latent, time-varying volatility that follows a stochastic evolution. Since h_t above depends on h_{t-1} , σ_t is a one-step ahead forecast that depends on σ_{t-1} . Moreover, σ_t has a variance that is not allowed to vary unrestrictedly with time, but rather is an autoregressive process of order one. We note that this model was fitted to each of the 205 days independently, thus allowing the parameters μ , ϕ , and λ_η to vary by day. This approach was deemed necessary in order to capture the day-to-day variation observed in the traffic record.

FUNCTIONAL DATA ANALYSIS (FDA) MODELING

An integral tool used to analyze traffic breakdown is functional data analysis (FDA), or functional data models. FDA models are used to extract the signal from the 15-minute aggregates which, although they are accurate representations of speed and flow, are still very ‘noisy.’ Furthermore, irregularities in the data collection process, or missing measurements, may result in aggregates calculated from few observations. Because these raw traffic observations can be erratic, because short intervals may identify traffic estimates that can only be sustained over the short term, aggregates based on few of these raw observations may be similarly noisy. To address this issue, functional data analysis (FDA) is employed to produce summaries, or smooth representations, of the data (both speed and flow) over a fine time scale, to extract the signal from the noise. These resulting interpolating smooths will be herein referred to as ‘traces’ (For further details regarding our FDA models and theory, the reader is directed to Ramsay and Silverman, 2005, for example). We note that in the previous discussing regarding the estimation of SV, FDA models were NOT used as their smoothness is believed to mask the true, observed variability in the traffic stream data. However, for other parts of our analysis (see below), the noise in our data is considered as nuisance variability and FDA traces are, in fact, preferable.

Data Traces, Capacity, and Breakdown

Traces derived from FDA models are especially beneficial for extracting capacity data. These traces represent trends of already aggregated data, and the peaks of these smooth curves necessarily correspond to *sustained* maxima which are natural estimates of capacity. Therefore, to help extract realistic signals from the noisy aggregates, daily flow maxima will be extracted from fitted FDA flow traces (smooths). Figure 2 below illustrates the usefulness of the FDA traces as the smooth curve helps to identify a realistic maximal value.

Because the censoring designation of daily flow maxima is dependent on daily breakdowns, and because these values directly relate to fitted capacity distributions, accurate assessment of breakdowns is essential. Like most traffic analyses, we define breakdown to be the transition between freely flowing traffic and congested conditions. This transition, or breakdown, occurs when persistent speeds above a fixed threshold are immediately followed by persistent speeds below the same threshold. This speed threshold was set to 48 mph based on a visually distinction between congested and freely flowing traffic regimes. By applying FDA, we are able to extract interpolated, intermediate speed and flow values based on the smooth representations of the data. Thus, traces address the two issues with our data: they provide precise (in time) estimates while maintaining smoothness. When sustained drops in traffic speeds do occur, associated daily maximal flows are considered observed capacity data. Otherwise, on days where traces do not identify sustained breakdown, associated maxima are censored values. Both these censored and observed capacity values will be used to calibrate the previously defined GEV model.

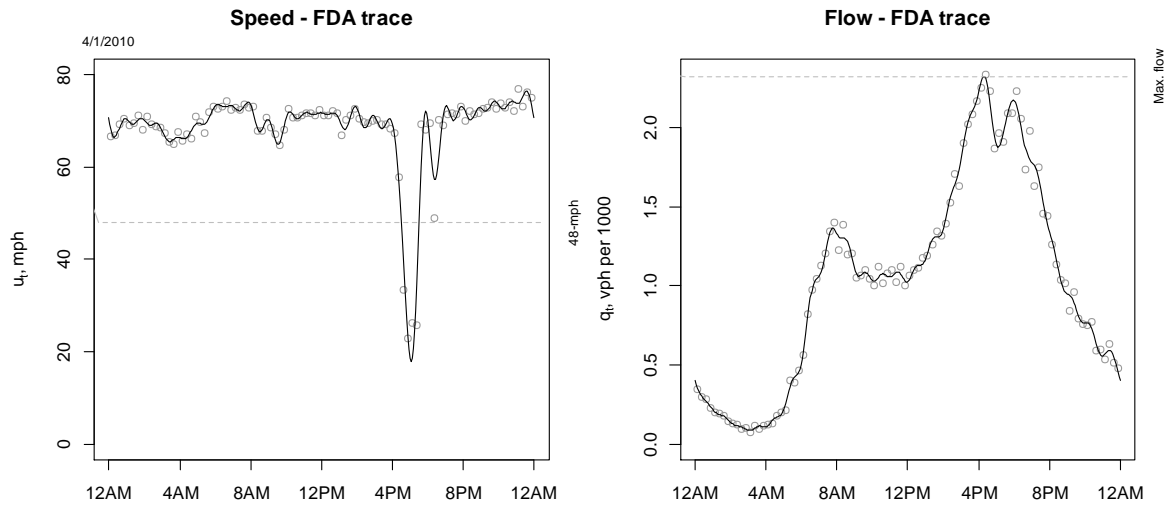


FIGURE 2 Speed (left) and flow (right) FDA traces for April 1, 2010. The small circles represent flow aggregates. Since a sustained speed below 48mph was observed (left), that day's maximal peak flow (right) was deemed a capacity value.

PROCEDURE (BAYESIAN MODEL FITTING)

Capacity Data

In general, daily maxima (capacity data) have two dominant characteristics: small sample sizes and a high number of censored values. Because of this, and because capacity model-fitting need not be done in real-time, a computational Bayesian approach was employed. Ozguven and Ozbay (2008) concluded that Bayesian estimation is far superior (more efficient) to non-parametric techniques for survival analyses with small samples and substantial amounts of censoring. The OpenBUGS statistically software was used for these applications, and the analysis and manipulation of all OpenBUGS output, the data containing the Bayesian samples of the parameters, was then performed with the R statistical software.

Because there is an absolute limit to the number of vehicles a road may carry, finite upper bounds were assumed for the capacity data, and, consequently, GEV shape parameters were assumed to be negative. The corresponding prior distribution on the shape parameter reflects this limitation. A somewhat diffuse prior distribution for the GEV scale parameter was chosen, but within a realistic range based on previous model-fitting. Similarly, a semi-informative prior distribution was chosen for the location parameter as the center of the distribution can be estimated within a reasonable range. Thus, the scale and shape priors were not over-specified, and we allowed the data to guide these posterior analyses.

Freeway capacities, C , are assumed to be generalized extreme value distributions (GEV)

$$C \sim GEV(\alpha, \beta, \xi)$$

with location, scale, and shape parameters α , β , and ξ , respectively. Collected data are the combined maximum daily traffic flows, each of which is classified as a censored or un-censored, capacity value based on the previously established definition. For the GEV model, the shape (ξ), scale (β), and location (α) parameters are assumed to follow uniform prior distributions on $(-.75, 0)$, $(0, 10)$, and $(3, 10)$, respectively.

Posterior results are based on 5,000 MCMC iterations, the first 2,000 discarded as a ‘burn-in’ period. Convergence and independence from the starting values were checked by CODA (distributions, traces, etc.), the standard tools in such cases. Also, in all cases, starting values for the sampling scheme were generated from the defined prior distributions.

Demand and Stochastic Volatility

Because of the complexity of the stochastic variability model discussed previously, parameter estimation is obtained via Bayesian MCMC methods. Fortunately, the R package ‘stochvol’ (Kastner, 2014) provides an efficient algorithm for such a Bayesian estimation of SV model parameters. To complete the setup, prior distributions for SV parameters μ , ϕ , and λ_n must be established. The parameter μ is assigned a diffuse normal distributions ($N(0, 100)$) as this prior specification is not typically influential on estimation (Kastner, 2014). For ϕ , the prior is specified by beta distribution hyperparameters (5, 1.5). Finally, following Kastner (2014) and Fruewirth-Schnatter and Wagner (2010), we assign λ_n a centered normal prior with hyperparameter equal to 1. Such a choice of hyperparameter value is not believed to be influential on our final results (Kastner, 2014). For further discussion regarding prior specification and the details of the process, the reader is directed to Kastner and Fruhwirth-Schnatter (2013).

The Bayesian sampling process is repeated for each time period (for each $t = 1, \dots, 96$) to yield distribution estimates for σ_t for each day (for each $i = 1, \dots, 205$). Posterior results are based on 10,000 MCMC iterations, with a ‘burn-in’ period of 1,000 iterations and further thinned. The ‘stochvol’ package converts its output to CODA-compatible objects, so convergence and independence from the starting values were checked by the typical methods (distributions, traces, etc.). In our case, the MCMC iterations indicated convergence to the stationary distribution of the chain.

RESULTS

SV for Individual Days

As stated, the SV model was fitted to each day independently. To observe the effect of flow volatility on the SV parameter estimates, plots of flows and SV estimates were compared for each day. Figures 3 and 4 below presents output for two days of our collection period: both the SV in terms of posterior quantiles of the latent volatilities in percent ($100 * \exp(\frac{h_t}{2})$) over time (top) as well as the corresponding daily flow profile on the same time axis.

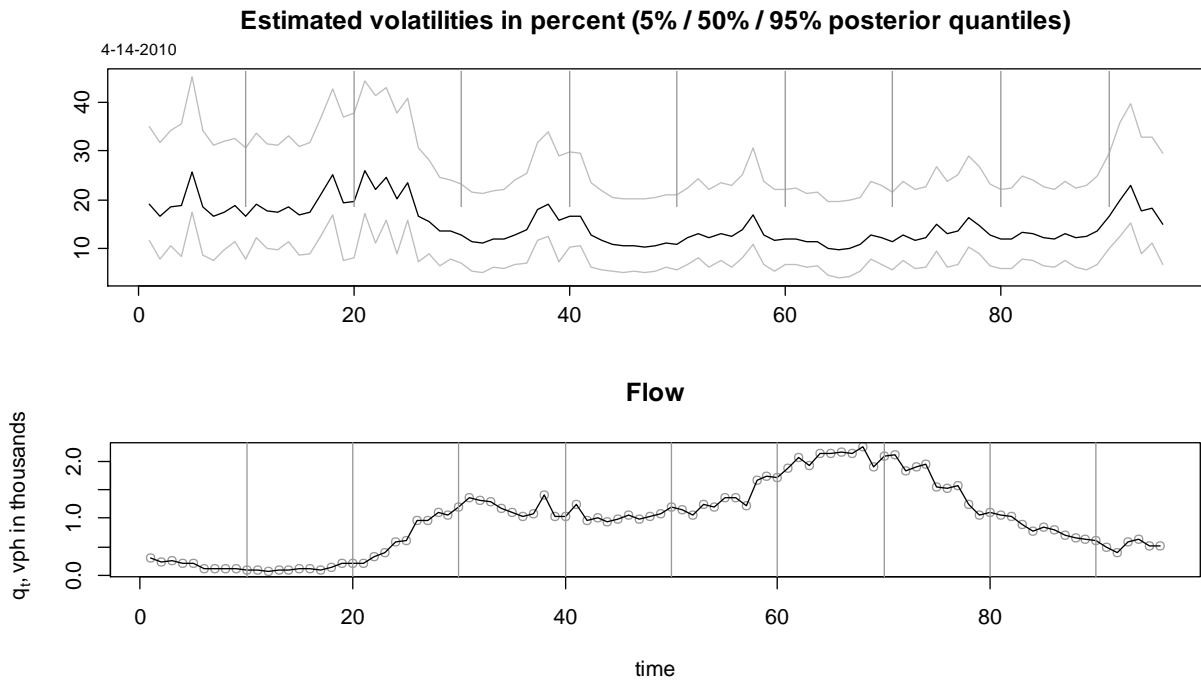


FIGURE 3 Plots of SV in terms of posterior quantiles of the latent volatilities (top) and daily flow values for April 14, 2010.

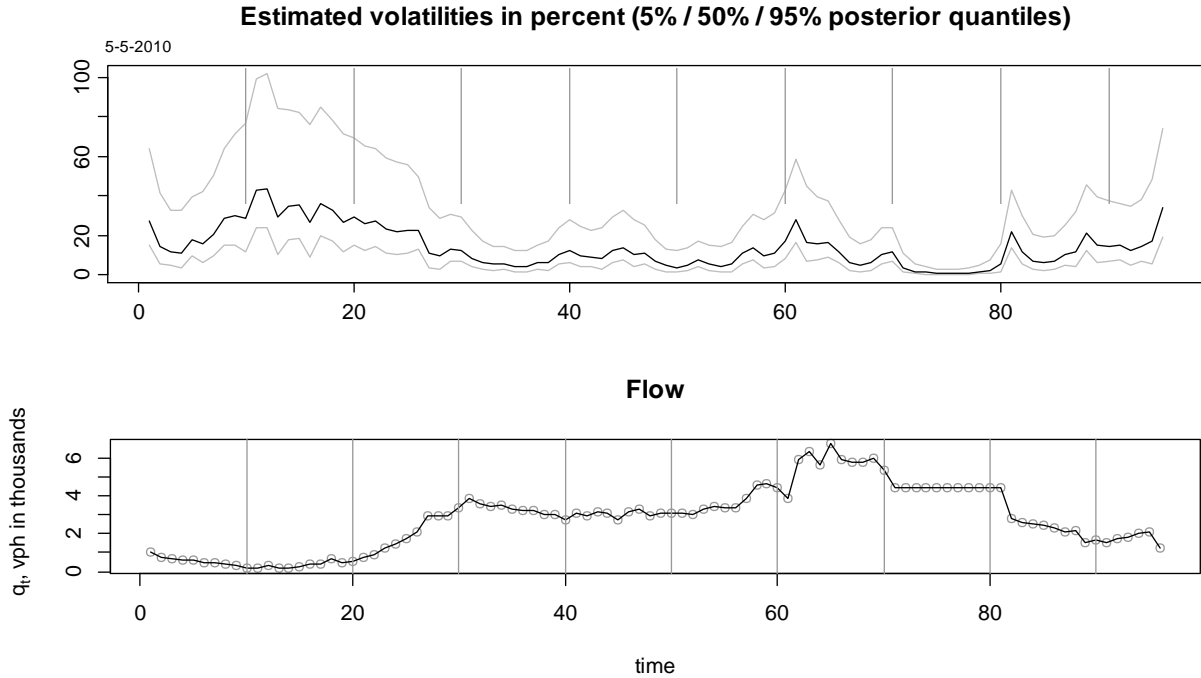


FIGURE 4 Plots of SV in terms of posterior quantiles of the latent volatilities (top) and daily flow values for May 5, 2010.

Figures 3 and 4 are typical of output for most days in that the SV model effectively identifies changes in flow volatility throughout the day. That is, from both plots we can observe strong correspondence between flow volatility and increases (spikes) in SV. Figure 4 is interesting as there was a span of missing flow values on this day (around time step 72-82) which were imputed based on the last observed flow. Observing the SV plot during this same period, we see SV estimates of zero.

SV by Time of Day

As stated previously, the SV was fitted to each day independently. Our procedure then produces an estimate of SV for each of 96 times steps for each of 205 days. Although day-to-day variability is observed in the flow data, most days seem to follow a somewhat similar pattern where volatility increases during morning and evening commuting hours. With this in mind, we have decided to group the daily SV estimates together and identify trends by time of day. More specifically, for the 205 estimate of SV at each of the 96 time steps, we have calculated both the mean confidence bounds. Figure 5 below presents these values as a time series.

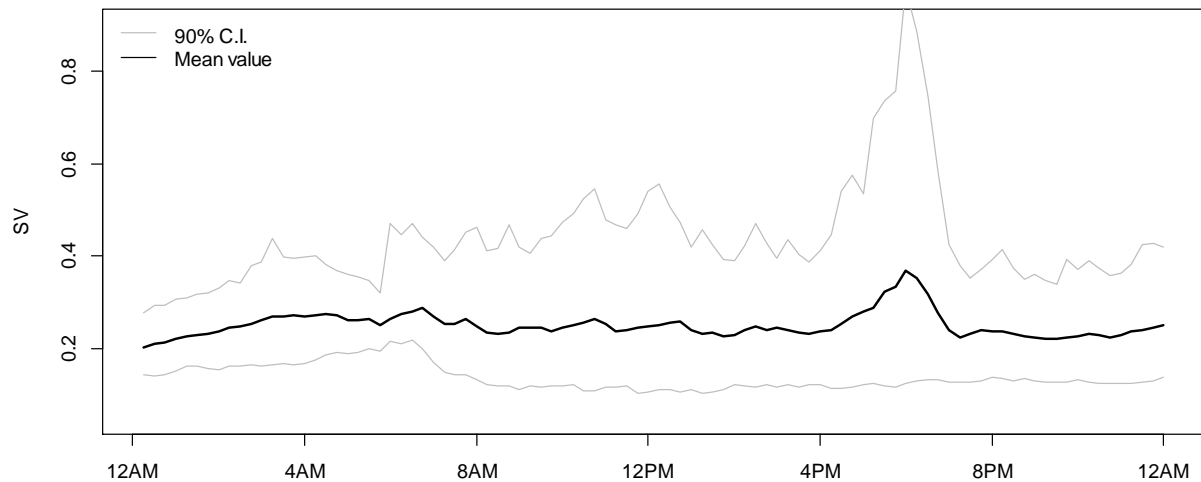


FIGURE 5 Mean stochastic volatility (SV) by time of day (black) along with upper and lower 5% bands (grey).

While the grouping and averaging process across all 205 days does seem to ‘wash away’ much of the variability, we can glean some information from this plot. Clearly, the increase in flow variability during the evening commute (around 6-7PM) is preserved in the SV plot and apparent in the somewhat sharp spike at this time. Since flow volatility is observed during this time on the majority of days, we expect the SV estimates to reflect this behavior. Also, we note that not only are the SV estimates during the evening commuting hours high on average, but the upper confidence band indicates that these values may be very large. Although morning commuting hours also tend to be volatile times, the averaging process seems to mask this behavior. We speculated that morning volatility probably occurs over a wider range of times when consider all days. If this is the case, our procedure would fail to preserve the spikes at

particular time steps and instead give smoother, elevated values. This does appear to be the case as SV estimates tend to be fairly high during the morning hours in comparison to other times of the day.

Breakdown Probabilities

For capacity, for the C model, our Bayesian approach yields posterior vector estimates of model parameters α , β , and ξ . Each vector, then, corresponds to a fitted GEV distribution for capacity. At each quantile of these many distributions, the median value was extracted to produce a single estimate of the GEV distribution form. Similarly, Bayesian models yield estimates for σ_t at all t and for each day. Once these parameters are identified, a simulation-based procedure is used to estimate the probability of breakdown, or breakdown risk. Such an approach takes advantage of the extraordinary wide variety of outcomes observed in the data set.

Before performing our simulation, we first collect flow signal estimates, $q_{i,t}$, for all i and t as identified via FDA traces. Then, we collect all σ_t for each day as identified from our SV model fitting procedure (above). Next we estimate the probability of breakdown by performing the following steps:

- 1) For some $t = t_0$, randomly select a $q_{t_0,i}$ ($i = 1, \dots, 205$).
- 2) For that same $t = t_0$, randomly select a $\sigma_{t_0,j}$ ($j = 1, \dots, 205$) and sample from $N(0, \sigma_{t_0,j})$ to determine d_{t_0} given by $D_{t=t_0} = q_{t=t_0,i} + N(0, \sigma_{t=t_0,j})$.
- 3) Draw a sample from the fitted GEV to obtain c , an estimate for the capacity of the roadway.
- 4) If $d_{t_0} > c$, then record this sample as a breakdown.
- 5) Repeat the process 1,000 times, recording the breakdown state for each sample. The number of breakdowns among the 1,000 samples is denoted $n_{t=t_0}$.
- 6) Calculate the empirical probability for breakdown, $P(D_{t=t_0} \geq C)$, at time $t = t_0$ given by $n_{t=t_0}/1000$. We denote this value, the probability of breakdown at time $t = t_0$, as $p_{t=t_0}$.
- 7) Repeat the process for all t ($t = 1, \dots, 96$) to find all p_t .

As a comparison, to determine the effect of SV on breakdown probability, a second simulation was performed. In this simulation, however, the $q_{t,i}$ were sampled from the raw flow aggregates (as opposed to the FDA signals) and no additional volatility component (SV) was considered. This flow value is then compared to a sampled capacity value (as above) to determine breakdown state. The process is repeated 1,000 times for each time step to identify empirical probabilities of breakdown. Finally, this breakdown probability was compared to that obtained using the SV. Figure 6 below compares the probability estimates for the two sampling procedures by time of day.

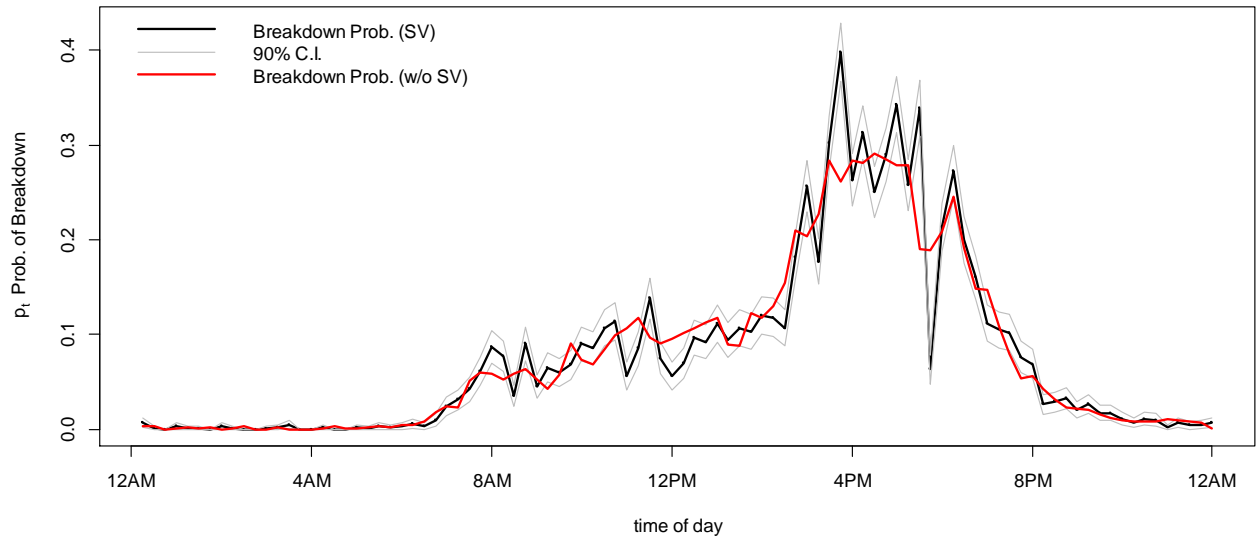


FIGURE 6 Probability of breakdown (p_t) by time of day as calculated by SV model (black) and sampling without considering SV (red). Confidence bounds for SV model predictions are included as well (grey lines).

The effect of introducing noise into our breakdown prediction is quite stark. When modeling the volatility separately and combining it with a flow signal, the prediction of breakdown itself becomes very noisy, much noisier, in fact, than probabilities based on the raw flow data. We observe from Figure 6 that the SV-based predictions of breakdown are not smooth, but tend to fluctuate wildly, especially at known volatile times of the day (morning/evening commutes). This seems to be in keeping with the ‘random’ nature of breakdown occurrence, where breakdowns cannot be predicted based solely on traffic flow.

CONCLUSION AND DISCUSSION

In this paper we have outlined a procedure to predict highway breakdowns based on flow values. The innovative part of our approach is the independent modeling of flow volatility through the use of a stochastic volatility model, a model form commonly used in economic time-series analyses. Functional data analysis (FDA) is used to extract realistic flow signals by time of day. These flow signals are combined with estimates of SV to produce component estimates of demand. In the end, when compared to estimates based on observed data, our estimation procedure yields dissimilar demands and ultimately dissimilar predictions of breakdown. We have simply introduced a procedure, but, going forward, a more thorough simulation-based study should be performed to evaluate the predictive ability of our approach. That is, many of our conclusions are based on visual analysis, and more thorough statistical testing of our model should be pursued. The application of our study is clear: estimation of probabilistic prediction of breakdown can be a valuable tool for decision-makers and administrators. At the very least, results suggest that volatility plays a role in this breakdown, and that the SV-model is a suitable approach to estimate this volatility.

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