

# STOCHASTIC APPROXIMATIONS FOR THE MACROSCOPIC FUNDAMENTAL DIAGRAM

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## 1. ABSTRACT

This paper proposes analytical estimates of a network Macroscopic Fundamental Diagram (MFD) using probabilistic methods. It produces a distribution for the MFD, found to depend mainly on two parameters. This abstract focuses in Fig. 1, which shows the comparison between the MFDs from an exact traffic simulation and the 90% probability interval from our analytical method.

Existing analysis methods for the MFD fall into three types: (i) empirical [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12], (ii) analytical [13, 14, 15], and (iii) simulation [16, 17, 18, 19, 20, 21, 22]. Analytical results are based on the method of cuts for homogeneous corridors where all intersections “see” the same set of cuts, so the cuts from a single intersection suffice to compute the MFD for the whole corridor. Despite this apparent simplicity, this approach quickly becomes intractable and one needs to resort to simple but numerical algorithms to identify all possible cuts. Additionally, the homogeneous corridor method cannot be scaled up without complications to estimate the network MFD mainly because a network path cannot be guaranteed to have constant offset all along, even for homogeneous networks.

In this paper we introduce the concept of probabilistic homogeneous corridors, where all intersections see the same distribution of cuts. This allows tackling deterministic inhomogeneous corridors with different block length, signal timing, etc.

The example presented in Fig. 1 are networks with random offsets, while the general method will be presented in the final paper. The general method can be used to randomize other variables of our problem such as block length and signal timing, which is important because it allows the analytical estimation of MFDs on inhomogeneous networks, for which analytical methods are currently unavailable.

Random offset networks could be a good approximation for networks with traffic actuated signals or without coordination, and Fig. 1 shows that it provides tight bounds for the deterministic problem. The dimensionless formulation of this problem provides new insights, e.g.: (i) only two parameters determine the MFD distribution, (ii) the distribution of network capacity depends on a single parameter, (iii) the existence of a symmetry line in the MFD, which is crucial for mathematical tractability, and (iv) the block length does not affect the MFD when it is larger than twice the critical block length.

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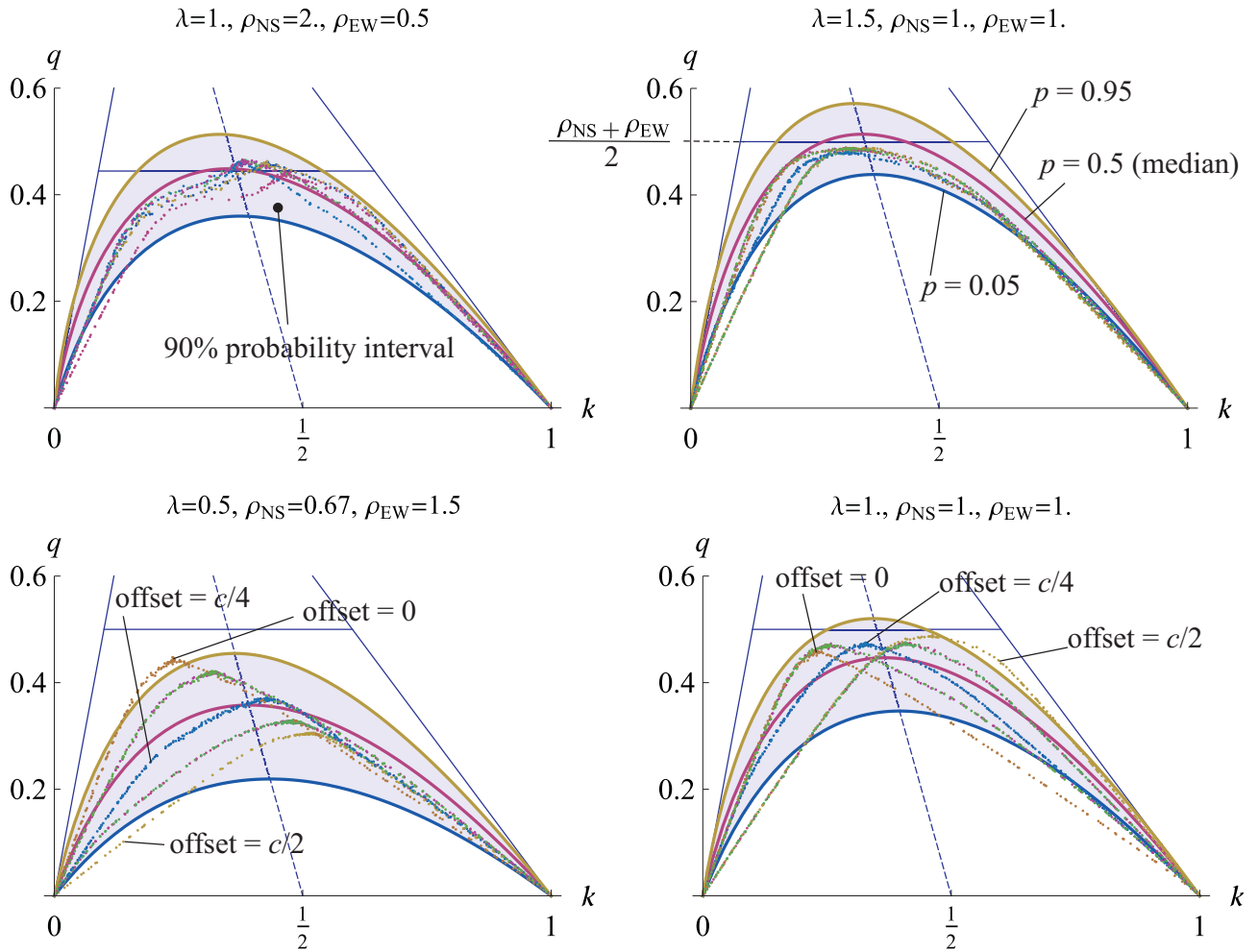


Figure 1: Comparison between the MFDs produced by an exact traffic simulation and the 90% probability interval produced by our analytical method.

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