Optimal Velocity Model with Dual Boundary Optimal Velocity Function

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Outline

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3 GENERAL DUAL BOUNDARY OPTIMAL VELOCITY MODEL

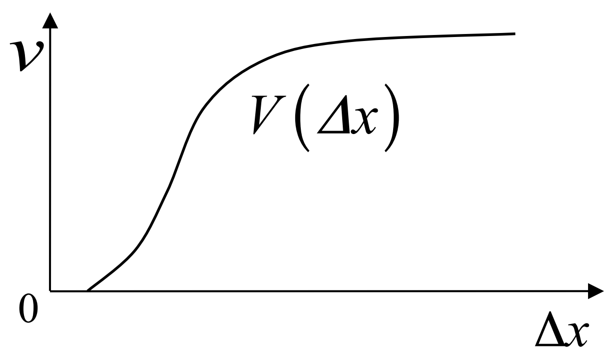
4 SUMMARIES

Optimal Velocity Model (OVM, *Bando et al. 1995*)

- Contains an equilibrium (steady) speed-spacing relation
- No explicit time delay
- Simple structure, convenient for analytical analysis

Governing Equation $$\ddot{x}_n(t) = \kappa\left\{V\left(\Delta x_n\right) - \dot{x}_n(t)\right\}$$

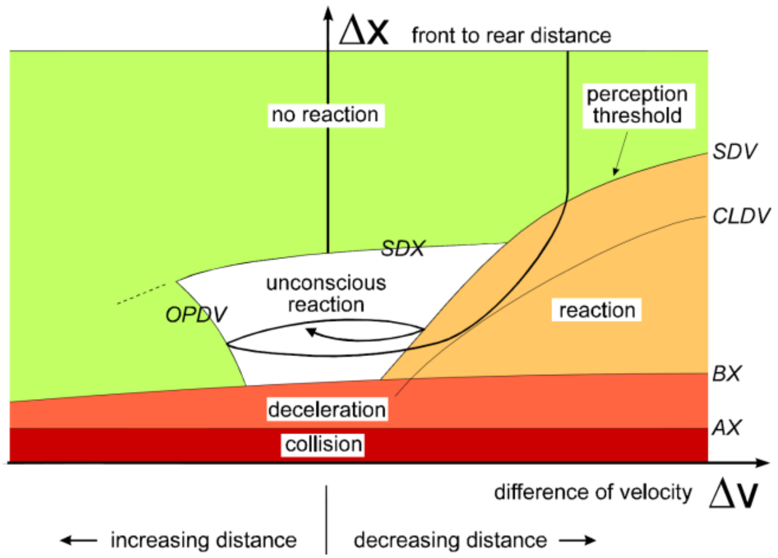
Optimal Velocity Fun. $$V(\Delta x) = V_1 + V_2 \tanh\left[C_1 \Delta x - C_2\right]$$

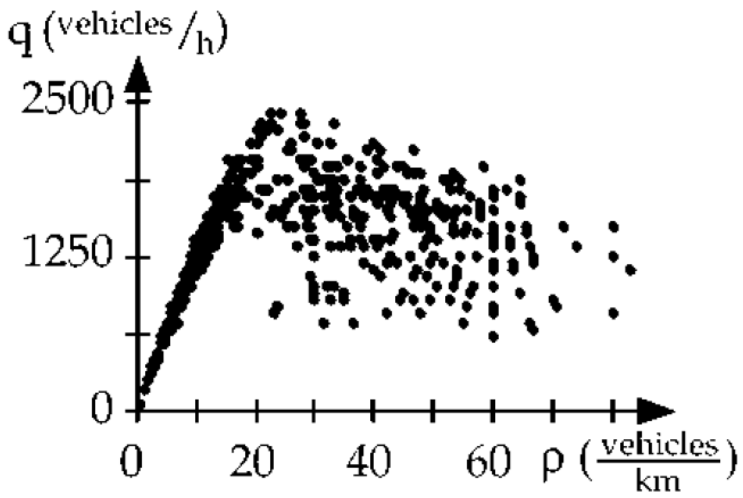


Assumption:
one to one correspondence between the spatial headway and the optimal driving speed in steady traffic state

Questions:

- **Drivers are satisfied with a range of conditions instead of an accurate optimal performance (*Boer 1999*)**
- **The two-dimension zone in the "spacing-relative speed" diagram (*Psychophysical or Action Point models*)**
- **Wide scattering in the fundamental diagram (*Kerner and Rehborn 1996~2003*)**



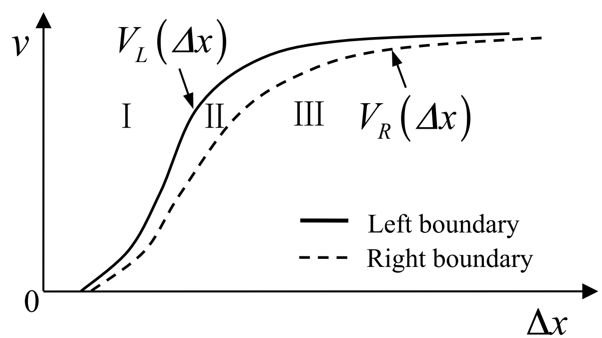


2. Basic DBOVM

Basic **Dual-Boundary-Optimal-Velocity-Model (DBOVM)** is proposed as follows:

$$\ddot{x}_n(t) = \begin{cases} \kappa\left\{V_L(\Delta x_n) - \dot{x}_n(t)\right\} & if: \dot{x}_n(t) > V_L(\Delta x_n) \\ 0 & if: V_R(\Delta x_n) \le \dot{x}_n(t) \le V_L(\Delta x_n) \\ \kappa\left\{V_R(\Delta x_n) - \dot{x}_n(t)\right\} & if: \dot{x}_n(t) < V_R(\Delta x_n) \end{cases}$$

Here $V_L(\Delta x_n)$ and $V_R(\Delta x_n)$ are the Optimal-Velocity-Functions (OVF) of left boundary and right boundary respectively:



2. Basic DBOVM

Bando et al. **proposed a *S*-shape Optimal-Velocity-Function in their work (*1995* & *1998*):**

$$V(\Delta x) = V_1 + V_2 \tanh\left[C_1 \Delta x - C_2\right]$$

Based on that, a simple Dual-Boundary-Optimal-Velocity-Function is established as follows:

$$V(\Delta x) = V_1 + V_2 \tanh\left[C_1 \Delta x - C_2\right] \qquad C_1 \in \left[C_{1R}, C_{1L}\right]$$

Left boundary: $$V(\Delta x) = V_1 + V_2 \tanh\left[C_{1L} \Delta x - C_2\right]$$

Right boundary: $$V(\Delta x) = V_1 + V_2 \tanh\left[C_{1R} \Delta x - C_2\right]$$

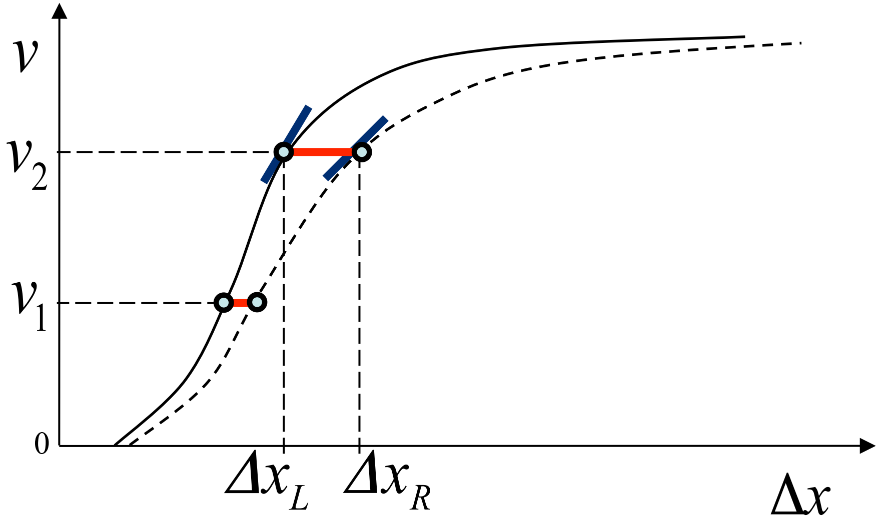
This simple **Dual-Boundary-Optimal-Velocity-Function** has two reasonable properties:

(1) The steady range increases with the speed increasing

$$\frac{d}{dv}\left(V_R^{-1}(v)-V_L^{-1}(v)\right)>0$$

(2) The derivative at the left boundary is larger than that at the right boundary

$$\frac{dV_L(\Delta x)}{d\Delta x}\Big|_{\Delta x=\Delta x_L} > \frac{dV_R(\Delta x)}{d\Delta x}\Big|_{\Delta x=\Delta x_R}$$



$$a_{dec}=\kappa\left[V_L(\Delta x_L-\delta)-v_e\right]$$

$$a_{dec}=-\kappa\delta V_L'(\Delta x_L)$$

$$a_{acc}=\kappa\delta V_R'(\Delta x_R)$$

$$|a_{dec}|\geq|a_{acc}|$$

Condition of the simulations:

- **Model parameters are set as in Bando's original studies:**

  $V_1 = 15.3\ \text{m/s}\quad V_2 = 16.8\ \text{m/s}\quad C_1 = 0.086\ \text{m}^{-1}\quad C_2 = 2.1\quad \kappa = 2.0\ \text{s}^{-1}\quad C_{1L} = 0.088\ \text{m}^{-1}\quad C_{1R} = 0.076\ \text{m}^{-1}$

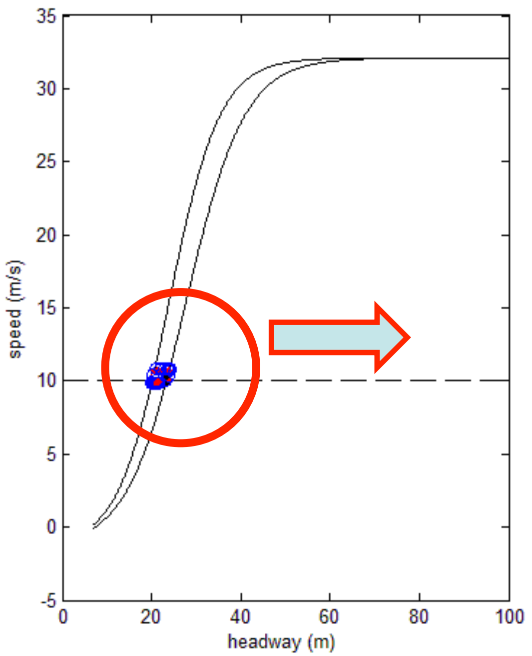
- **Three vehicles are considered in the local stability studies**

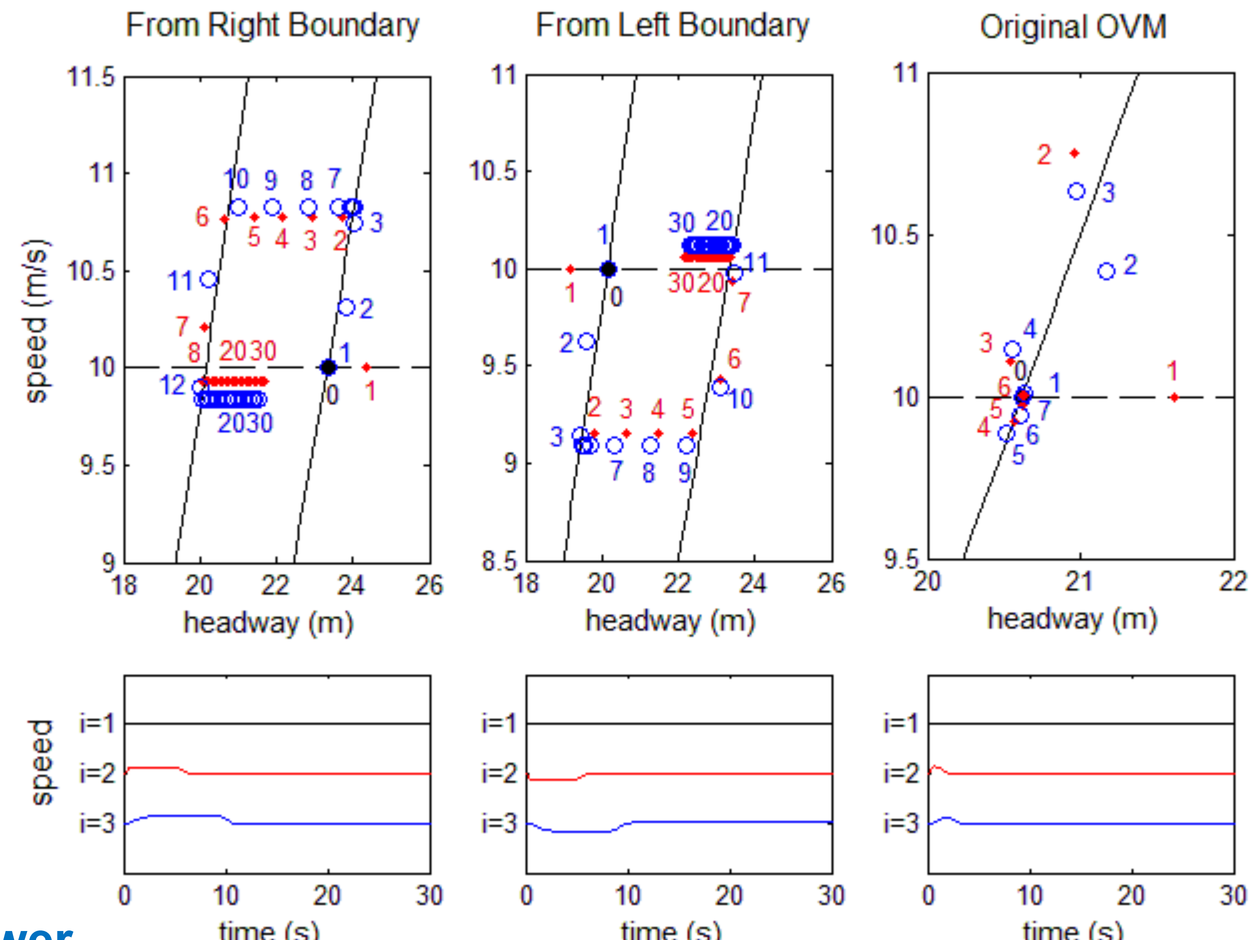
- **The initial state of vehicles should satisfy either the left or the right boundary optimal velocity function (initial speed = 10 m/s)**

- **A small perturbation is added on the leading vehicle by giving its position an instantaneous change (either increasing by 1 m or reducing by 1m)**

- **Time step = 0.1 sec**

Results of the simulations:



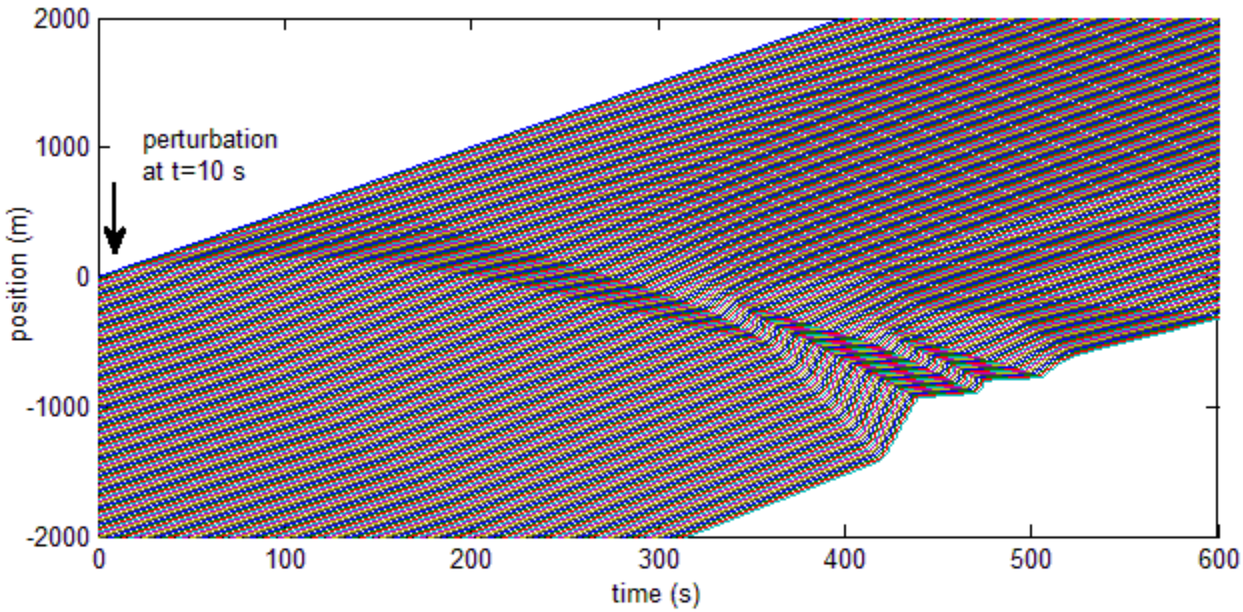


Red: the first follower
Blue: the second follower

Condition of the Simulations:

- Same model parameters as in local stability studies
- 200 vehicles in the platoon
- Initial speed = 5m/s; initial spacing = 18.2 m (on the right boundary)
- The initial condition satisfies the string stability criterion of OVM $V'(\Delta x) \le \kappa/2$

Stop-and-go is found during the evolution



Model Modification

- Introducing the speed adjustment mechanism into Basic DBOVM
- Drivers are allowed to adjust their driving speeds towards the speed of leading vehicles within the dual boundary steady region
- It reduces to Basic DBOVM when $\lambda = 0$

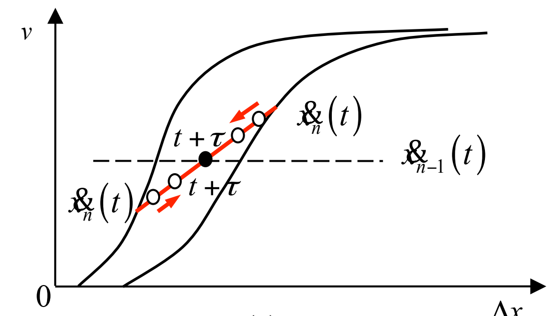
$$\ddot{x}_n(t) = \begin{cases} \kappa\left\{V_L(\Delta x_n) - \dot{x}_n(t)\right\} & if: \dot{x}_n(t) > V_L(\Delta x_n) \\ \lambda\left\{\dot{x}_{n-1}(t) - \dot{x}_n(t)\right\} & if: V_R(\Delta x_n) \le \dot{x}_n(t) \le V_L(\Delta x_n) \\ \kappa\left\{V_R(\Delta x_n) - \dot{x}_n(t)\right\} & if: \dot{x}_n(t) < V_R(\Delta x_n) \end{cases}$$

Path of State Transition

Follower's acceleration: $\ddot{x}_n(t) = \lambda\left\{\dot{x}_{n-1}(t) - \dot{x}_n(t)\right\}$

Speeds at next time step: $\dot{x}_n(t+\tau) = \dot{x}_n(t) + \tau\ddot{x}_n(t)$

$$\dot{x}_{n-1}(t+\tau) = \dot{x}_{n-1}(t) + \tau\ddot{x}_{n-1}(t)$$



(a)

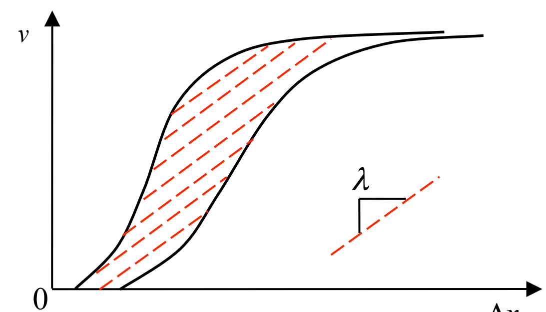
Spacing at next time step:

$$\Delta x_n(t+\tau) - \Delta x_n(t) = \tau\left\{\dot{x}_{n-1}(t) - \dot{x}_n(t)\right\} + \frac{\tau^2}{2}\left\{\ddot{x}_{n-1}(t) - \ddot{x}_n(t)\right\}$$

Slope of the state transition path:

$$\beta = \frac{\dot{x}_n(t+\tau) - \dot{x}_n(t)}{\Delta x_n(t+\tau) - \Delta x_n(t)}$$

$$\boxed{\beta = \frac{1}{\frac{1}{\lambda} + \frac{\tau}{2}\left(\frac{\ddot{x}_{n-1}(t)}{\ddot{x}_n(t)} - 1\right)} \approx \lambda}$$

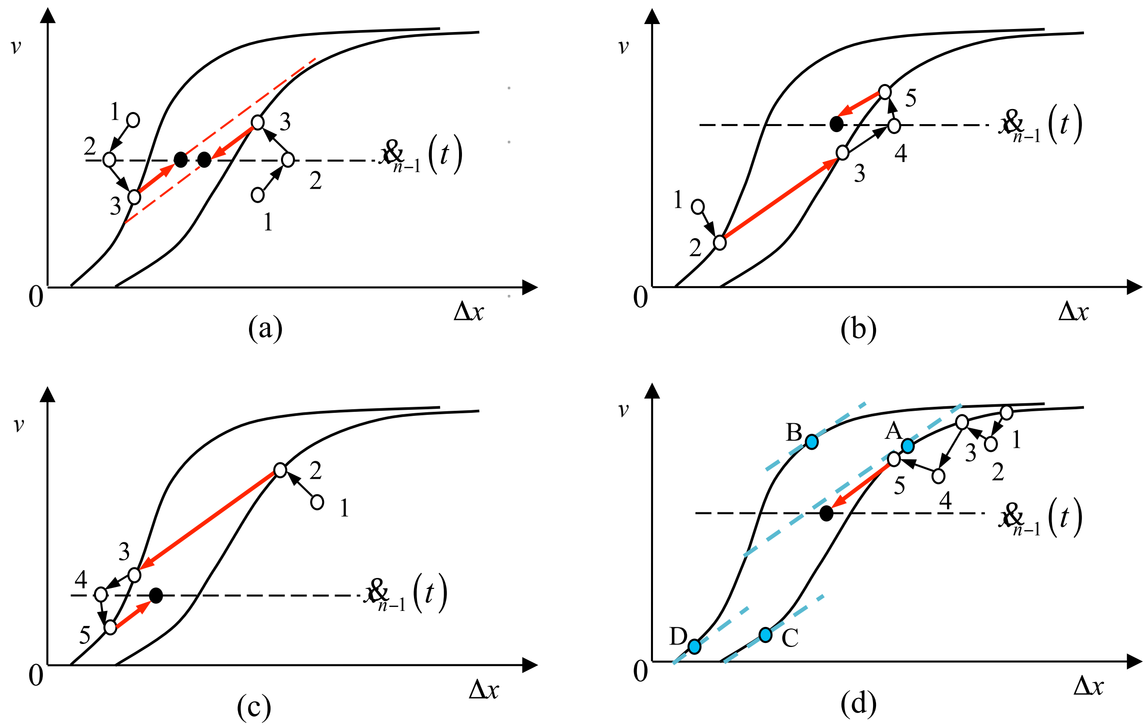


(b)

The slope approximates to λ when (1) time step τ approaches zero; (2) follower and leader have similar accelerations

Some Typical Scenarios of State Transition

- (Fig. a) The follower reaches the steady state directly
- (Fig. b & Fig. c) The traffic state first moves across the dual-boundary-region, and then reaches the steady state
- (Fig. d) The traffic state goes zigzag against the boundary



Convergence of Traffic State

Suppose $\dot{x}_n(t_0) > \dot{x}_{n-1}(t_0)$

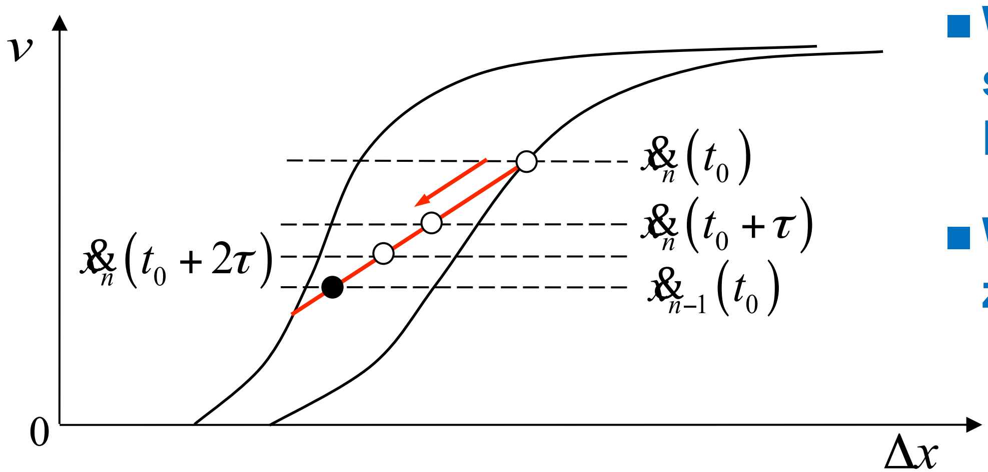
Then, $\dot{x}_n(t_0+\tau) = \dot{x}_n(t_0) - \lambda\tau\left\{\dot{x}_n(t_0) - \dot{x}_{n-1}(t_0)\right\}$

By recursive method, speed of the *n*th vehicle at time $t_0 + m\tau$

$$\dot{x}_n(t_0+m\tau) = \dot{x}_n(t_0)(1-\lambda\tau)^m - \dot{x}_{n-1}(t_0)\left\{(1-\lambda\tau)^m - 1\right\}$$

The speed difference at time $t_0 + m\tau$ can be expressed as:

$$\boxed{\Delta\dot{x}_n(t_0+m\tau)} = \dot{x}_{n-1}(t_0) - \dot{x}_n(t_0+m\tau) = \left\{\dot{x}_{n-1}(t_0) - \dot{x}_n(t_0)\right\}(1-\lambda\tau)^m \boxed{= \Delta\dot{x}_n(t_0)(1-\lambda\tau)^m}$$

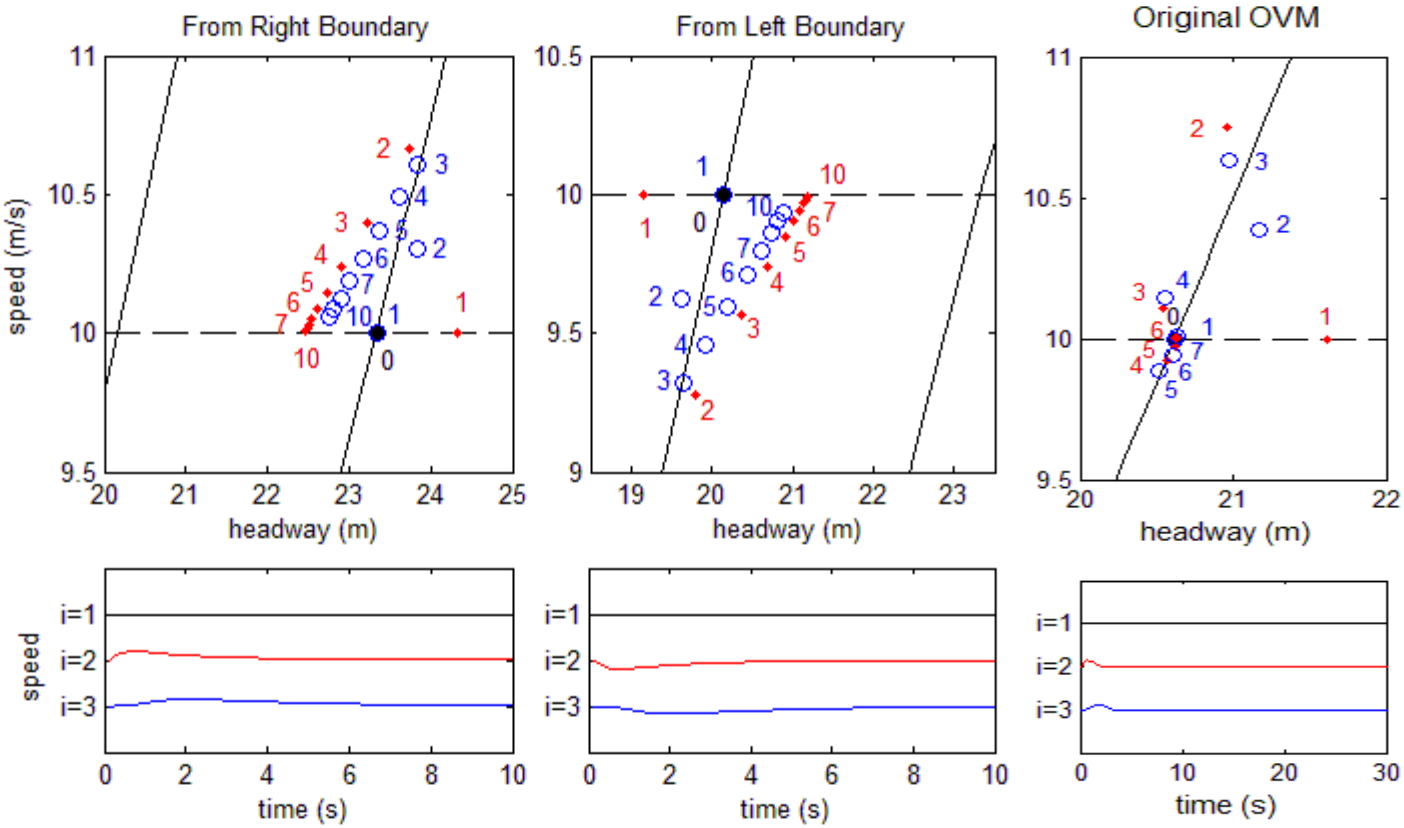


- **When λ=0.5, τ=1sec, the speed difference drops by half per second .**
- **When time step approaches zero**

$$\boxed{\lim_{\tau\to 0}(1-\lambda\tau)^{\frac{1}{\tau}} = \frac{1}{e^{\lambda}}}$$

3. General DBOVM

- Use the same model parameters as in aforementioned studies. Additionally, let λ=0.5.
- The new model shows a reasonable performance in convergence to the steady state.



Red: the first follower

Blue: the second follower

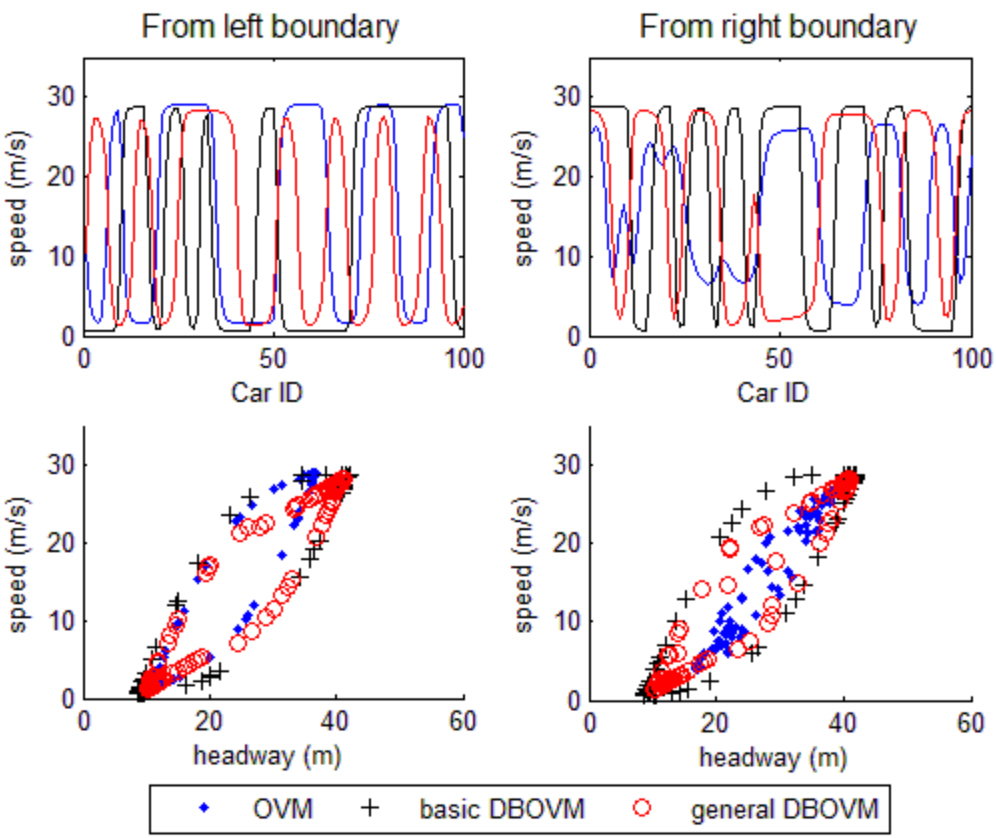
Condition of the Simulations:

- The periodical ring road is used as the simulation condition for studies on string stability.
- 100 vehicle are involved in the ring road simulation.
- Both the basic DBOVM (λ=0) and the general DBOVM (λ=0.5) are simulated.
- Time step = 0.1 sec

Two Studies:

- Comparison of the hysteresis features across the original OVM, basic DBOVM and general DBOVM.
- Comparison of string stability regions across the three models.

- The initial speed of the platoon is 16 m/s, which is sting unstable for both the left boundary and the right boundary under the law of OVM.
- The hysteresis loop produced by the basic DBOVM is the largest one, the general DBOVM is smaller, and the original OVM is the smallest .



Snapshots at t=5000

- The dual boundary steady region in the DBOVM has similar effect as the explicit delay for the original OVM (*Bando et al. 1998*).
- The speed adjustment mechanism in the general DBOVM restrains such effect.

- Find the unstable regions of DBOVM by trial and error .
- A long simulation time (100000 s) is used, in order to ensure that the perturbation has sufficient time for evolution.
- The final steady speed is higher than the initial one in the order of 0.1 m/s, when the platoon starts from the right boundary. For the left boundary, the situation is quite the reverse.

TABLE 1 String Unstable Regions in General DBOVM and Basic DBOVM

	Left Boundary OVF $V(\Delta x)=15.3+16.8\tanh[0.088\Delta x-2.1]$	Right Boundary OVF $V(\Delta x)=15.3+16.8\tanh[0.076\Delta x-2.1]$
General DBOVM $\lambda=0.5$	$\Delta x\in(17.9,\ 31.7)$	$\Delta x\in(21.1,\ 31.8)$
	$v\in(7.2,\ 25.3)$	$v\in(7.6,\ 20.5)$
Basic DBOVM $\lambda=0$	$\Delta x\in(14.3,\ 34.9)$	$\Delta x\in(15.4,\ 40.3)$
	$v\in(3.8,\ 27.9)$	$v\in(3.0,\ 27.8)$
OVM	$\Delta x\in(16.5,\ 31.2)$	$\Delta x\in(22.0,\ 35.2)$
	$v\in(5.7,\ 24.9)$	$v\in(8.5,\ 24.0)$

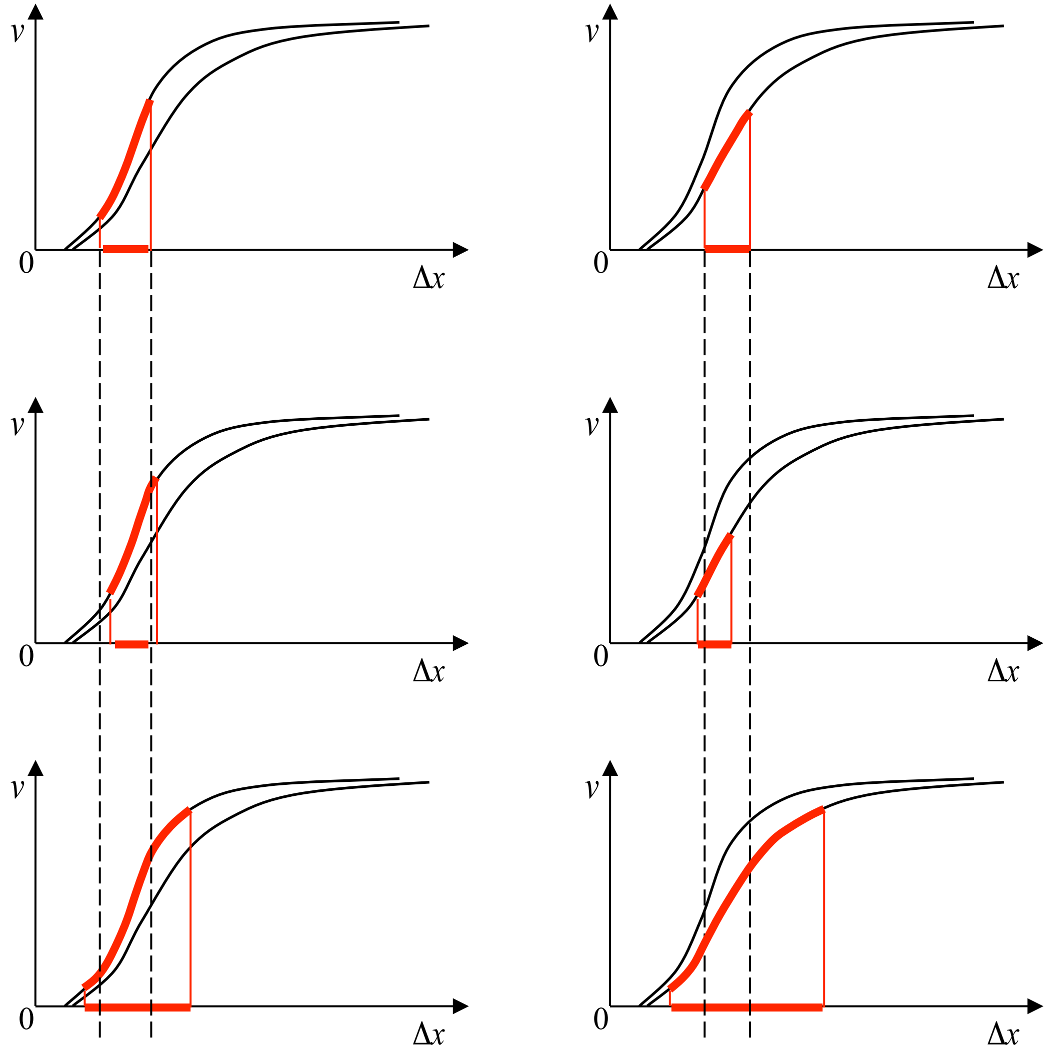
String Stability Features

Left boundary | Right boundary

OVM

General DBOVM
λ=0.5

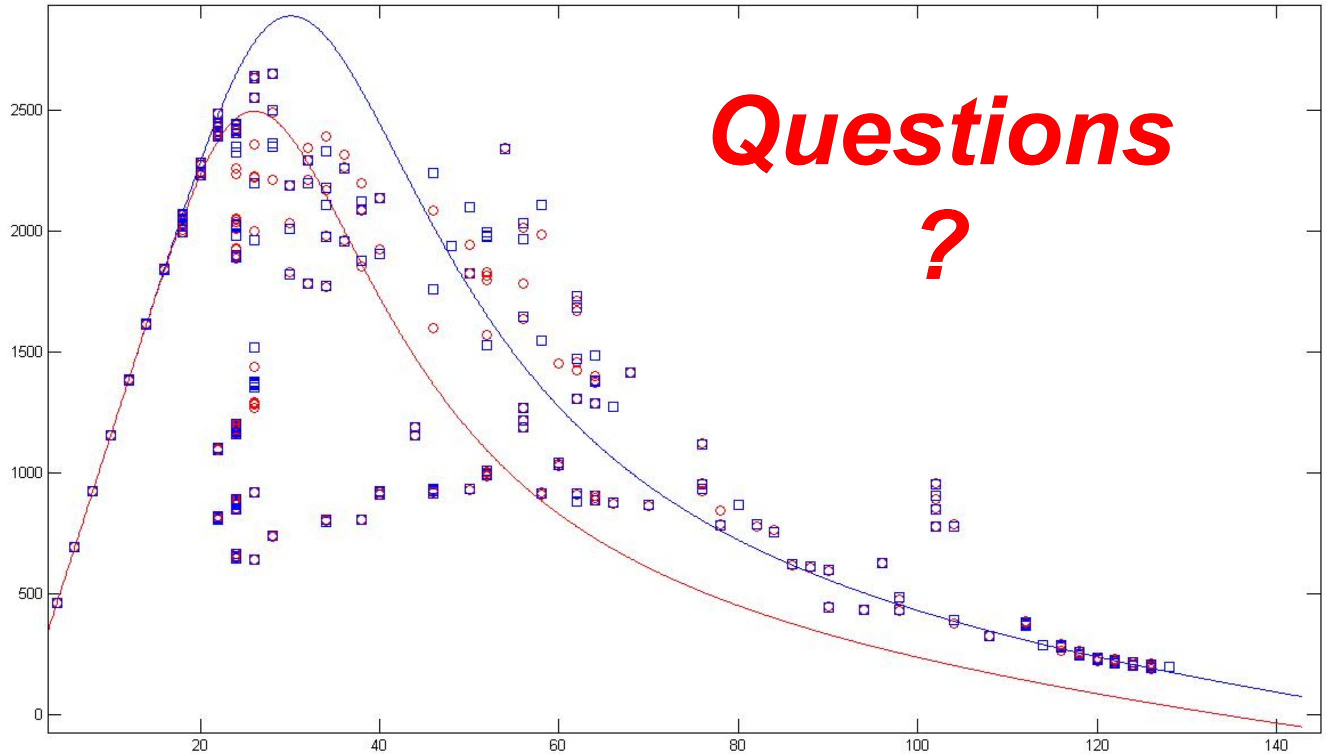
Basic DBOVM
λ=0



4. Summaries

- This paper propose a Dual-Boundary-Optimal-Velocity-Model (DBOVM), which contains a dual-boundary-steady-region instead of the optimal speed-spacing relationship in the original OVM .
- The speed adjustment mechanism is an important part in the DBOVM, which allows traffic reaches the steady state everywhere inside the dual-boundary-steady-region.
- The path of traffic state transition is found within the dual-boundary-steady-region, with the slope equals to the sensitivity parameter of the speed adjustment term.
- The dual-boundary-steady-region in DBOVM has the hysteresis effect. The wider the region is, the stronger the hysteresis effect will be.
- The speed adjustment effect in general DBOVM restrains the hysteresis and improves the stability of traffic.
- The dual-boundary-steady-region in the general DBOVM allows the traffic flow to reach a new steady state slightly apart from the initial one under the effect of small perturbation.

- Explore the string stability condition of the DBOVM in an analytical way
- Calibrate the DBOVM with real traffic data
- Use the DBOVM to simulate real traffic phenomena.



Questions

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