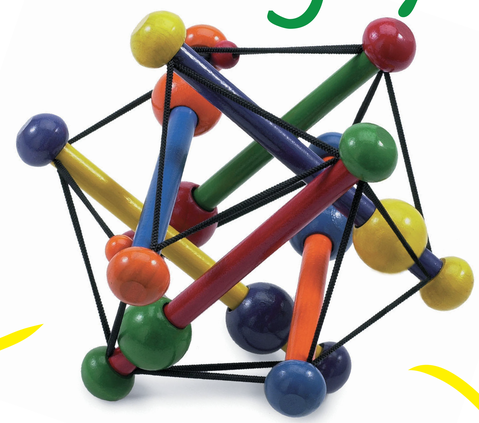
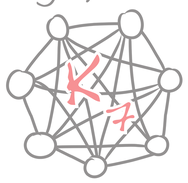
planar graphs

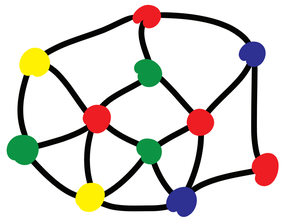


minor-excluded graphs



exclude $K_{3,3}$ and K_5

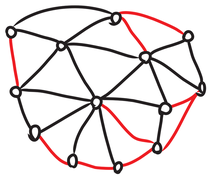
4-colourable



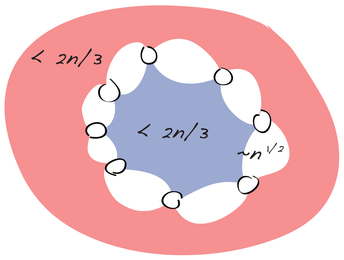
Euler's formula

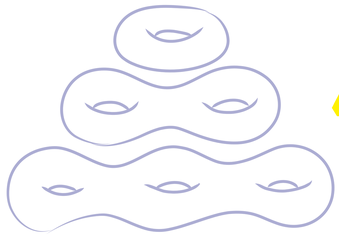
=> $n < 3m$
(sparse)

EX: linear-time MST by repeatedly contracting min-weight edge adjacent to lowest-degree vertices



small separators

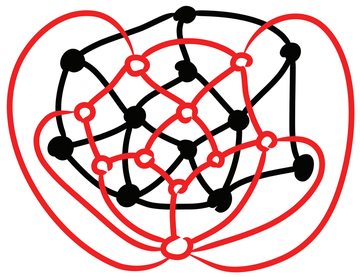




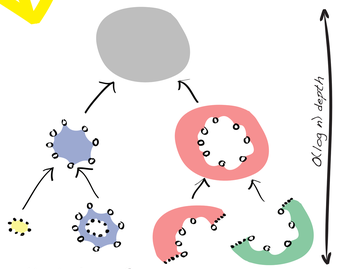
embedded graphs

= planar!

duality

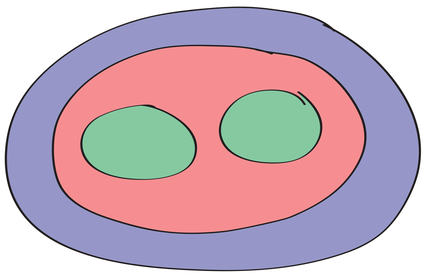


power networks



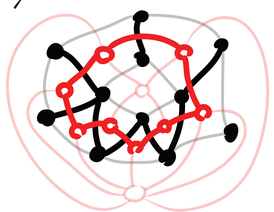
divide & conquer

decomposition



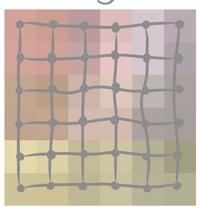
break graph into pieces using dual BFS
solve problem separately in each piece

cycle-cut duality

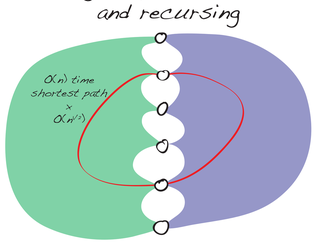


dual spanning trees

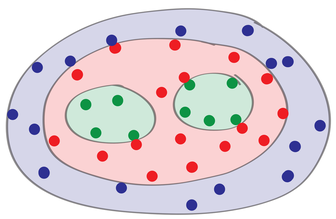
images



EX: $O(n^{1.5} \log n)$ time for girth by finding shortest cycles through separator vertices and recursing

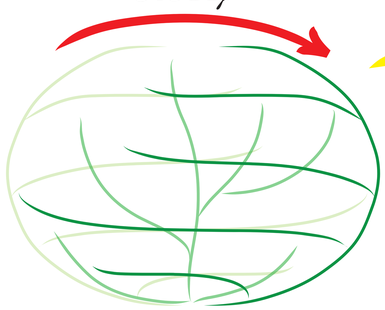


EX: 1-eps approximation for vertex cover, etc.



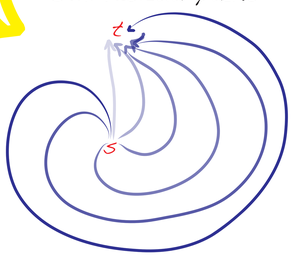
bigger pieces = less error, longer to solve

sweep



build solution by sweeping through graph guided by dual spanning trees

EX: $O(n \log n)$ max flow by saturating $O(n)$ residual paths



road networks

Glencora Borradaile -- glencora.org