

**Directions:** You have 30 minutes to complete this quiz. A score of 70% or above will receive full credit.

1. In each of the following cases, state whether  $f = O(g)$ ,  $f = \Theta(g)$ , or  $f = \Omega(g)$ .

(a)  $f(n) = \log n$        $g(n) = \log(n^2)$

(b)  $f(n) = 10\sqrt{n}$        $g(n) = (\log n)^8$

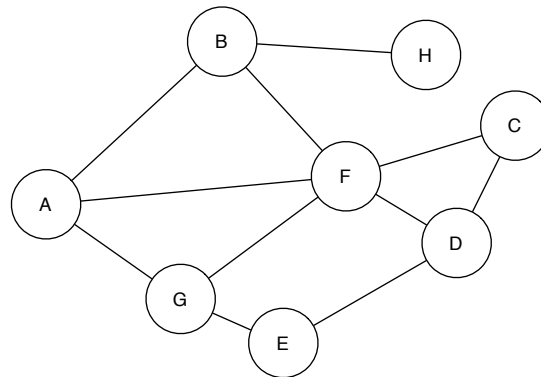
(c)  $f(n) = n! = (n)(n-1)(n-2) \dots (3)(2)(1)$        $g(n) = 2^n$

2. Solve the following recurrence relations.

(a)  $T(n) = 8T(n/2) + O(n^2)$  and  $T(1) = O(1)$

(b)  $T(n) = T(n-1) + n$  and  $T(1) = O(1)$

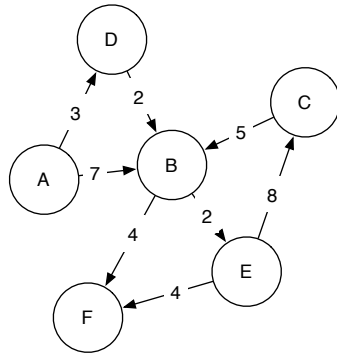
3. Consider the following graph.



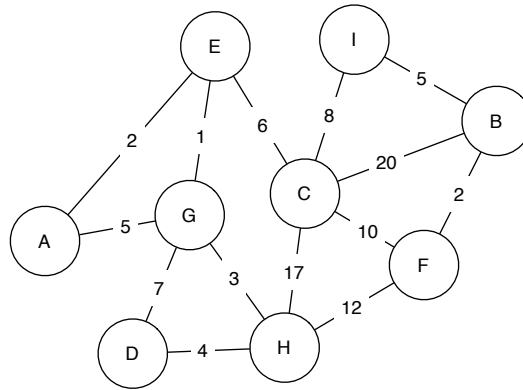
- (a) Perform depth-first search from vertex  $A$ . When there is a choice of vertices pick the one that is alphabetically first ( $A, B, C, D, E, F, G, H$ ).
- List the vertices in the order they are first visited (ie. by pre-order).
  - List the vertices in the order they are last visited (ie. by post-order).

- (b) Perform breadth-first search starting with vertex  $A$ . List the vertices in the order they are visited.

4. Find the shortest path tree of the following graph rooted at the vertex *A*.



5. Find a minimum spanning tree of the following graph. What is the cost of this tree?



6. Suppose you were going to prove the statement: “In a complete binary tree, the number of leaves is exactly one more than the number of internal nodes.” by induction. What would your inductive hypothesis and base case be?