This gives an example of a formal proof as expected in projects and tests.

A perfect binary tree is a binary tree in which every level is completely filled. The depth of a tree is the maximum number of hops a vertex is from the root.

Claim 1. *A perfect binary tree with n leaves has depth $\lfloor \log_2 n \rfloor$.*

Proof. Let n be an **arbitrary** integer ≥ 1 and let T be a perfect binary tree with n leaves.

Base case: If $n = 1$, then the tree is a single vertex has depth $\lfloor \log_2 1 \rfloor = 0$.

Inductive hypothesis: Assume that a perfect binary tree with k leaves where $1 \leq k < n$ has depth $\lfloor \log_2 k \rfloor$.

Applying the axiom of induction: Remove the leaves of the tree to create a new tree T'. T' has $n/2$ leaves, since each leaf of T' has exactly two children in T because T is perfect.

T has depth one more than that of T' and the depth of T' is $\lfloor \log_2 \frac{n}{2} \rfloor$ by either the base case (if $n = 2$) or the I.H. (if $n > 2$). The depth of T is therefore:

$$\lfloor \log_2 \frac{n}{2} \rfloor + 1 = \lfloor \log_2 n - 1 \rfloor + 1 = \lfloor \log_2 n \rfloor$$

$\square$