Practice Assignment 6

Due: Thursday, March 12 at 2PM to TEACH

To get credit, each student must submit their own solutions (which need not be typeset) to TEACH by the due date above – no late submissions are allowed. Note that while you must each submit solutions individually, you are free to work through the problem sets in groups. Solutions will be posted shortly after class and may be discussed in class. These will be not be graded on the basis of *completion* alone, not *correctness*.

Note: it would make more sense to attempt question 2 after Tuesday's lecture.

1. (a) I have a problem X that I can solve by using 3SAT as a black box, with an additional polynomial amount of time. (That is, problem X poly-time reduces to 3SAT.) Are the following statements true or false/unknown?
   i. X is in NP.
   ii. X is NP-hard.

   (b) I have two problems, X and Y. 3SAT reduces to problem X and problem X reduces to problem Y. (These reductions are poly-time reductions.) Is the following statement true or false/unknown?
   i. Y is NP-hard.

   (c) Problem X and problem Y are both NP-complete. Is the following statement true or false/unknown?
   i. Problem X reduces to problem Y and vice versa.

2. We are feeling experimental and want to create a new dish. There are various ingredients we can choose from and we'd like to use as many of them as possible, but some ingredients don't go well with others. If there are n possible ingredients (numbered 1 to n), we write down an $n \times n$ binary matrix where the (i, j) entry is 1 if i and j can go together and 0 otherwise. Notice that this matrix is necessarily symmetric; and that the diagonal entries are always 0.

   We wish to solve the following problem:

   EXPERIMENTAL CUISINE :

   input: n, the number of ingredients to choose from; B, the $n \times n$ binary matrix that encodes which items go well together

   output: the maximum number of ingredients which can be selected together

   Show that if EXPERIMENTAL CUISINE can be solved in polynomial time, then P=NP.