

water-gun induction Consider the following game for an odd number of school children. Each child stands at a point such that *no two distances between children are equal*. Each child has a water gun. Someone yells “go” and every child (simultaneously) sprays the person who is nearest to them. Prove that at least one person stays dry. Do this by proving that the statement *there is a dry survivor in every odd water-gun fight with $2n - 1$ children* is true by induction on n . In the inductive step, how can you build a solution for $n = k + 1$ from $n = k$?

internal nodes and leaves Prove the following statement by induction:

In any complete binary tree (a tree in which every vertex either has two children or is a leaf), the number of leaves is exactly one more than the number of internal nodes.

A node is a leaf if it has no children; otherwise, it is an internal node. *Clearly indicate the three steps in your inductive proof.*