Cross-Layer Optimized Conditions for QoS Support in Multi-Hop Wireless Networks with MIMO Links

Bechir Hamdaoui and Parameswaran Ramanathan

***Abstract*— Recent advances in antenna technology made it possible to build wireless devices with more than one antenna at affordable costs. Because multiple antennas offer wireless networks a potential capacity increase, they are expected to be a key part of next-generation wireless networks to support the rapidly emerging multimedia applications characterized by their high and diverse QoS requirements. In this paper, we develop methods that exploit the benefits of multiple antennas to enable multi-hop wireless networks with flow-level QoS capabilities. We first propose a cross-layer table-driven statistical approach that allows each node to determine the amount of spatial reuse and/or multiplexing, offered by the multiple antennas, that are available to it. We then use the developed statistical approach to derive sufficient conditions under which flow rates are guaranteed to be feasible. The derived conditions are multi-layer aware in the sense that they account for cross-layer effects between the PHY and the MAC layers to support QoS at higher layers. We evaluate and compare the derived sufficient conditions via extensive simulations. We show that the conditions result in high flow acceptance rates when used in multi-hop wireless networking problems such as QoS routing and multicommodity flow problems. We also demonstrate the importance and the effect of considering cross-layer couplings into the development of flow acceptance methods.**

***Index Terms*— Cross-layer optimization, acceptance tests, QoS, MIMO, multi-hop wireless networks.**

I. Introduction

MULTIMEDIA applications have quality of service (QoS) needs typically expressed in terms of maximum allowed time delays and/or minimum required data rates. It is imperative that next generation wireless networks have mechanisms that provide multimedia applications with QoS assurances. Until recently, wireless networks were composed of nodes that are equipped with single omnidirectional antennas. However, due to advances in technology, it is now possible to build wireless nodes with more than one antenna. Multiple antennas provide wireless networks with significantly more capabilities than single antennas. They can potentially increase overall network throughput via spatial reuse of the spectrum by allowing multiple simultaneous communications in the same vicinity and/or via spatial division multiplexing by achieving high data rates. Because of their potential benefits, multiple antennas are expected to be an essential part of next-generation wireless networks to face and support the rapidly emerging multimedia applications characterized by their high and diverse QoS requirements.

There have been numerous studies on multiple antennas [1–5]. Most of the reported work, however, studied the benefits of multiple antennas from a physical layer standpoint. The study of ways on how to exploit these benefits at higher layers is more recent and still in its infancy [6–12]. In general, these recent studies have focused on developing medium access control (MAC) protocols that are suited for wireless networks when equipped with the multiple antenna technology [7, 8, 11, 12]. The proposed MACs aim at exploiting some of the offered benefits such as spatial reuse [11, 12] and spatial multiplexing [13] to provide better network utilization. In this paper, we develop methods that exploit the benefits of multiple antennas to enable multi-hop wireless networks with QoS capabilities. To the best of our knowledge, exploiting the multiple antenna benefits to achieve flow-level QoS in multi-hop wireless networks has not been addressed yet. We derive sufficient condition sets under which flow rates are feasible in that once the conditions are met, these rates are guaranteed to be achievable. The derived conditions are also multi-layer aware in the sense that they account for cross-layer effects between the PHY and the MAC layers to provide QoS routing capabilities at higher layers. Specifically, the conditions provide sufficiency of flow-level feasibility while accounting for physical constraints such as the nodes' maximum transmit powers, the multipath nature of the wireless environment, and the errors associated with the technique used by nodes to estimate the channel coefficients.

The derived cross-layer sufficient condition sets can then be used as acceptance (admission control) tests that QoS routing schemes can depend on to control flow admissibility into the network [14–16]. Another major area where the derived sufficient conditions can also be useful is the multicommodidy flow routing [17–21]. Multicommodity flow routing problems in multi-hop wireless networks are typically formulated as linear programs. For example, the work in [17, 18] uses the multicommodity flow technique to provide routing solutions for end-to-end flows with QoS requirements in multi-channel multi-hop wireless networks. These linear programming formulations, however, require models that capture network constraints such as those due to radio and interference limitations so that QoS needs are guaranteed to be satisfied. For the single

Manuscript received May 15, 2006; revised December 1, 2006. Bechir Hamdaoui is with the EECS Department at University of Michigan, Ann Arbor (e-mail: hamdaoui@eecs.umich.edu).

Parameswaran Ramanathan is with the ECE Department at University of Wisconsin, Madison (e-mail: parmesh@ece.wisc.edu).

Digital Object Identifier 10.1109/JSAC.2007.070504.

antenna equipped wireless networks, conditions capturing such constraints have been developed in [18, 22] and used, for example, in [17] to find feasible QoS routes. This work derives conditions that are suited for multi-hop wireless networks when equipped with multiple antennas. In this paper, we make the following contributions:

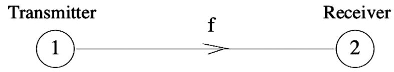
- We develop a table-driven statistical approach that allows nodes to determine the amount of spatial reuse and/or multiplexing, offered by the multiple antennas, that are available to them in their vicinities. The novelty of the approach is two-fold: (1) the approach accounts for cross-layer effects between PHY and MAC layers, and (2) the table can be computed off-line and indexed by some physical parameters that nodes are able to measure/monitor at runtime.
- We derive and prove three different sufficient condition sets for flow rates feasibility in multi-hop wireless networks using multiple antennas. The derived sets account for PHY and MAC coupling effects that are captured through the entries of the runtime-indexable table.
- We evaluate, compare, and analyze the three derived condition sets via simulations. We show that these derived sets result in high flow acceptance rates when used in multi-hop wireless networking problems such as QoS routing and multicommodity flow problems. We also demonstrate the importance and the effect of considering cross-layer couplings into the development of QoS methods.
- We discuss the applicability of these sets to multi-hop networking problems. Based on the obtained numerical and theoretical results, we make recommendation usage of each one of the three sets; this could serve as a design guideline.

In Section II, we provide insightful illustrations of the multiple antennas benefits that are useful for understanding the framework developed later in the paper. In Section III, we propose a statistical approach that determines amounts of spatial reuse and/or multiplexing in nodes' vicinities. In Section IV, we develop the flow-level sufficient condition sets. The effectiveness of the derived condition sets is carried out via extensive simulations in Section V. In Section VI, we discuss the applicability of these sets. Finally, we conclude the paper in Section VII.

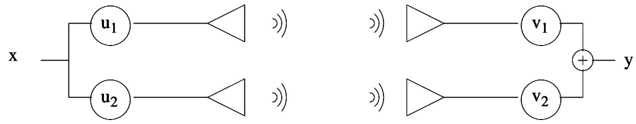
II. MIMO LINKS

A. Overview of Antenna Array Processing

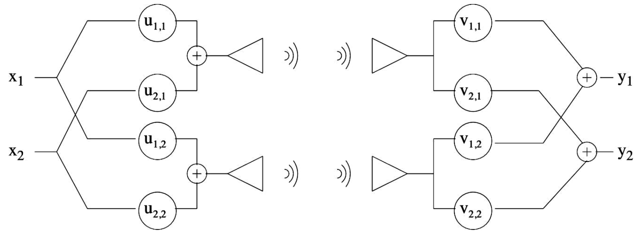
Consider the multiple-input multiple-output (MIMO) link, shown in Fig. 1(a), that consists of a transmitter and a receiver each with $K = 2$ antennas. To transmit a signal $x(t)$ over the 2-antenna array, the transmitter sends two weighted copies of the signal, one on each antenna. That is, $u_1 x(t)$ is sent over antenna 1, and $u_2 x(t)$ is sent over antenna 2; the vector $\mathbf{u} = [u_1\ u_2]^T$ (^{T} means matrix transpose), formed by the weights u_1 and u_2, is called *the transmitting weight vector*. At the receiver, the received two signals (one at each antenna) are weighted with a *receiving weight vector* $\mathbf{v} = [v_1\ v_2]^T$ and summed to produce $y(t)$. This is illustrated in Fig. 1(b). If we let $\mathbf{H}$ denote the matrix of channel coefficients between



(a) Node topology graph.



(b) One-stream flow f.



(c) Two-stream flow f.

Fig. 1. Antenna array processing.

the transmitter and the receiver, then one can write $y(t) = (\mathbf{u}^T\mathbf{H}\mathbf{v})x(t)$. Now by choosing appropriate weight vectors $\mathbf{u}$ and $\mathbf{v}$, one can ensure that the received signal $y(t)$ achieves a unit gain ($\mathbf{u}^T\mathbf{H}\mathbf{v} = 1$) if it is received at the desired receiver or a zero gain ($\mathbf{u}^T\mathbf{H}\mathbf{v} = 0$) if it is received at a non-desired receiver. Hence, with multiple antennas, a node can successfully communicate with its desired receiver while allowing other undesired nearby receivers to also successfully receive their signals. This is known as *spatial reuse*.

Multiple antennas can also be exploited to send multiple-stream signals. As shown in Fig. 1(c), the transmitter can send two signals (streams), $x_1(t)$ and $x_2(t)$, each of which is weighted over both antennas using the transmitting weight vectors $\mathbf{u}_1 = [u_{1,1}\ u_{1,2}]^T$ and $\mathbf{u}_2 = [u_{2,1}\ u_{2,2}]^T$, respectively. At the receiver, two separate streams, $y_1(t)$ and $y_2(t)$, are constructed by weighting the two received signals (one on each antenna) by two receiving weight vectors $\mathbf{v}_1 = [v_{1,1}\ v_{1,2}]^T$ and $\mathbf{v}_2 = [v_{2,1}\ v_{2,2}]^T$. Hence, in terms of equations, one can write $y_1(t) = (\mathbf{u}_1^T\mathbf{H}\mathbf{v}_1)x_1(t) + (\mathbf{u}_2^T\mathbf{H}\mathbf{v}_1)x_2(t)$ and $y_2(t) = (\mathbf{u}_1^T\mathbf{H}\mathbf{v}_2)x_1(t) + (\mathbf{u}_2^T\mathbf{H}\mathbf{v}_2)x_2(t)$. In this case, with an appropriate choice of all the weight vectors and under the assumption that $\mathbf{H}$ is a full-ranked matrix (we clarify this assumption later in the paper), one can ensure that $\mathbf{u}_1^T\mathbf{H}\mathbf{v}_1 = 1$ and $\mathbf{u}_2^T\mathbf{H}\mathbf{v}_1 = 0$ to correctly decode $y_1(t)$, and $\mathbf{u}_1^T\mathbf{H}\mathbf{v}_2 = 0$ and $\mathbf{u}_2^T\mathbf{H}\mathbf{v}_2 = 1$ to correctly decode $y_2(t)$. Hence, multiple antennas can also be exploited to send multiple-stream signals (two streams in the example). This is known as *spatial division multiplexing*.

In this paper, we assume that each node in the network is capable of estimating the coefficients of the wireless channel between it and any of its neighbors. Channel coefficient estimation methods are beyond the scope of this work; nodes

Fig. 2. Node topology graph

can use one of the traditional techniques to estimate channel coefficients [23]. We let σ_E^2 be the estimation error variance associated with the channel estimation technique. The variance σ_E^2 can be computed as $\frac{\sigma_n^2}{LE}$ where L is the length of pilot sequence per antenna, E is the transmit energy per pilot symbol per antenna, and σ_n^2 is the noise variance of channel [23]. We also assume that channel conditions exhibit only long term variability; i.e., channels coefficients can be assumed to be constant over durations in the order of those of packet communications; i.e., at the start of each packet communication, nodes update their channel coefficients and assume them to be the same until the communication of the packet ends. We further assume that the wireless environment is multipath (rich scattering conditions) and hence the matrix $\mathbf{H}_{m,n}$ of channel coefficients between any pair (m,n) of nodes is of full rank. Under the multipath assumption, the elements of $\mathbf{H}_{m,n}$ can be modelled as Gaussian i.i.d random variables with zero mean and unit variance [24]. Throughout, we use $\mathbf{u}_{m,i}$ to denote the $K_m \times 1$ transmitting weight vector of node m that is used to transmit its i^{th} stream of data, where K_m is the number of elements of m's antenna array. The j^{th} element $(u_{m,i,j})$ of the vector $\mathbf{u}_{m,i}$ is used by m to weigh the i^{th} transmitted stream on the j^{th} element of the antenna array. If only one stream of data is being transmitted by m, the notation $\mathbf{u}_m$ will then be used to denote the transmitting weight vector. Also, $\mathbf{v}_{m,i}$ will be used to denote the $K_m \times 1$ receiving weight vector of node m that is used to receive its i^{th} stream of data. The j^{th} element $(v_{m,i,j})$ of the vector $\mathbf{v}_{m,i}$ is used by m to weigh the i^{th} received stream on the j^{th} element of the antenna array. If only one stream of data is being received by m, $\mathbf{v}_m$ will then be used instead.

B. Node Topology Graph

In this work, we study a multi-hop wireless network that consists of a finite set $\mathcal{N}$ of nodes where each node $m \in \mathcal{N}$ is equipped with an antenna array of K_m elements that the node uses to transmit and receive signals. Each node is also characterized by a transmission range defined as the furthest distance that the node's transmitted signal can reach. We let $\mathcal{L}$ denote the set of all pairs (m,n) of distinct nodes in $\mathcal{N}$ such that m and n are within each other's transmission range. An ordered pair of nodes (m,n) in $\mathcal{L}$ is said to form a flow f if node m needs to transmit to node n. We refer to node m as the transmitter of flow f and node n as the receiver of flow f. f is said to be *active* if m is currently transmitting to n; otherwise, f is said to be *inactive*. Let $\mathcal{F}$ denote the set of all flows. Hereafter, we model the studied network as the undirected graph $G = (\mathcal{N}, \mathcal{L}, \mathcal{F})$, referred to as *node topology graph*.

C. Benefits and Limitations of MIMO

Consider the multi-hop wireless network $G = (\mathcal{N}, \mathcal{L}, \mathcal{F})$, shown in Fig. 2, where $\mathcal{N} = \{1,2,3,4\}$, $\mathcal{L} = \{(1,3),(2,4),(1,4)\}$, and $\mathcal{F} = \{f_1, f_2\}$. Assume that the number of antennas are such that $K_1 = K_3 = 1$ and $K_2 = K_4 = 2$. Also, assume that f_1 and f_2 are the only two flows in the network; i.e., node 1 has data to send to node 3 and node 2 has data to send to node 4. Let's now suppose that while node 1 is transmitting to node 3 (i.e., flow f_1 is active), node 2 wants to transmit to node 4. Let $\mathbf{u}_1$ be the (1×1) transmitting weight vector that node 1 is currently using to weigh its transmitted signal and $\mathbf{v}_3$ be the (1×1) receiving weight vector that node 3 is currently using to weigh the signal received from node 1. In order for node 4 to receive an interference-free signal from node 2, it must design its receiving weight vector $\mathbf{v}_4$ in such a way that it suppresses the interference caused by node 1's transmission while assuring an acceptable gain (e.g., unit) of its intended signal coming from node 2. In terms of equations, these constraints can be written as $(\mathbf{u}_2^T \mathbf{H}_{2,4})\mathbf{v}_4 = 1$ and $(\mathbf{u}_1^T \mathbf{H}_{1,4})\mathbf{v}_4 = 0$ where $\mathbf{u}_2 = [u_{2,1}\ u_{2,2}]^T$ is the transmitting weight vector of node 2 and $\mathbf{v}_4 = [v_{4,1}\ v_{4,2}]^T$ is the receiving weight vector of node 4. Because the elements of $\mathbf{H}_{1,4}$ and $\mathbf{H}_{2,4}$ are i.i.d, there exists a unique solution $\mathbf{v}_4^*$ to the system of these two equations; i.e., knowing $\mathbf{H}_{1,4}$, $\mathbf{H}_{2,4}$, $\mathbf{u}_1$, and $\mathbf{u}_2$, node 4 can solve the system of the two equations to determine $\mathbf{v}_4^*$. If there is no limit on node 4's power, it is always possible for node 4 to receive one interference-free stream of data from node 2 concurrently with node 1's transmitted signal. Hence, multiple antennas may increase the spatial reuse by allowing concurrent flows in the same vicinity; i.e., because of node 4's two antennas, f_1 and f_2 can both be active simultaneously.

Now suppose that node 1 is not currently transmitting. Node 4 can then use its both antennas to receive two streams of data concurrently. To design its receiving weight vectors $\mathbf{v}_{4,1} = [v_{4,1,1}\ v_{4,1,2}]^T$ and $\mathbf{v}_{4,2} = [v_{4,2,1}\ v_{4,2,2}]^T$, node 4 will then have to solve the following two systems each of two linear equations: $(\mathbf{u}_{2,1}^T \mathbf{H}_{2,4})\mathbf{v}_{4,1} = 1$, $(\mathbf{u}_{2,2}^T \mathbf{H}_{2,4})\mathbf{v}_{4,1} = 0$ and $(\mathbf{u}_{2,1}^T \mathbf{H}_{2,4})\mathbf{v}_{4,2} = 0$, $(\mathbf{u}_{2,2}^T \mathbf{H}_{2,4})\mathbf{v}_{4,2} = 1$. The vectors $\mathbf{u}_{4,1} = [u_{4,1,1}\ u_{4,1,2}]^T$ and $\mathbf{u}_{4,2} = [u_{4,2,1}\ u_{4,2,2}]^T$ are the two transmitting weight vectors used by node 2 to transmit its two streams. Again under the rich scattering environment assumption, each of the above two systems has a unique solution, $\mathbf{v}_{4,1}^*$ and $\mathbf{v}_{4,2}^*$. Without power limitation, node 4 can always receive two concurrent data streams by weighing its received signal using the two solutions $\mathbf{v}_{4,1}^*$ and $\mathbf{v}_{4,2}^*$. Note that now node 1 cannot transmit without causing interference at node 4 (spatial reuse is not possible now). This is because both of node 4's degrees of freedom are used to receive the two-stream flow via exploitation of the spatial division multiplexing offered by its antenna array.

Therefore, a node's degrees of freedom—number of antennas—can be exploited in one of three ways: (1) all degrees are used to send a multiple-stream flow of data by exploiting the spatial division multiplexing of the antenna array; (2) all degrees of freedom are used to increase the spatial reuse of the spectrum by allowing multiple concurrent streams in the same vicinity; (3) some of the degrees are used to send a multiple-steam flow while the others are used to allow for concurrent streams in the same neighborhood. It is important to recognize that the level of exploitation of the spatial reuse and/or multiplexing is, however, contingent on physical limitations such as node's power, multipath, and/or channel estimation errors. For instance, referring to the above example for illustration, note that it is due to the existence of the solution $\mathbf{v}_4^*$ that node 4 is able to receive its interference-free signal concurrently with node 1's transmission. If, for example, we impose a power limit, say P_4, on node 4's receiving capability and $\mathbf{v}_4^*$ happens to be such that $||\mathbf{v}_4^*||^2 > P_4$, then node 4 can no longer receive an interference-free signal; in this case, node 4's available power is not enough to combat the interference coming from flow f_1. Hence, due to power limitation, it is not possible now for flows f_1 and f_2 to be active simultaneously without interfering with each other. Throughout, we use P_m to denote the maximum transmit/receive power of node m normalized to the noise power which we will refer to as *maximum normalized power* of node m.

To summarize, because of physical limitations, when a node is equipped with K antennas, it does not mean that K concurrent streams (spatial reuse and/or multiplexing) can occur within the node's vicinity; in fact, physical limitations may restrict the number of possible concurrent streams to be less than K. In the next section, we propose a statistical method that wireless nodes can distributively use to compute this number. In Section IV, we derive flow-level optimized acceptance conditions under which flow rates are guaranteed to be feasible. These derived conditions rely on the proposed statistical method to capture the effects of the physical limitations.

III. Cross-Layer Optimization

Unlike elastic applications such as the Web and FTP, multimedia applications are bandwidth-sensitive; i.e., the network must provide these applications with bandwidth guarantees in order to achieve their acceptable QoS level. On the other hand, multimedia applications are loss-tolerant; i.e., an acceptable QoS level can still be achieved if only occasional packets are lost or erroneous. Let ξ_m denote the packet loss/error rate tolerated by node m's multimedia application; i.e., if at most $\xi_m\%$ of the packets are lost or erroneous, the QoS is still considered acceptable. Throughout this section, m will designate any transmitter (equipped with an antenna array of size K_m) that wants to send data to its neighbor n (equipped with antenna array of size K_n), forming then a flow f.

A. Effective Transmit and Receive Degrees of Freedom

Let's suppose that there are β streams currently received by nodes located within m's transmission range, and γ streams currently transmitted by nodes located within n's reception range. Let's also suppose that m wants to transmit an α-stream flow f of data to its neighbor n. As illustrated in Section II, due to physical limitations, although node m is left with $K_m-\beta$ degrees of freedom that it can exploit to send a multiple-stream signal with up to $K_m-\beta$ streams, it may actually be able to transmit only $\alpha < K_m-\beta$ streams. In other words, the number $(\alpha+\beta)$ of possible concurrent streams in the vicinity of m is likely to be less than the number of its actual antenna elements K_m. We will refer to the number $(\alpha+\beta)$ as the *effective transmit degrees of freedom* of flow f and denote it by $M_t(f)$. Due to similar reasons, node n may not be able to successfully receive $\alpha = K_n-\gamma$ streams even though n is left with $K_n-\gamma$ degrees of freedom. Hence, the number $(\alpha+\gamma)$ of possible concurrent streams in n's vicinity is also likely to be less than its total number of antennas K_n. We will refer to the number $(\alpha+\gamma)$ as the *effective receive degrees of freedom* of flow f and denote it by $M_r(f)$. The objective of this section is to determine the two numbers $M_t(f) \le K_m$ and $M_r(f) \le K_n$ given both the nodes' and the network's physical limitations and constraints. In Section IV, $M_t(f)$ and $M_r(f)$ will be coupled with MAC-layer characteristics to derive flow-level acceptance conditions.

Let's now provide a method that m and n can use to determine $M_t(f)$ and $M_r(f)$. In [25], we proved that the probability $\mathcal{P}_t(\alpha,\beta)$ of m successfully transmitting an α-stream signal to n without causing interference to any of the β nearby streams currently received by nodes located within m's transmission range is

$$\mathcal{P}_t(\alpha,\beta) = \int_{\frac{\alpha(1+\Delta P_n')}{P_m}}^{\infty} c_{K_m-\alpha-\beta+1}(x)dx$$

and the probability $\mathcal{P}_r(\alpha,\gamma)$ of n receiving an interference-free α-stream signal from m provided that there are γ streams currently transmitted by nodes located within its reception range is[1]

$$\mathcal{P}_r(\alpha,\gamma) = \int_{\frac{\alpha}{P_n}}^{\infty} c_{K_n-\alpha-\gamma+1}(x)dx$$

where $c_i(x)$ is the central chi-squared distribution with i degrees of freedom for $i \in \{K_m-\alpha-\beta+1, K_n-\alpha-\gamma+1\}$, $\Delta P_n' = (\frac{\alpha-1}{\alpha}+\kappa(n))P_{max}\sigma_E^2$, P_{max} is the maximum transmit power of all nodes, and $\kappa(n)$ is the number of node n's neighbors. Now, given that at most $\xi_m\%$ of a node m's packets can be lost/erroneous while having an acceptable transmitted QoS, $M_t(f)$ can be calculated as

$$M_t(f) = \max_{\alpha,\beta \le K_m} \{\alpha+\beta : \mathcal{P}_t(\alpha,\beta) \ge 1-\xi_m\}. \quad (1)$$

Likewise, given that at most $\xi_n\%$ of a node n's packets can be lost/erroneous while having an acceptable QoS of the received signal, $M_r(f)$ can be calculated as

$$M_r(f) = \max_{\alpha,\gamma \le K_n} \{\alpha+\gamma : \mathcal{P}_r(\alpha,\gamma) \ge 1-\xi_n\}. \quad (2)$$

It should be clear that in order to establish an α-stream flow, both the transmitter and the receiver must have enough degrees of freedom that can support the flow.

B. Table-Driven Approach for Determining Degrees of Freedom

We now present a table-driven approach that each node can use to determine its effective transmit and/or receive degrees of freedom given the statistics of the wireless channel, the channel estimation errors, the nodes' power availability, and the network topology. Note that details regarding how and when nodes should exchange information among each

[1]Note that both probabilities are derived while accounting for: (1) the statistics of the wireless channel ($\mathbf{H}$), (2) the channel estimation errors (σ_E^2), (3) the nodes' power availability (P), and (4) the network topology (G).

TABLE I
EFFECTIVE TRANSMIT DEGREES OF FREEDOM: $M_t(f)$

$K_m = 8$, $\xi_m = 1.5\%$, $P_{max} = 60$ dB	ERRONEOUS ESTIMATION $\sigma_E^2 > 10^{-1}$			GOOD ESTIMATION $10^{-3} < \sigma_E^2 < 10^{-1}$			PERFECT ESTIMATION $\sigma_E^2 < 10^{-3}$		
	NETWORK CONNECTIVITY			NETWORK CONNECTIVITY			NETWORK CONNECTIVITY		
	DENSE $\kappa(n) > 15$	AVERAGE $\kappa(n) \approx 10$	SPARSE $\kappa(n) < 5$	DENSE $\kappa(n) > 15$	AVERAGE $\kappa(n) \approx 10$	SPARSE $\kappa(n) < 5$	DENSE $\kappa(n) > 15$	AVERAGE $\kappa(n) \approx 10$	SPARSE $\kappa(n) < 5$
LOW POWER $P_m < 10$	0	0	3	0	2	3	3	3	3
MEDIUM POWER $10 < P_m < 40$	0	0	6	2	4	6	5	6	6
HIGH POWER $40 < P_m < 60$	0	3	7	5	6	7	7	7	7

other to construct their tables are omitted here to keep the presentation focused. For all illustration purposes, we assume that $K_m = K_n = 8$, $\xi_m = \xi_n = 1.5\%$, and $P_{max} = 60$ dB.

Effective Transmit Degrees of Freedom Table: As shown in Eq. (1), $M_t(f)$ depends on: (1) the transmitter's level of available normalized power P_m, (2) the error variance associated with the channel estimation method σ_E^2, and (3) the receiver n's number of neighbors, $\kappa(n)$. We propose to divide P_m into three levels: LOW, MEDIUM, and HIGH; σ_E^2 into three categories: ERRONEOUS, GOOD, and PERFECT; and $\kappa(n)$ into three types: DENSE, AVERAGE, and SPARSE. Each node maintains a three-dimensional table, whose entries can be computed off-line using Eq. (1), that can be indexed by the three parameters, P_m, σ_E^2, and $\kappa(n)$, to determine $M_t(f)$ as illustrated in Table I. The idea here is that, by monitoring P_m, σ_E^2, and $\kappa(n)$, m can use its table to determine its effective transmit degrees of freedom in real-time. For example, if m's power level is MEDIUM, estimation method is GOOD, and n's number of neighbors is SPARSE, then Table I indicates that the number of possible concurrent transmissions (including those from m to n) in m's vicinity can at most be 6 ($M_t(f) = 6$).

Note that the size of the table (number of levels, types, and/or categories) is intended to be a parameter that needs be made by network designers. As one can see, this parameter depends on how accurate effective degrees of freedom need to be; the higher the size of the table, the more accurate the entries, but also the more memory space/time needed to store/lookup the entries.

Effective Receive Degrees of Freedom Table: Similarly, $M_r(f)$ can be determined by using Eq. (2). Note that unlike $M_t(f)$, $M_r(f)$ depends only on node n's power level, P_n, and not on σ_E^2 nor on $\kappa(n)$. Readers can refer to [25] for mathematical and intuitive explanations on why this is the case. Here, a node n can use a table-driven approach, also computed off-line, to determine its $M_r(f)$ in real-time as illustrated in Table II. For instance, if node n's power level is MEDIUM, then $M_r(f) = 6$ (see Table II). The idea here is that by monitoring its nearby transmitters and its power level, a node can use its table to decide on how many streams it can receive successfully.

There are several points that are worth noting regarding the above table-driven approach. First, as mentioned earlier, $M_t(f)$ and $M_r(f)$ are always less than the number of antennas which equals 8 in this example. Second, as expected, the smaller the channel estimation error and/or the higher the power levels, the greater the effective degrees of freedom. Third, the effective degrees of freedom may vary over time due to variation of channel conditions and variation of power levels. Fourth, the effective degrees of freedom may also vary over space due to the fact that different nodes may have different number of neighbors and/or different power levels. Finally, a communication can successfully occur only if both the transmitter and the receiver, not just one of them, have enough degrees of freedom.

TABLE II
EFFECTIVE RECEIVE DEGREES OF FREEDOM: $M_r(f)$

$K_n = 8$, $\xi_n = 1.5\%$	LOW POWER $P_n < 10$	MEDIUM POWER $10 < P_n < 40$	HIGH POWER $40 < P_n < 60$
$M_r(f)$	3	6	7

C. Flow Interference Graph

Let's now associate each flow f in $\mathcal{F}$ with the pair $(M_t(f), M_r(f))$ of the effective transmit and receive degrees of freedom as determined in Sections III-A and III-B. We represent the multi-hop wireless network via two graphs: *node topology graph* and *flow interference graph*. The former graph is an extension of the node topology graph $G = (\mathcal{N}, \mathcal{L}, \mathcal{F})$ defined in Section II-B to include all pairs $(M_t(f), M_r(f))$ associated with all flows $f \in \mathcal{F}$; whereas the latter models the set $\mathcal{F}$ of all flows in the network as a directed graph $H = (\mathcal{F}, \mathcal{E}, \mathcal{D})$ where $\mathcal{E} \subseteq \mathcal{F} \times \mathcal{F}$, and $\mathcal{D}$ is the set of all pairs $(M_t(f), M_r(f))$ for all $f \in \mathcal{F}$. An ordered pair $(f, g) \in \mathcal{F} \times \mathcal{F}$ belongs to $\mathcal{E}$ if and only if (1) f and g do not share a node between them and (2) the transmission of flow f causes interference at the receiver of flow g. Note that if $(f, g) \in \mathcal{E}$, it does not necessarily mean that $(g, f) \in \mathcal{E}$. For every $f \in \mathcal{F}$, we let $\mathcal{E}_r(f)$ and $\mathcal{E}_t(f)$ denote the sets of outdegree and indegree flows of f in H. That is, the receiver of every flow in $\mathcal{E}_r(f)$ interferes with the transmitter of f; while the transmitter of every flow in $\mathcal{E}_t(f)$ interferes with the receiver of flow f.

IV. FLOW-LEVEL ACCEPTANCE TESTS

In the previous section, we developed a cross-layer model that captures the transmission/reception capabilities of nodes given the nodes' and network's physical limitations and constraints. In this section, we use the model to derive flow-level sufficient conditions under which the rates of all current admitted flows in the network are statistically guaranteed to be met.

Let's assume that each f in $\mathcal{F}$ flows data traffic at a rate of $x(f) \times W$ bps where W denotes the link bandwidth of the wireless medium; i.e., a maximum of τW bits a one-stream communication can attain in the interval $[0, \tau]$ seconds. Let $\mathbf{x} = [x(f)]_{f \in \mathcal{F}}$ be the vector, referred to as *flow rate vector*, representing the normalized (w.r.t W) data rates of all flows in $\mathcal{F}$. The vector $\mathbf{x}$ is said to be *feasible* in H if there exists a time schedule in which the rates of all flows are satisfied. Formally, $\mathbf{x}$ is feasible in H if there exists a time schedule $S = [0, \tau]$ of length $\tau > 0$ in which every flow $f \in \mathcal{F}$ communicates $\tau \times x(f) \times W$ bits.

There are two types of constraints that affect the feasibility of a flow rate vector: interference-based and radio-based. As already mentioned, due to interference, nodes cannot have more communications in their vicinity than what their effective degrees of freedom allow. On the other hand, due to radio's limited capabilities, a node cannot receive and transmit at the same time. In this paper, however, a node is allowed to transmit multiple streams to different receivers as long as the transmitter and all receivers each has enough effective degrees of freedom. Likewise, a node is allowed to receive multiple streams each coming from a different transmitter as long as the receiver and all transmitters each has enough effective degrees of freedom. In the remainder of this paper, a vector $\mathbf{x} = [x(f)]_{f \in \mathcal{F}}$ in H is considered to be a flow rate vector if it satisfies the necessary conditions given in [25] (radio-based constraints). This work focuses on deriving sufficient condition sets guaranteeing flow rate feasibility under interference-based constraints. These sets, referred to as *acceptance tests*, can be used in (1) QoS routing protocols to control flow admissibility into the network, and (2) multicommodity flow formulations to model network constraints.

Theorem 1: $\mathbf{x} = [x(f)]_{f \in \mathcal{F}}$ is feasible in H if $x(f) \leq \min\{M_t(f) - \sum_{g \in \mathcal{E}_r(f)} x(g), M_r(f) - \sum_{g \in \mathcal{E}_t(f)} x(g)\}$ for all $f \in \mathcal{F}$. We will refer to the set of these conditions as *Acceptance Test I*, and denote it by `AT-I`.

PROOF: Let F denote $|\mathcal{F}|$, and suppose that $\forall f \in \mathcal{F}$, $x(f) \leq \min\{M_t(f) - \sum_{g \in \mathcal{E}_r(f)} x(g), M_r(f) - \sum_{g \in \mathcal{E}_t(f)} x(g)\}$. Without loss of generality, let's arrange the flows in $\mathcal{F}$ as $\{1, 2, \ldots, F\}$ such that $x(i) \leq x(j)$ for all $1 \leq i \leq j \leq F$, and let $\mathcal{F}^i$ denote the set of flows $\{1, 2, \ldots, i\}$. Let $S = [0, \tau]$ be a time schedule of length $\tau > 0$ seconds. We show by induction that for all $n = 1, 2, \ldots, F$, the flows in the $\mathcal{F}^n$ are scheduleable in S, which proves that $\mathcal{F}$ is scheduleable since $\mathcal{F} = \mathcal{F}^F$.

BASIS: $\mathcal{F}^1 = \{1\}$. Since both $\mathcal{E}_r(1) = \emptyset$ and $\mathcal{E}_t(1) = \emptyset$, $x(1) \leq \min\{M_t(1), M_r(1)\}$. Hence $\mathcal{F}^1$ is schedulable in S.

INDUCTION STEP: Suppose that $\forall\ i = 1, 2, \ldots, n-1$, $\mathcal{F}^i$ is schedulable in S and prove that $\mathcal{F}^n$ is also schedulable in S. Since $\mathcal{F}^n = \mathcal{F}^{n-1} \cup \{n\}$, then it suffices to prove that flow n can be scheduled, provided that all flows in $\mathcal{F}^{n-1}$ are already scheduled. By hypothesis, $x(n) \leq \min\{M_t(n) - \sum_{g \in \mathcal{E}_r(n)} x(g), M_r(n) - \sum_{g \in \mathcal{E}_t(n)} x(g)\}$. By letting $\Phi_r(n) = \mathcal{F}^{n-1} \cap \mathcal{E}_r(n)$, it follows that $x(n) \leq M_t(n) - \sum_{g \in \Phi_r(n)} x(g)$. Hence even when all the flows in $\Phi_r(n)$ are scheduled disjointly, the transmitter of flow n can still schedule flow n for transmission without interference. Similarly, by letting $\Phi_t(n) = \mathcal{F}^{n-1} \cap \mathcal{E}_t(n)$, it follows that $x(n) \leq M_r(n) - \sum_{g \in \Phi_t(n)} x(g)$, and hence the receiver of flow n can schedule n's reception without interference. ■

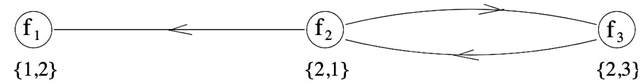


Fig. 3. Flow interference graph for illustration of Theorems 1 and 2

Theorem 2: $\mathbf{x} = [x(f)]_{f \in \mathcal{F}}$ is feasible in H if $x(f) \leq \min\left\{\frac{M_t(f)}{|\mathcal{E}_r(f)|+1}, \frac{M_r(f)}{|\mathcal{E}_t(f)|+1}\right\}$ for all $f \in \mathcal{F}$. We will call the set of these conditions *Acceptance Test II*, and denote it by `AT-II`.

PROOF: Let $x(f) \leq \min\left\{\frac{M_t(f)}{|\mathcal{E}_r(f)|+1}, \frac{M_r(f)}{|\mathcal{E}_t(f)|+1}\right\}$ for all $f \in \mathcal{F}$. Again, let's arrange the flows in $\mathcal{F}$ as $\{1, 2, \ldots, F\}$ ($F = |\mathcal{F}|$) such that $x(i) \leq x(j)$ for all $1 \leq i \leq j \leq F$, and let $\mathcal{F}^i$ denote the set of flows $\{1, 2, \ldots, i\}$. Let $S = [0, \tau]$ be a time schedule of length $\tau > 0$ seconds. We show by induction that for all $n = 1, 2, \ldots, F$, $\mathcal{F}^n \subseteq \mathcal{F}$ is scheduleable in S.

BASIS: $\mathcal{F}^1 = \{1\}$. Since $|\mathcal{E}_r(1)| \geq 0$ and $x(1) \leq \frac{M_t(1)}{|\mathcal{E}_r(1)|+1}$, then $x(1) \leq M_t(1)$. Similarly, since $|\mathcal{E}_t(1)| \geq 0$ and $x(1) \leq \frac{M_r(1)}{|\mathcal{E}_t(1)|+1}$, then $x(1) \leq M_r(1)$. Thus, $\mathcal{F}^1$ is scheduleable in S.

INDUCTION STEP: Suppose that $\forall\ i = 1, 2, \ldots, n-1$, $\mathcal{F}^i$ is scheduleable in S. We'll show that $\mathcal{F}^n$ is also scheduleable in S. Since $\mathcal{F}^n = \mathcal{F}^{n-1} \cup \{n\}$, then it suffices to prove that flow n can be scheduled, provided that all flows in $\mathcal{F}^{n-1}$ are already scheduled. Let $\Phi_r(n) = \mathcal{F}^{n-1} \cap \mathcal{E}_r(n)$ and $\Phi_t(n) = \mathcal{F}^{n-1} \cap \mathcal{E}_t(n)$. Note that for all $k \in \Phi_r(n) \cup \Phi_t(n)$, $x(k) \leq x(n)$ (since flow k is already scheduled). Hence, since $x(n) \leq \min\left\{\frac{M_t(n)}{|\mathcal{E}_r(n)|+1}, \frac{M_r(n)}{|\mathcal{E}_t(n)|+1}\right\}$, then so is $x(k)$ for all $k \in \Phi_r(n) \cup \Phi_t(n)$.

From flow n's transmitter perspective, each $x(k)$ can then at most be $\frac{M_t(n)}{|\mathcal{E}_r(n)|+1}$. The worst case occurs when all of the $|\mathcal{E}_r(n)|$ receivers that are nearby flow n's transmitter happen to be in $\mathcal{F}^{n-1}$ and also scheduled disjointly. In such case, they will occupy a total rate of at most $|\mathcal{E}_r(n)| \times \frac{M_t(n)}{|\mathcal{E}_r(n)|+1}$. Now since the effective transmit degrees of freedom of n is $M_t(n)$, then $M_t(n)$ interference-free communications can be carried out in the vicinity of the transmitter of n. Thus, the transmitter of n is able to transmit $M_t(n) - |\mathcal{E}_r(n)| \times \frac{M_t(n)}{|\mathcal{E}_r(n)|+1} = \frac{M_t(n)}{|\mathcal{E}_r(n)|+1} \geq x(n)$ without interfering with any of its nearby receivers and hence flow n is feasible from the transmitter's standpoint. Similarly, one can prove that flow n's receiver can receive $x(n)$ without interfering with any of its nearby transmitters. ■

One point that requires attention is that neither `AT-I` nor `AT-II` can be said to be "stronger" (less conservative) than the other. That is, flow rate vectors passing `AT-II` do not necessarily pass `AT-I` and vice versa. This is illustrated by the following example of the flow interference graph $H = (\mathcal{F}, \mathcal{E}, \mathcal{D})$, given in Fig. 3, where $\mathcal{F} = \{f_1, f_2, f_3\}$, $\mathcal{E} = \{(f_2, f_1), (f_2, f_3), (f_3, f_2)\}$ and $\mathcal{D} = \{(1,2), (2,1), (2,3)\}$ ($M_t(f_1) = M_r(f_2) = 1$, $M_r(f_1) = M_t(f_2) = M_t(f_3) = 2$ and $M_r(f_3) = 3$). Consider the flow rate vectors $\mathbf{x}' = (\frac{3}{4}, \frac{1}{2}, \frac{3}{4})$ and $\mathbf{x}'' = (1, \frac{3}{4}, \frac{1}{4})$. Table III shows that $\mathbf{x}'$ passes

TABLE III
NUMERICAL VERIFICATION UNDER $\mathbf{x}'$

$\mathbf{x}'$	Conditions of Theorem 2	Conditions of Theorem 1
$x'(f_1) = \frac{3}{4}$	$\min\{\frac{1}{1}, \frac{2}{2}\} = 1$	$\min\{1-0, 2-\frac{1}{2}\} = 1$
$x'(f_2) = \frac{1}{2}$	$\min\{\frac{2}{3}, \frac{1}{2}\} = \frac{1}{2}$	$\min\{2-\frac{3}{2}, 1-\frac{3}{4}\} = \frac{1}{4}$
$x'(f_3) = \frac{3}{4}$	$\min\{\frac{2}{2}, \frac{3}{2}\} = 1$	$\min\{2-\frac{1}{2}, 3-\frac{1}{2}\} = 1\frac{1}{2}$

TABLE IV
NUMERICAL VERIFICATION UNDER $\mathbf{x}''$

$\mathbf{x}''$	Conditions of Theorem 2	Conditions of Theorem 1
$x''(f_1) = 1$	$\min\{\frac{1}{1}, \frac{2}{2}\} = 1$	$\min\{1-0, 2-\frac{3}{4}\} = 1$
$x''(f_2) = \frac{3}{4}$	$\min\{\frac{2}{3}, \frac{1}{2}\} = \frac{1}{2}$	$\min\{2-\frac{5}{4}, 1-\frac{1}{4}\} = \frac{3}{4}$
$x''(f_3) = \frac{1}{4}$	$\min\{\frac{2}{2}, \frac{3}{2}\} = 1$	$\min\{2-\frac{3}{4}, 3-\frac{3}{4}\} = 1\frac{1}{4}$

`AT-II` (e.g., $x'(f_1) = \frac{3}{4} \leq \min\{\frac{1}{1}, \frac{2}{2}\} = 1$, etc) and Table IV shows that $\mathbf{x}''$ passes `AT-I` (e.g., $x''(f_1) = 1 \leq \min\{1-0, 2-\frac{3}{4}\} = 1$, etc). However, $\mathbf{x}'$ does not pass `AT-I` since $x'(f_2) = \frac{1}{2} > \min\{2-\frac{3}{2}, 1-\frac{3}{4}\} = \frac{1}{4}$. Likewise, $\mathbf{x}''$, which passes `AT-I`, fails `AT-II` since $x(f_2) = \frac{3}{4} > \min\{\frac{2}{3}, \frac{1}{2}\} = \frac{1}{2}$. In fact, from a mathematical standpoint, one cannot draw any conclusion stating that one set of conditions performs better than the other set in the sense that when used, for example, in an admission control mechanism, that better set would always result in a higher acceptance rate. This performance comparison can, however, be done through simulations as shown in Section V. On the other hand, one can widen the space of acceptable rate vectors by testing both `AT-I` and `AT-II` whenever one needs to decide whether to accept a flow vector; i.e., if a to-be-admitted flow vector passes either `AT-I` or `AT-II`, then the network accepts the vector. Clearly, this approach results in a higher acceptance rate when compared with that based on either `AT-I` or `AT-II`, but not both. Referring to the above example for illustration, testing each of $\mathbf{x}'$ and $\mathbf{x}''$ on both `AT-I` and `AT-II` would result in each of them being accepted; whereas if only one test is being considered, then only one vector will be admitted (either $\mathbf{x}'$ or $\mathbf{x}''$ depending on which test is being considered). The question that naturally arises now is whether there is a set of conditions (i.e., an acceptance test) that would capture rate vectors that fail both `AT-I` and `AT-II`. The following theorem states a set of sufficient conditions that are stronger than those stated by Theorems 1 or 2 in that it results in accepting vectors that fail both `AT-I` and `AT-II` while accepting any vector that pass `AT-I` and/or `AT-II`.

Theorem 3: $\mathbf{x} = [x(f)]_{f \in \mathcal{F}}$ is feasible in H if

$$x(f) \leq \max\{\frac{M_t(f)}{|\mathcal{E}_r(f)|+1}, M_t(f) - \sum_{g \in \mathcal{E}_r(f)} x(g)\},$$

and

$$x(f) \leq \max\{\frac{M_r(f)}{|\mathcal{E}_t(f)|+1}, M_r(f) - \sum_{g \in \mathcal{E}_t(f)} x(g)\}$$

for all $f \in \mathcal{F}$. We will refer to this set of conditions as *Acceptance Test III*, and denote it by `AT-III`.

PROOF: Let $\mathbf{x} = [x(f)]_{f \in \mathcal{F}}$ be a flow rate vector satisfying the sufficient conditions stated by the theorem. Let's again arrange the flows in $\mathcal{F}$ as $\{1, 2, \ldots, F\}$ ($F \equiv |\mathcal{F}|$) such that $x(i) \leq x(j)$ for all $1 \leq i \leq j \leq F$, and let $\mathcal{F}^i$ denote the set of flows $\{1, 2, \ldots, i\}$. We show by induction that for all $n = 1, 2, \ldots, F$, $\mathcal{F}^n$ is feasible.

BASIS: $\mathcal{F}^1 = \{1\}$. Since there is only one flow, flow 1, then it follows from the hypothesis that $x(1) \leq \min\{M_t(1), M_r(1)\}$ and hence $\mathcal{F}^1$ is feasible by both its transmitter and its receiver.

INDUCTION STEP: Suppose that $\forall\ i = 1, 2, \ldots, n-1$, $\mathcal{F}^i$ is feasible and prove that $\mathcal{F}^n$ is also feasible. Since $\mathcal{F}^n = \mathcal{F}^{n-1} \cup \{n\}$, then it suffices to prove that both the transmitter and the receiver of flow n can schedule n provided that all flows in $\mathcal{F}^{n-1}$ are already scheduled. Let $\Phi_t(n) = \mathcal{F}^{n-1} \cap \mathcal{E}_t(n)$ and $\Phi_r(n) = \mathcal{F}^{n-1} \cap \mathcal{E}_r(n)$. We will show for the transmitter case only; the receiver case can be proven similarly. Suppose that $\frac{M_t(n)}{|\mathcal{E}_r(n)|+1} \leq M_t(n) - \sum_{g \in \mathcal{E}_r(n)} x(g)$. Hence, $x(n) \leq M_t(n) - \sum_{g \in \mathcal{E}_r(n)} x(g)$. It then follows that $x(n) \leq M_t(n) - \sum_{g \in \Phi_r(n)} x(g)$ because $\Phi_r(n) \subseteq \mathcal{E}_r(n)$. Thus, even when all the flows in $\Phi_r(n)$ are scheduled disjointly, the transmitter of flow n can still schedule flow n for transmission without interference. Now suppose that $\frac{M_t(n)}{|\mathcal{E}_r(n)|+1} > M_t(n) - \sum_{g \in \mathcal{E}_r(n)} x(g)$. Note that for all $k \in \Phi_r(n)$, $x(k) \leq x(n) \leq \frac{M_t(n)}{|\mathcal{E}_r(n)|+1}$ (since flow k is already scheduled). When all of the $|\mathcal{E}_r(n)|$ receivers that are nearby the transmitter of flow n happen to be in $\mathcal{F}^{n-1}$ and also scheduled disjointly, they will occupy a total rate of at most $|\mathcal{E}_r(n)| \times \frac{M_t(n)}{|\mathcal{E}_r(n)|+1}$. Now since the effective transmit degrees of freedom of n is $M_t(n)$, then $M_t(n)$ interference-free communications can be carried out in the vicinity of the transmitter of n. Thus, the transmitter of n is able to transmit $M_t(n) - |\mathcal{E}_r(n)| \times \frac{M_t(n)}{|\mathcal{E}_r(n)|+1} = \frac{M_t(n)}{|\mathcal{E}_r(n)|+1} \geq x(n)$ without interfering with any of its nearby receivers and hence flow n is feasible from the transmitter's standpoint.■

As mentioned earlier, the sufficient conditions stated in `AT-III` are always "stronger" than those stated in either `AT-I` and/or `AT-II` in the sense that all flow rate vectors that pass `AT-I` and/or `AT-II` also pass `AT-III`. However, a vector passing `AT-III` does not necessarily pass `AT-I` or `AT-II`. Referring to the same example again, note that while $\mathbf{x}''' = (1, \frac{3}{4}, \frac{3}{4})$ does not pass neither `AT-I`, nor `AT-II`, it does pass `AT-III`. When all flows, however, have identical rates, all theorems are equivalent. Hence, we state the following theorem.

Theorem 4: If $x(f) = x(g)$ for all $f, g \in \mathcal{F}$, then `AT-I`, `AT-II` and `AT-III` are equivalent.

PROOF: If $x(f) = x(g)$ for all $f, g \in \mathcal{F}$, then $\forall f \in \mathcal{F}$, $x(f) \leq M_t(f) - \sum_{g \in \mathcal{E}_r(f)} x(g) \Leftrightarrow x(f) \leq \frac{M_t(f)}{|\mathcal{E}_r(f)|+1}$ and $x(f) \leq M_r(f) - \sum_{g \in \mathcal{E}_t(f)} x(g) \Leftrightarrow x(f) \leq \frac{M_r(f)}{|\mathcal{E}_t(f)|+1}$. This implies that `AT-I` $\Leftrightarrow$ `AT-II` $\Leftrightarrow$ `AT-III`. ■

There are two important points that are worth mentioning regarding the tests: `AT-I`, `AT-II`, and `AT-III`. First, since `AT-III` always performs better than `AT-I` and/or `AT-II`, one may suggest to always use `AT-III`. The issue, however, is that due to its non-linearity, `AT-III` is not well suited for all problems. For instance, `AT-III` cannot be used in the multicommodity flow routing problems formulated in [17, 19–21] to model radio and interference constraints. In these instances, one may choose to use `AT-I` and/or `AT-II` since they are both linear, or formulate the instances as integer programs and then use AT-III. As one can see, there is a clear tradeoff between complexity (non-linearity) and efficiency (higher acceptance rates); AT-III is more complex, but also more efficient; whereas, AT-I and AT-II are simpler, but less efficient. From a mathematical perspective, AT-III is proven to be always superior than the other two. However, one cannot mathematically determine how much superior AT-III is. Note the importance of knowing the gap between the levels of efficiency if one wants to know whether, for example, the complexity added as a result of choosing AT-III instead of AT-I or AT-II is worth the gain in efficiency. This will be answered in the next section.

The second point that is also worth noting is that in terms of acceptance capabilities, neither AT-I, nor AT-II can be said to be always superior than the other. How much and under what circumstances each one of the two would be superior than the other will also be answered via simulations in the next section.

V. Performance Evaluation

We now evaluate and compare the performances of the three acceptance tests: AT-I, AT-II, and AT-III. We consider three mechanisms each uses one of the three tests as the basis for deciding on the admissibility of flows into the network. We study the effect of (1) traffic load variability, and (2) physical constraints such as nodes' powers and channel estimation errors on the effectiveness of the mechanisms. While the complexity of each test is obvious, the efficiency gap among them is not. The objective here is then to evaluate and measure such gap. All simulations are run until the measured metrics converge to within 5% of the real value at a confidence level of 98%.

A. Simulation Method & Parameter Setting

We generate and simulate random multi-hop wireless networks each consisting of $N = 50$ nodes. The capacity of the medium, defined to be the maximum number of bits that a one-antenna equipped node can transmit in one second, is set to $W = 54$ Mbps. Nodes are uniformally distributed in a cell of size $A = 100 \times 100$ meters square where two nodes are considered neighbors if the distance between them does not exceed d meters. Each node is equipped with $K = 8$ antennas, and associated with a maximum normalized power (normalized to the noise power) selected from a uniform distribution in the range $\bar{P}[1-\sqrt{3}C_P, 1+\sqrt{3}C_P]$ where $\bar{P}$ and C_P are respectively the average and the coefficient of variation of the powers. During the course of simulations, end-to-end flows are generated randomly according to a Poisson process with arrival rate λ. Each end-to-end flow is characterized by (1) a random pair (source–destination) of nodes; (2) a chain of one-hop flows constituting the shortest path between these two nodes; (3) a flow data rate selected from a uniform distribution in the range $\bar{R}[1 - \sqrt{3}C_R, 1 + \sqrt{3}C_R]$ where $\bar{R} = 3\%W$ and C_R are respectively the average and the coefficient of variation of the data rates; and (4) an exponentially distributed duration of rate μ. The value of ξ is set to 3%. The simulator, implementing the above method, is written in MATLAB.

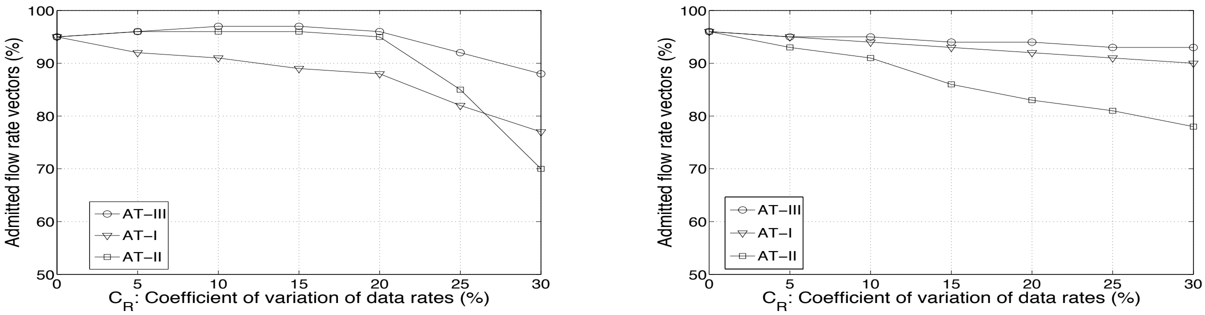
Let $\mathbf{x}$ and $\mathbf{p}$ denote the vectors representing respectively the data rates and the maximum normalized powers of all flows. For every hundred combinations (a combination consists of a random graph, a random flow rate vector $\mathbf{x}$ and a random power vector $\mathbf{p}$), we collect the percentage of *admitted flow rate vectors* under each one of the three proposed acceptance tests. A vector flow rate is admitted if it passes the corresponding acceptance test. Provided P, σ_E^2, and the number of neighbors κ, each node uses the table-driven approach to determine its effective transmit and receive degrees of freedom to and from each one of its neighbors. These effective degrees of freedom are then used in AT-I, AT-II, and AT-III to decide on accepting rate flows.

B. Effect of Traffic Load Characteristics

The percentage of admitted flow rate vectors as a function of the coefficient of variation of the data rates C_R is shown in Fig. 4. In this simulation scenario, we fix $\bar{P} = 20$ dB (MEDIUM LEVEL), $\sigma_E^2 = 10^{-2}$ (GOOD ESTIMATION), and we consider two topologies: SPARSE (Fig. 4(a)) and DENSE (Fig. 4(b)). A network is considered SPARSE (resp. DENSE) when its average node degree is below 5 (resp. above 15). These average node degrees are monitored through appropriate choices of d.

General Analysis: First, note that as expected from our developed theory, AT-III always accepts more vectors than AT-I and AT-II. Second, observe that while the percentage of admitted vectors is the same for each test when rates have no variation (as stated in Theorem 4), it decreases as C_R increases and regardless of the test being used. This is true for both topologies, SPARSE and DENSE, and explains as follows. Vectors with high data rate variability (high C_R) are likely to result in an unbalanced rate distribution in the network; i.e., some regions of the network will have to support higher data rates than other regions. When this happens, the data rate requirements of some flows are likely not to be satisfied, which results in a lower percentage of admitted rate vectors. The figure also shows that the percentage of admitted vectors is above 90% for low to medium ranges of C_R. Because the derived conditions are all sufficient, but not necessary, a rate vector not passing an acceptance test does not tell whether the vector is feasible or not. In other words, we do not know whether failing the test is because the conditions are not necessary (vector is feasible, but fails the set) or the vector is not feasible in the first place. Our results show that about 10% of all vectors tested the condition sets do not pass these acceptance tests. Recall that rate vectors are randomly generated, and expected to be a mix of feasible and unfeasible ones. Hence, some of the 10% may have failed because they are not feasible (cannot be supported by the network), and thus, should not be counted when measuring the percentage of admitted vectors. Hence, under these simulation scenarios, our tests result in an acceptance rate of at least 90%.

AT-I vs. AT-II Analysis: It is important and interesting to note that while in SPARSE topologies AT-II outperforms AT-I for low to medium data rate variability (Fig. 4(a)), the exact opposite behavior is observed when topologies are DENSE (Fig. 4(b)). This explains as follows. First, by



(a) SPARSE topology: $\kappa<5$

(b) DENSE topology: $\kappa>15$

Fig. 4. Effect of the data rate variability C_R on flow acceptance capabilities: power level (MEDIUM), channel estimation (GOOD), C_P (10%).

examining Theorems 1 and 2, note that a node's conditions under AT-II do not depend on its neighbors' rates; i.e., a node's neighbor's rate does affect the node's decision on whether to accept a flow. In the case of AT-I, however, the conditions depend on neighbors' rates. Therefore, when applying AT-I in DENSE topologies, lowering the rates of some flows would allow others to have their rates increased without violating the upper bound constraints. This is not the case for AT-II since conditions do not depend on neighbors' rates. Hence, flow rate vectors with high variation are more likely to pass AT-I than AT-II in DENSE topologies. In SPARSE topologies, even if we still allow flows to use the "unused" rates in their vicinity whenever their neighbors are not using it (i.e., in AT-I), it would not be beneficial in terms of accepting more flow rate vectors since nodes in SPARSE topologies do not have that many neighbors, and hence not much "unused" rate is available, in the first place. This explains why AT-II outperforms AT-I in SPARSE topologies. Also note that, in SPARSE topologies, AT-II performs close to AT-III for low to average variations of data rates ($C_R \leq 20\%$); whereas, in DENSE topologies, AT-I performs close to AT-III.

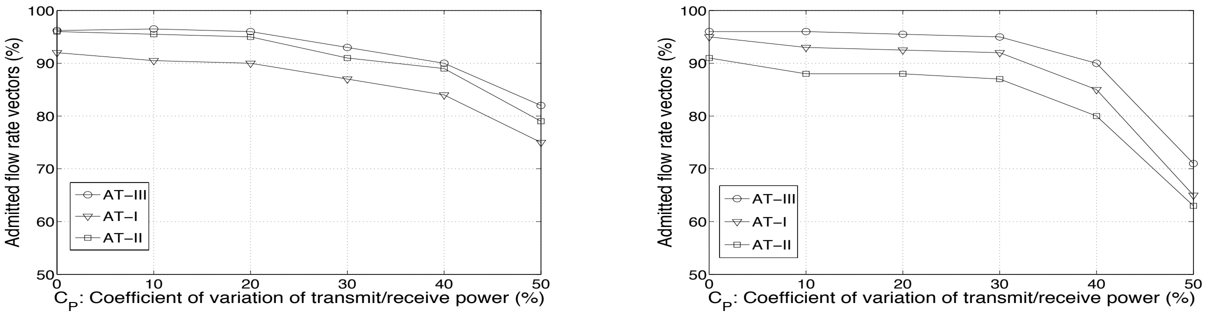
Findings: The results of this section can be summarized as follows. First, when rates are substantially different, AT-III is definitely the one to use as it outperforms both AT-I and AT-II significantly. This is true in both topologies: SPARSE and DENSE. Here, the gain in efficiency is worth the added complexity. Second, when the rates of flows are close to each other, AT-II is best suited for SPARSE topologies; whereas, AT-I is best suited for DENSE topologies. Also, since the gap between AT-II and AT-III in SPARSE topologies and that between AT-I and AT-III in DENSE topologies are small, then the efficiency gained by using AT-III instead of AT-I or AT-II is not worth the added complexity.

C. Effect of Physical Constraints

Effect of power variability C_P: Fig. 5 shows the percentage of admitted flow rate vectors for different values of the coefficient of variation of powers C_P. In this simulation scenario, we fix $\bar{P} = 20$ dB (MEDIUM LEVEL), $\sigma_E^2 = 10^{-2}$ (GOOD ESTIMATION), $C_R = 10\%$, and we consider two topologies: SPARSE (Fig. 5(a)) and DENSE (Fig. 5(b)). There are two points that one can make regarding the effect of power on acceptance capabilities of the proposed tests. First, note that, regardless of the acceptance test, the higher the power variation is, the lower the acceptance rate is. This is because a high value of C_P implies that it is likely that some of the nodes have fewer degrees of freedom than others. High variation in the number of degrees of freedom results in turn in some nodes not being able to admit flows and thus in lower acceptance rates of flow rate vectors. The second point to observe is that variations in power do not affect the performances of the tests with respect to each other. The gap between the acceptance rates remains almost the same regardless of power levels and variations.

Effect of channel estimation error σ_E^2: In Fig. 6, for each acceptance test, we show the percentage of admitted data rates for different values of σ_E^2. The data rate and the power variations are both set to 10% during the course of these simulations; whereas, $\bar{P}$ is set to 20 dB. As expected, the figure shows that the acceptance rate of the conditions decreases as σ_E^2 increases. Note that, for reasonable channel estimation errors $\sigma_E^2 \simeq 10^{-3}$, the acceptance rate of all tests is above $\approx 90\%$. When σ_E^2 is high, even though nodes may have high power levels, it may not be possible for them to successfully transmit/receive due to enormous interference caused by imperfect estimation of the channel coefficients. In terms of the derived sufficient conditions, this results in smaller transmit/receive degrees of freedom and hence higher rejections of flows.

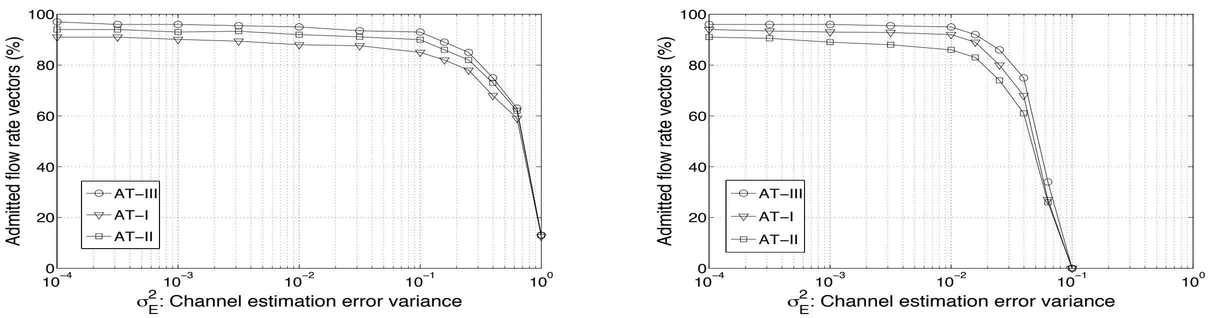
Another observation to recall from Fig. 6 is that all acceptance rates are more sensitive to σ_E^2 in DENSE topologies than in SPARSE topologies. In fact, Fig. 6 shows that the acceptance rates under all tests remain above $\approx 80\%$ even when σ_E^2 reaches as high as $\approx 10^{-1}$ when topologies are SPARSE; whereas those rates drop quickly below $\approx 80\%$ as soon as σ_E^2 reaches about $\approx 10^{-2}$ when topologies are DENSE. This is because nodes are likely to have more interferers in DENSE networks than in SPARSE ones; hence, in scenarios with ERRONEOUS estimations (σ_E^2 is high), nodes are likely to accept more flows in SPARSE networks (less interferers) than in DENSE ones. Of course, this argument holds given that powers are kept the same.



(a) SPARSE topology: κ<5

(b) DENSE topology: κ>15

Fig. 5. Effect of the maximum normalized power variability C_P on on flow acceptance capabilities: channel estimation (GOOD), C_R (10%), $\bar{P}$ (20dB).



(a) SPARSE topology: κ<5

(b) DENSE topology: κ>15

Fig. 6. Effect of σ_E^2 on flow acceptance capabilities: power level (MEDIUM), C_R (10%), $\bar{P}$ (20dB), C_P (10%).

VI. Discussion: Applicability of Acceptance Tests

Multicommodity End-to-End Flow Routing: One major networking problem in which the developed acceptance tests can be used is the multicommodidy flow routing problem in multi-hop wireless MIMO networks. A multicommodity flow routing instance consists of finding feasible routes for end-to-end flows that maximizes a given utility function while satisfying some design and network constraints. For example, the work in [26] solves an energy-efficient routing problem for wireless sensor networks with data rate constraints using multicommodity, where the objective is to increase the lifetime of sensor networks while meeting the QoS requirements. Mesh networks is another example in which multi-hop routing can be formulated as multicommodity where the objective here is typically to maximize throughput while ensuring QoS requirements of end users [17]. All these instances, however, require models to capture network constraints such as those due to radio and interference limitations so that QoS (i.e., data rates) needs are guaranteed to be satisfied. If these instances fail to impose appropriate conditions that capture network constraints, then routing solutions may be such that the shared medium may not be able to provide the net data rate required to support these flows. The developed acceptance tests, `AT-I`, `AT-II`, and `AT-III`, can be used to model such constraints. Recall that the conditions expressed in `AT-III` are not linear; whereas, those expressed in `AT-I` and `AT-II` are linear. Based on the findings of the evaluation section, we make the following recommendations (summarized in Table V).

TABLE V

Recommendation Usage in Multicommodity Flow Problems

	SPARSE NETWORKS	DENSE NETWORKS
HIGH RATE VARIABILITY	`AT-III` is recommended. Efficiency gain is worth added complexity	
MEDIUM RATE VARIABILITY	`AT-II` is recommended.	`AT-I` is recommended.
LOW RATE VARIABILITY	Added complexity is not worth efficiency gain	Added complexity is not worth efficiency gain

QoS End-to-End Flow Routing: Another application where the developed tests can also be used is end-to-end QoS routing in multi-hop wireless MIMO networks. Multi-hop routing schemes such as AODV [27] can be extended to include one of these tests to serve as the basis for admission control when supporting end-to-end flows with QoS requirements such as multimedia applications. For example, the link-bandwidth calculation mechanism, proposed in [25], can use the developed acceptance tests to decide on whether to admit a new flow into the network when supporting end-to-end QoS flows. One, however, needs to always use `AT-III` instead of `AT-I` or `AT-II` because non-linearity is no longer an issue when using these tests in QoS routing.

VII. Conclusion

This paper develops a set of three acceptance tests that exploit the benefits of multiple antennas to enable multi-hop wireless MIMO networks with QoS capabilities. We first propose a table-driven statistical approach that allows each node to determine the amount of spatial reuse and/or multiplexing, offered by MIMO, that are available to it in its vicinity. This method is then used to derive the three tests that, once passed, flow rates are guaranteed to be feasible. The acceptance tests are useful in networking problems such as QoS routing and multicommodity flow problems to control admissibility of multimedia/QoS flows into the network. The tests are multi-layer aware in the sense that they account for cross-layer effects between the PHY and the MAC layers to provide QoS capabilities at higher layers.

REFERENCES

[1] S. K. Jayaweera and H. Vincent Poor, "Capacity of multiple-antenna systems with both receiver and transmitter channel state information," *IEEE Tran. on Infor. Theory*, vol. 49, no. 10, pp. 2697–2709, Oct. 2003.

[2] R. Narasimhan, "Spatial multiplexing with transmit antenna and constellation selection for correlated MIMO fading channels," *IEEE Tran. on Signal Processing*, vol. 51, no. 11, pp. 2829–2838, November 2003.

[3] R. S. Blum, "MIMO capacity with interference," *JSAC*, June 2003.

[4] D. Gore, R. Heath, and A. Paulraj, "Statistical antenna selection for spatial multiplexing systems," in *ICC*, 2002.

[5] Q. H. Spencer, A. Lee Swindlehust, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Tran. on Signal Processing*, vol. 52, no. 2, pp. 461–471, Feb. 2004.

[6] S. D. Blostein and H. Leib, "Multiple antenna systems: Their role and impact in future wireless access," *IEEE Comm. Magazine*, July 2003.

[7] Y. B. Ko, V. Shankarkumar, and N. Vaidya, "Medium access control protocols using directional antennas in ad hoc networks," in *INFOCOM 2000*.

[8] A. Nasipuri, S. Ye, J. You, and R. E. Hiromoto, "A MAC protocol for mobile ad hoc networks using directional antennas," in *Proc. of IEEE Wireless Comm., and Net. Conf. (WCNC)*, Sep. 2000, pp. 1214–1219.

[9] R. Ramanathan, "On the performance of ad hoc networks with beamforming antennas," *MOBIHOC*, 2001.

[10] L. Bao and J. J. Garcia-Luna-Aceves, "Transmission scheduling in ad hoc networks with directional antennas," in *MOBICOM 2002*.

[11] R. R. Choudhury, X. Yang, R. Ramanathan, and N. H. Vaidya, "Using directional antennas for medium access control in ad hoc networks," in *MOBICOM 2002*.

[12] T. Korakis, G. Jakllari, and L. Tassiulas, "A MAC protocol for full exploitation of directional antennas in ad-hoc wireless networks," in *Proc. of ACM MOBIHOC*, 2003, pp. 98–107.

[13] K. Sundaresan, R. Sivakumar, M. A. Ingram, and T-Y Chang, "A fair medium access control protocol for ad-hoc networks with MIMO links," in *INFOCOM 2004*.

[14] R. Sivakumar, P. Sinha, and V. Bharghavan, "CEDAR: A core-extraction distributed ad hoc routing algorithm," *JSAC*, August 1999.

[15] J. Sobrinho and A. S. Krishnakumar, "Quality-of-service in ad hoc carrier sense multiple access wireless networks," *JSAC*, August 1999.

[16] C. Zhu and M. S. Corson, "Qos routing for mobile ad hoc networks," in *INFOCOM 2002*.

[17] M. Alicherry, R. Bhatia, and L. Li, "Joint channel assignment and routing for throughput optimization in multi-radio wireless mesh networks," in *MOBICOM 2005*.

[18] M. Kodialam and T. Nandagopal, "Characterizing the capacity region in multi-radio multi-channel wireless mesh networks," in *MOBICOM 2005*.

[19] M. Marsan, A. Nucci C. Chiasserini, G. Carello, and L. Giovanni, "Optimizing the topology of bluetooth wireless personal area networks," in *INFOCOM 2002*.

[20] J-H. Chang and L. Tasiulas, "Energy conserving routing in wireless ad hoc networks," in *INFOCOM 2000*.

[21] J-H. Chang and L. Tassiulas, "Routing for maximum system lifetime in wireless ad-hoc networks," in *Proceedings of 37th Annual Alterton Conference on Communication, Control and Computing*, September 1999.

[22] B. Hamdaoui and P. Ramanathan, "Sufficient conditions for flow admission control in wireless ad-hoc networks," *Mobile Computing and Commucations Review*, vol. 9, no. 4, 2005.

[23] J. C. Mundarath, P. Ramanathan, and B. D. Van Veen, "NULLHOC: A MAC protocol for adpative antenna array based wireless ad hoc netowrks in multipath environments," in *GLOBECOM 2004*.

[24] T. Marzetta and B. M. Hochwald, "Capacity of mobile multiple-antenna communication link in rayleigh flat fading," *IEEE Tran. on Information Theory*, vol. 45, no. 1, pp. 139–157, January 1999.

[25] B. Hamdaoui and P. Ramanathan, "A cross-layer admission control framework for wireless ad-hoc networks using multiple antennas," *To Appear in IEEE Transactions on Wireless Communications*.

[26] B. Hamdaoui and P. Ramanathan, "Energy-efficient and MAC aware routing for data aggregation in wireless sensor networks," *IEEE Sensor Network Operations*, March 2005.

[27] *Ad-hoc On-demand Distant Vector Routing*, IETF RFC 3561.

Bechir Hamdaoui received the Diploma of Graduate Engineer from the National School of Engineers at Tunis (BAC+6+DEA, ENIT), Tunisia, in 1997. He also received M.S. degrees in both Electrical & Computer Engineering and Computer Sciences, and Ph.D. degree in Computer Engineering all from the University of Wisconsin at Madison in 2002, 2004, and 2005, respectively. From 1998 to 1999, he worked as a quality control and planning engineer on power generation plant project under the supervision of FIAT Avio. He was an intern at Telcordia during the summer of 2004. Since his graduation in August of 2005, he joined Real-Time Computing Lab at the University of Michigan at Ann Arbor as a postdoctoral researcher. His research spans various disciplines in the area of wireless networking. Specifically, he focuses on developing and designing protocols and methods that provide next-generation wireless networks with the capabilities of energy-aware routing, admission control and QoS, spectrum agile access and sharing, and efficient usage of network resources.

Parameswaran Ramanathan received the B. Tech degree from the Indian Institute of Technology, Bombay, India, in 1984, and the M. S. E. and Ph. D. degrees from the University of Michigan, Ann Arbor, in 1986 and 1989, respectively. Since 1989, Dr. Ramanathan has been faculty member in the Department of Electrical & Computer Engineering, University of Wisconsin, Madison, where is presently a Full Professor and the Department Chair. He leads research projects in the areas of sensor networks and next generation cellular technology. In 1997-98, he took a sabbatical leave to visit research groups at AT&T Laboratories and Telcordia Technologies. Dr. Ramanathan's research interests include wireless and wireline networking, real-time systems, fault-tolerant computing, and distributed systems. He is presently an Associate Editor for IEEE Transactions on Mobile Computing. He has also served as an Associate Editor for IEEE Transactions on Parallel and Distributed Computing (1996-1999) and Elsevier Ad-Hoc Networks Journal (2002-2005). He has also served on program committees of conferences such as Mobicom, Mobihoc, International Conferences on Distributed Systems and Networks, Distributed Computing Systems, Fault-tolerant Computing Symposium, Real-time Systems Symposium, Conference on Local Computer Networks, and International Conference on Engineering Complex Computer Systems. He was the Finance and Registration Chair for the Fault-tolerant Computing Symposium (1999). He was the program co-chairman of the Workshop on Dependendability Issues in Wireless Ad Hoc Networks and Sensor Networks, Workshop on Sensor Networks and Applications (2003), Broadband Wireless (2004), Workshop on Architectures for Real-time Applications, 1994 and the program vice-chair for the International Workshop on Parallel and Distributed Real-time Systems, 1996. He is a member of Association of Computing Machinery and a senior member of IEEE.