Statistical Characterization of Uplink Interference in Two-Tier Co-Channel Femtocell Networks

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***Abstract*—Femtocell (FC) is a new networking paradigm that differs from the traditional, macrocell (MC) network in many ways: size/coverage, random deployment, autonomous operation, signal propagation environment, etc. Inter-cell interference is a major issue in FCs operating over the same channel as the underlying MC. In this paper, we present an analytic study of the uplink (UL) physical interference and other related metrics, namely the signal to interference ratio (SIR) and the outage probability in FCs. We consider a stochastic model in which the spatial distribution of the femto users (FUs) and the macro users (MUs) is described by two independent, homogeneous Poisson Point Processes (PPPs). We first characterize the UL interference at the FC by deriving its first and second order statistics, its probability density function (PDF) and its tail distribution. Second, we derive the PDF of the per-FU SIR, its temporal auto-correlation and the outage probability. Finally, we validate the derived outage probability via Monte-Carlo simulations and show how this probability constrains the system capacity in terms of number (or density) of FUs that could be accepted in the MC.**

Index Terms—Femtocells, Poisson Point Process, Large Deviations, SIR, Temporal Correlation, Outage Probability.

I. INTRODUCTION

Femtocell is a new networking paradigm that has emerged as a response to the wireless operators needs of providing high capacity and high coverage for wireless users. A FC is a low power, small-area-covering wireless cellular network consisting of one Femto Access Point (FAP) and stationary or low-mobility FUs deployed in an indoor environment such as a home or an office environment. Characterizing the communication performance of single-hop transmissions from a FU to its associated FAP is a fundamental step towards understanding the phenomena that may affect the FC system performance and designing efficient strategies to combat them. It is well known that the presence of co-channel interference represents the most important cause of performance degradation in wireless systems. Therefore, it has been the focus of some recent research works [1]–[7]. The study of the interference in two-tier FC/MC networks presents new challenges mainly due to: the particularity of the propagation environments to be considered (Indoor, Indoor-to-Outdoor, Outdoor-to-Indoor) [8], the presence of cellular users (FUs and MUs) with different transmission capabilities and degrees of autonomy, the spatial and the temporal distribution of these users, etc. Some of the recent works [2]–[4] addressed the downlink (DL) physical interference in two-tier FC/MC networks. For instance, in [2], the authors derived the PDF of the DL SIR for FC networks. Although they took into account the shadowing effect, they did not consider the spatial nor the temporal distribution of the cellular users (CUs) in their analysis. In [3], Chu et al. derived the DL outage probabilities for the CUs conditioned on their distances from the macro base station (MBS). In [4], the authors studied the performances of a two-tier FC network in terms of DL outage probability, under the assumption of Poisson-spatially-distributed FAPs. Other works [5]–[7] addressed the UL interference in two-tier FC networks. In [7], Chandrasekhar and Andrews investigated the UL capacity in overlaid FC/MC CDMA systems through the derivation of lower bounds on the per-tier outage probability. In [5], the authors used a hierarchical static geometric model to characterize the UL interference at the FCs. Jeney [6] also presented an analytical characterization of the UL interference and outage probability while assuming uniformly-distributed MUs and FAPs locations. In this paper we present a statistical characterization of the UL co-channel physical interference, the SIR and the outage probability at the FAPs in two-tier time division multiple access (TDMA) FC/MC networks. In our analysis, we take into consideration the wireless environment specificities (shadowing and pathloss), the spatial distribution and the temporal distribution of the FUs and MUs. We also study the tail probability distribution/asymptotic behavior of the interference based on the large deviations theory. Finally, we characterize the temporal auto-correlation of the SIR for the case of stationary CUs and slowly-moving CUs. The remainder of this paper is organized as follows. Section II describes the system model. Section III characterizes the UL interference of FC network. Section IV studies the SIR, its temporal auto-correlation and the outage probability. Section V presents system evaluation via simulations. Section VI concludes the paper.

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II. NETWORK MODEL

We consider a single-carrier two-tier cellular system consisting of FCs (with coverage radius R) overlaid on one MC (with coverage radius $R_M >> R$), where both of them operate over an identical carrier frequency f. We model the spatial distribution of the MUs and the FUs using two independent homogeneous PPPs, ϕ_1 and ϕ_2, in the two-dimensional plane, with intensities λ_1 and λ_2 respectively. For a PPP with intensity λ, the probability of n nodes being inside a region Z depends only on the total area A_Z of Z and is given by:

$$\mathbb{P}(n \in Z) = \frac{(\lambda A_Z)^n}{n!} e^{-(\lambda A_Z)}$$

Here λ is the spatial density of interfering nodes (in our case λ_1 for FUs and λ_2 for MUs), in nodes per unit area. In this work, we consider the UL communication stream; i.e., communication from the MUs to the MBS and from the FUs to their corresponding FAPs. We assume that TDMA is used by the CUs (MUs and FUs) to access the wireless channel, and that the UL communications at the FCs are synchronized with those at the MC [9][1], and consequently are mutually synchronized. We further assume that FUs residing in the same FC do not interfere with each other since they are scheduled in different time slots. In our model, each CU is in one of two states: On or Off. We denote $\delta_i(t)$ the indicator of the activity of user i:

$$\delta_i(t) = \begin{cases} 1 & \text{if user i is active (On) at time t} \\ 0 & \text{if user i is inactive (Off) at time t} \end{cases}$$

And, we assume that all FUs and MUs have the same average activity rate, which we denote by $\overline{\delta}$. According to our model, there is only one active FU (FU_i) in each femtocell FC_i at a given time slot t. Hence, we are interested in the interference caused by the neighboring active FUs and the neighboring active MUs at FAP_i. Using the PPP assumption about node locations, we model the interference's spatial distribution as follows: We consider that the FAP (FAP_i) is located at the center of a disk of radius R representing the area of FC_i covered by FAP_i. Since only FU_i is active at time slot t at FC_i, then the interference at FAP_i originates from the active FUs located in the annulus Z_1, delimited by the radii R and R_1 (with respect to the center FAP_i of FC_i), and also from the active MUs confined in the annulus Z_2, delimited by the radii R and R_2 from FAP_i. R_1 and R_2 are chosen such that the interference due to FUs beyond R_1 (respectively MUs beyond R_2) is negligible. In our analysis, we assume that the physical channel gain is represented by a combination of path-loss and log-normal shadowing in compliance with the ITU specification [10].

III. Interference Analysis

In this section, we present a statistical characterization of the interference in FC networks. We first determine its average and variance, based on PPP properties. Then, we derive its probability density function (PDF). We finally study/characterize the asymptotic behavior of the interference, i.e. its tail probability distribution in order to determine the factors that may cause a large deviation of the interference at a FAP. In our FC network, we assume a TDMA operation where only one FU is active per FC per time slot. However, when the femto user FU_i is communicating with its associated FAP_i at time slot t, its signal may be affected by the transmissions of the neighboring active FUs and MUs. Hence the interference at FAP_i at time slot t could be expressed as:

$$I(t) = \sum_{j \in Z_{F1}} \delta_j(t) r_j^{-\alpha_1} S_j(t) P_j(t) + \sum_{k \in Z_{M2}} \delta_k(t) r_k^{-\alpha_2} S_k(t) P_k(t)$$

[1]Once turned on and before initiating any communication, FCs get synchronized to the cellular core network using an asymmetric communication link such as xDSL thanks to an enhanced version of IEEE 1588 [9].

The interference expression consists of two sums: the first one is over the set of neighboring active FUs, Z_{F1}, confined in the region Z_1, and the second one is over the set of neighboring active MUs, Z_{M2}, confined in the region Z_2. In this expression, $\alpha_1 > 2$ and $\alpha_2 > \alpha_1 > 2$ denote the path loss exponents associated with the FUs and the MUs respectively, S_j denotes the log-normal shadowing amplitude related to user j, P_j denotes the transmission power of user j, and r_j its distance from FAP_i. Assuming that all the nodes transmit with constant power, we denote $X_j(t) = S_j(t)P_j(t)$, $\forall j \in Z_{F1} \cup Z_{M2}$. $X_j(t)$ are independent log-normal random variables $\forall j \in Z_{F1} \cup Z_{M2}$. We further assume that $X_j(t)$ are i.i.d (independent identically distributed) with mean μ_1 and variance σ_1^2, $\forall j \in Z_{F1}$. Likewise, $X_k(t)$ are i.i.d with mean μ_2 and variance σ_2^2, $\forall k \in Z_{M2}$. Note here j and k represent the interfering FUs and MUs indices respectively. Hence, I(t) can be written as:

$$I(t) = \sum_{j \in Z_{F1}} \delta_j(t) r_j^{-\alpha_1} X_j(t) + \sum_{k \in Z_{M2}} \delta_k(t) r_k^{-\alpha_2} X_k(t) \quad (1)$$

A. Statistical Characterization

For ease of derivation, we use the following notation: $I(t) = I_1(t) + I_2(t)$, with $I_1(t) = \sum_{j \in Z_{F1}} \delta_j(t) r_j^{-\alpha_1} X_j(t)$ and $I_2(t) = \sum_{k \in Z_{M2}} \delta_k(t) r_k^{-\alpha_2} X_k(t)$.

Theorem 1: The average μ_I and the variance σ_I^2 of the interference at FAP_i can be expressed as

$$\begin{aligned} \mu_I &= \frac{2\Pi\lambda_1}{\alpha_1 - 2}\overline{\delta}\mu_1\left(\frac{1}{R^{\alpha_1-2}} - \frac{1}{R_1^{\alpha_1-2}}\right) \\ &+ \frac{2\Pi\lambda_2}{\alpha_2 - 2}\overline{\delta}\mu_2\left(\frac{1}{R^{\alpha_2-2}} - \frac{1}{R_2^{\alpha_2-2}}\right) \\ \sigma_I^2 &= \overline{\delta}(\sigma_1^2 + \mu_1^2)\frac{\Pi\lambda_1}{\alpha_1 - 1}\left(\frac{1}{R^{2(\alpha_1-1)}} - \frac{1}{R_1^{2(\alpha_1-1)}}\right) \\ &+ \overline{\delta}(\sigma_2^2 + \mu_2^2)\frac{\Pi\lambda_2}{\alpha_2 - 1}\left(\frac{1}{R^{2(\alpha_2-1)}} - \frac{1}{R_2^{2(\alpha_2-1)}}\right) \end{aligned}$$

Proof: The proof of this theorem uses the law of total expectation, the law of total variance and Campbell's theorem for PPP [11]. We have $\mu_I \triangleq \mathbb{E}[I(t)] = \mathbb{E}[I_1(t)] + \mathbb{E}[I_2(t)]$. Moreover the two sums $I_1(t)$ and $I_2(t)$ are independent since the two PPPs ϕ_1 and ϕ_2 are independent, the activity of MUs and FUs are independent, and the shadowing factors of the different interfering users are also mutually independent. Hence, $\sigma_I^2 \triangleq \mathbb{V}[I(t)] = \mathbb{V}[I_1(t)] + \mathbb{V}[I_2(t)]$. In the rest of this proof, we will only present the derivation of $\mathbb{E}[I_1(t)]$ and $\mathbb{V}[I_1(t)]$ (the derivation of $\mathbb{E}[I_2(t)]$ and $\mathbb{V}[I_2(t)]$ uses exactly the same techniques). Using the law of total expectation we have:

$$\mathbb{E}[I_1(t)] = \mathbb{E}_r[\mathbb{E}_\delta[\mathbb{E}_X[I_1(t)|r,\delta]]] = \mathbb{E}_r[\mu_1\overline{\delta}\sum_{j \in Z_{F1}} r_j^{-\alpha_1}]$$

By applying Campbell's Theorem, we get:

$$\begin{aligned}\mathbb{E}[I_1(t)] &= \mu_1\overline{\delta}\int_R^{R_1}\frac{1}{r^{\alpha_1}}2\Pi\lambda_1 r\,dr\\ &= \frac{2\Pi\lambda_1\mu_1\overline{\delta}}{\alpha_1-2}(\frac{1}{R^{\alpha_1-2}}-\frac{1}{R_1^{\alpha_1-2}})\end{aligned}$$

On the other hand, using the law of total variance we have $\mathbb{V}[I_1(t)] = \mathbb{E}[\mathbb{V}[I_1(t)|r,\delta]] + \mathbb{V}[\mathbb{E}[I_1(t)|r,\delta]]$ with:

$$\begin{aligned}\mathbb{E}[\mathbb{V}[I_1(t)|r,\delta]] &= \mathbb{E}[\sigma_1^2\sum_{j\in Z_{F1}}(\delta_j(t)r_j^{-\alpha_1})^2]\\ &= \sigma_1^2\overline{\delta}\mathbb{E}[\sum_{j\in Z_{F1}}(r_j^{-\alpha_1})^2]\\ \mathbb{V}[\mathbb{E}[I_1(t)|r,\delta]] &= \mathbb{V}[\mu_1\sum_{j\in Z_{F1}}\delta_j(t)r_j^{-\alpha_1}]\\ &= \mu_1^2\{\mathbb{E}[\mathbb{V}[\sum_{j\in Z_{F1}}\delta_j(t)r_j^{-\alpha_1}|r_j]]\\ &+ \mathbb{V}[\mathbb{E}[\sum_{j\in Z_{F1}}\delta_j(t)r_j^{-\alpha_1}|r_j]]\}\\ &= \mu_1^2\{(\overline{\delta}-\overline{\delta}^2)\mathbb{E}[\sum_{j\in Z_{F1}}(r_j^{-\alpha_1})^2]\\ &+ \overline{\delta}^2(\mathbb{E}[(\sum_{j\in Z_{F1}}r_j^{-\alpha_1})^2]-\mathbb{E}[\sum_{j\in Z_{F1}}r_j^{-\alpha_1}]^2)\}\end{aligned}$$

On the other hand, we have:

$$\begin{aligned}\mathbb{E}[(\sum_{j\in Z_{F1}}r_j^{-\alpha_1})^2] &= \mathbb{E}[\sum_{j\in Z_{F1}}(r_j^{-\alpha_1})^2]+\mathbb{E}[\sum_{i\neq j\in Z_{F1}}\frac{1}{r_i^{\alpha_1}r_j^{\alpha_1}}]\\ &= \mathbb{E}[\sum_{j\in Z_{F1}}(r_j^{-\alpha_1})^2]\\ &+ \int_{Z_{F1}}\int_{Z_{F1}}r_1^{-\alpha_1}r_2^{-\alpha_1}\,\phi_1(dr_1)\,\phi_1(dr_2)\\ &= \mathbb{E}[\sum_{j\in Z_{F1}}(r_j^{-\alpha_1})^2]+\mathbb{E}[\sum_{j\in Z_{F1}}r_j^{-\alpha_1}]^2\end{aligned}$$

Hence, $\mathbb{V}[\mathbb{E}[I_1(t)|r,\delta]] = \mu_1^2\overline{\delta}\mathbb{E}[\sum_{j\in Z_{F1}}(r_j^{-\alpha_1})^2]$. Thus:

$$\begin{aligned}\mathbb{V}[I_1(t)] &= \overline{\delta}(\sigma_1^2+\mu_1^2)\mathbb{E}[\sum_{j\in Z_{F1}}(r_j^{-\alpha_1})^2]\\ &= \overline{\delta}(\sigma_1^2+\mu_1^2)\frac{\Pi\lambda_1}{\alpha_1-1}(\frac{1}{R^{2(\alpha_1-1)}}-\frac{1}{R_1^{2(\alpha_1-1)}})\end{aligned}$$

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The knowledge of the statistics of the UL interference may have many interesting applications in the design of FC networks and the improvement of its PHY layer performance such as the reduction of the aggregate interference, or QoS-aware network design, etc. Another important feature of interference is its PDF. In fact, in some noncooperative systems, the estimation of the interference value is of a paramount importance [12]. And, we believe that the characterization of the interference's PDF would be helpful for the estimation of its realization per time slot (this is not the scope of this work, but we are planning to investigate it in some future work). It would also help us study the asymptotic behavior of the interference and provide some insights about the factors that induce a large deviation of this interference.

We notice that the expression of the interference at FAP_i at any time slot t is nothing but the sum of independent log-normal random variables. Hence, using the Fenton-Wilkinson approximation [13] about the distribution of the sum of log-normal random variables we conclude the following:

Corollary 1: At any time slot t, $I(t)$ is a log-normal random variable characterized by the following PDF:

$$f_I(x) = \frac{1}{\sqrt{2\Pi}x\sigma_{eq}}\exp(\frac{-(\ln x-\mu_{eq})^2}{2\sigma_{eq}^2}) \quad (2)$$

Where: $\mu_{eq} = \ln(\frac{\mu_I^2}{\sqrt{\sigma_I^2+\mu_I^2}})$ and $\sigma_{eq}^2 = \ln(\frac{\sigma_I^2+\mu_I^2}{\mu_I^2})$.

B. Asymptotic Behavior

In this section, we study the probability of the interference blowing up as a function of the number of interferers and their transmission power. Therefore, we resorted to some large deviation theory results developed for the heavy tailed type of distributions, more particularly the log-normal distribution. Based on the fact that the mean and the standard deviation of the shadowing of the MUs are higher than those of the FUs [14], and using the large deviation lemma for the sum of log-normal distributions [15], we derived the following theorem.

Theorem 2: Let n_1 and n_2 denote the number of FU interferers and MU interferers respectively, and let $n = n_1 + n_2$ be the total number of interferers. Let $I_{(n)}$ denote the interference at FAP_i in the presence of n interferers. Then, for $\alpha_2 = 4$, we have as $x \to \infty$:

$$\mathbb{P}(I_{(n)} > x) \sim n_2\overline{F}_{\mu_{max},\sigma_{max}^2}(x) \quad (3)$$

$\overline{F}_{\mu_{max},\sigma_{max}^2}(x)$ is the Complementary Cumulative Distribution Function (CCDF) of the log-normal distribution with parameters μ_{max} (mean) and σ_{max}^2 (variance), where

$$\begin{aligned}\sigma_{max}^2 &= \ln(\frac{\sigma_2^2+\mu_2^2}{\mu_2^2})\\ \mu_{max} &= \ln(\frac{\mu_2^2}{\sqrt{\sigma_2^2+\mu_2^2}})+\ln\{\frac{e^{-\lambda_2\Pi R}}{R}-\frac{e^{-\lambda_2\Pi R_2}}{R_2}\\ &+ \ln(\frac{R_2}{R})+\sum_{n=0}^{\infty}\frac{(-\lambda_2\Pi)^n(R_2^n-R^n)}{n.n!}\}\end{aligned}$$

Proof: The proof of this theorem follows from the lemma in [15]. This lemma implies that the sum of independent log-normally distributed random variables is asymptotically equivalent to that of their maximum. In our case, the interference is consisting of two sums: the first one is over the log-normal random variables $(\frac{1}{r^{\alpha_1}}X_j)_{j\in Z_{F1}}$ associated with the FUs, the second one is over the log-normal random variables $(\frac{1}{r^{\alpha_2}}X_k)_{k\in Z_{M2}}$ associated with the MUs. In what follows, for ease of derivation we substitute $\frac{1}{r^{\alpha_1}}$ and $\frac{1}{r^{\alpha_2}}$ with their means $\mathbb{E}[\frac{1}{r^{\alpha_1}}]$ and $\mathbb{E}[\frac{1}{r^{\alpha_2}}]$ respectively. By applying the lemma in [15], we get $\mathbb{P}(I_{(n)} > x) \sim m_n\overline{F}_{\mu_{max},\sigma_{max}^2}(x)$, with m_n is the number of summands in the interference expression whose variances are equal to σ_{max}^2, and μ_{max} is their corresponding mean. In a two-tier MC/FC network, the mean and the standard deviation of the shadow fading of the MUs are higher than those of the FUs [14]. Hence, $m_n = n_2$ and:

$$\begin{aligned}\sigma_{max}^2 &\triangleq \max(\mathbb{V}[\ln(\mathbb{E}[\frac{1}{r^{\alpha_1}}]X_j)]_{Z_{F1}}, \mathbb{V}[\ln(\mathbb{E}[\frac{1}{r^{\alpha_2}}]X_k)]_{Z_{M2}})\\ &= \mathbb{V}[\ln(X_k)]_{k\in Z_{M2}} = \ln(\frac{\sigma_2^2+\mu_2^2}{\mu_2^2})\\ \mu_{max} &= \mathbb{E}[\ln(\mathbb{E}[\frac{1}{r^{\alpha_2}}]X_k)]_{k\in Z_{M2}}\\ &= \ln(\mathbb{E}[\frac{1}{r^{\alpha_2}}]) + \ln(\frac{\mu_2^2}{\sqrt{\sigma_2^2+\mu_2^2}})\end{aligned}$$

Moreover, in a network where node positions are distributed according to PPP with intensity λ_2, the distance between the origin (in our case the FAP) and any other node is Rayleigh-distributed with mean $\frac{1}{2\sqrt{\lambda_2}}$. Hence, we have for $\alpha_2 > 2$:

$$\begin{aligned}\mathbb{E}[\frac{1}{r^{\alpha_2}}] &\triangleq \int_R^{R_2} \frac{1}{r^{\alpha_2}} 2\Pi\lambda_2 r e^{-\lambda_2\Pi r^2}\, dr\\ &= \lambda_2\Pi\{\frac{e^{-\lambda_2\Pi R}}{R^{(\frac{\alpha_2}{2}-1)}} - \frac{e^{-\lambda_2\Pi R_2}}{R_2^{(\frac{\alpha}{2}-1)}} + \int_R^{R_2} \frac{e^{-\lambda_2\Pi r}}{r^{(\frac{\alpha}{2}-1)}}\, dr\}\end{aligned}$$

Particularly, for $\alpha_2 = 4$ we have:

$$\begin{aligned}\int_R^{R_2} \frac{e^{-\lambda_2\Pi r}}{r^{(\frac{\alpha_2}{2}-1)}}\, dr &= \int_R^{R_2} \frac{e^{-\lambda_2\Pi r}}{r}\, dr\\ &= \ln(\frac{R_2}{R}) + \sum_{n=0}^{\infty} \frac{(-\lambda_2\Pi)^n(R_2^n - R^n)}{n.n!}\end{aligned}$$

In this derivation we have considered $\alpha_2 = 4$ for ease of computation. Moreover, $\alpha_2 = 4$ is a typical value of the pathloss for MUs in two tier FC/MC networks [10]. ■

We notice that the probability of the interference deviating from its mean is fully characterized by the interferers with the highest signal amplitude mean and variance. That is, a large value of the UL interference at a FC is mainly caused by the interfering transmitters with maximum transmitted signal amplitudes, which happen to be the MUs in our case, rather than the total number of interferers.

IV. Signal to Interference Ratio and Outage Probability

In this section, we first derive some statistical characteristics of the UL signal to interference ratio (SIR) that allowed us characterize the link outage probability. Then, we study the temporal auto-correlation of the SIR for the case of stationary CUs and the case of slowly-moving CUs using the uniform mobility model.

A. Statistical Characterization

The SIR of FU_i transmitting at time slot t to its associated FAP_i could be written as:

$$\gamma(t) = \frac{L_i P_i(t) r_i^{-2}}{I(t)}$$

where $P_i(t)$ represents the transmission power of FU_i and L_i represents the attenuation corresponding to the wall loss (assumed constant), and r_i represents the distance between FU_i and FAP_i. In this analysis we assume that r_i does not vary much with time and that FU_i transmits with a constant power for a certain number of time slots. Moreover, since FU_i is located inside the femtocell FC_i (indoor environment) and close enough to FAP_i, we assume that there aren't enough obstructions to affect its transmitted signal. Therefore, we use the free space propagation model (no shadowing) to model the transmission from FU_i to FAP_i, and we denote $K_i = L_i P_i(t) r_i^{-2}$ the signal received at FAP_i from FU_i.

Theorem 3: The PDF of the SIR corresponding to the transmission of FU_i in FC_i to FAP_i is:

$$f_\gamma(u) = \frac{1}{\sqrt{2\Pi} u \sigma_{eq}} \exp(\frac{-(\ln(u) - (\ln(K_i) - \mu_{eq}))^2}{2\sigma_{eq}^2}) \quad (4)$$

And the average and variance of the SIR are:

$$\mu_s = e^{(\ln(K_i) - \mu_{eq} + \frac{1}{2}\sigma_{eq}^2)}$$

$$\sigma_s^2 = (e^{\sigma_{eq}^2} - 1) e^{(2(\ln(K_i) - \mu_{eq}) + \sigma_{eq}^2)}$$

Proof: Note that $\gamma(t) = \frac{K_i}{I(t)}$ is a decreasing function of $I(t)$ whose PDF expression is derived in (2). Hence, applying the statistical transformation method gives

$$\begin{aligned}f_\gamma(u) &= f_I(\frac{K_i}{u})|\frac{d(\frac{K_i}{u})}{du}|\\ &= \frac{1}{\sqrt{2\Pi} u \sigma_{eq}} \exp(\frac{-(\ln(u) - (\ln(K_i) - \mu_{eq}))^2}{2\sigma_{eq}^2})\end{aligned}$$

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In addition, we assume that the transmission from FU_i to FAP_i fails if its SIR (γ) is below a certain defined threshold γ^{th}. This is the case if the interference at FAP_i is high enough compared to the amplitude of the signal transmitted by FU_i, so that this FAP cannot detect it.

Corollary 2: The outage probability $P_o \triangleq \mathbb{P}(\gamma < \gamma^{th})$ of FU_i's transmission to FAP_i is:

$$P_o = \frac{1}{2} erfc(-\frac{\ln(\gamma^{th}) - [\ln(K_i) - \mu_{eq}]}{\sqrt{2\sigma_{eq}^2}}) \quad (5)$$

B. The Temporal Auto-Correlation of the SIR

In many link outage analysis works, the realizations of the SIR are assumed independent across time. However, this is not always the case, especially when the interferers positions are correlated across time. In our analysis, we assume that the nodes are fixed (stationary) or are (at most) moving slowly. Therefore, in the following we derive the temporal autocorrelation of the SIR in two different time slots s and t while distinguishing between two cases:

Case(1)—Mobile interferers: We consider that the interferers, the MUs and the FUs, are moving with constant speeds $\overline{v_1}$ and $\overline{v_2}$ respectively, and their displacement direction is described by an angle θ uniformly distributed in $[0, 2\Pi]$.

Case(2)—Stationary interferers: We consider $\overline{v_1} = \overline{v_2} = 0$.

Theorem 4: The temporal autocorrelation of the SIR (γ) corresponding to the transmission of FU_i to FAP_i at the time slots s and t ($s < t$) is:

Under case(1)—Mobile interferers:

$$R_\gamma(\tau) = \frac{K_i^2 \mathbb{E}[\frac{1}{\overline{\delta}^2(\beta_1 X_j + \beta_2 X_k)(\beta_3 X_j + \beta_4 X_k)}] - \mu_s^2}{\sigma_s^2} \quad (6)$$

where $\tau = t - s$, X_j and X_k denote the log-normal shadowing coefficients related to the FUs and MUs respectively (as defined in (1)), and

$$\beta_1 = \frac{2\Pi\lambda_1}{\alpha_1 - 2}(\frac{1}{R^{\alpha_1 - 2}} - \frac{1}{R_1^{\alpha_1 - 2}})$$

$$\beta_2 = \frac{2\Pi\lambda_2}{\alpha_2 - 2}(\frac{1}{R^{\alpha_2 - 2}} - \frac{1}{R_2^{\alpha_2 - 2}})$$

$$\beta_3 = \int_R^{R_1} \int_0^{2\Pi} \frac{\lambda_1 r}{(r^2 + (\overline{v_1}\tau)^2 + 2\overline{v_1}\tau r \cos(\theta))^{\frac{\alpha_1}{2}}} \, dr \, d\theta$$

$$\beta_4 = \int_R^{R_2} \int_0^{2\Pi} \frac{\lambda_2 r}{(r^2 + (\overline{v_2}\tau)^2 + 2\overline{v_2}\tau r \cos(\theta))^{\frac{\alpha_2}{2}}} \, dr \, d\theta$$

Under case(2)—Stationary interferers:

$\beta_1 = \beta_3$ and $\beta_2 = \beta_4$, thus:

$$R_\gamma(\tau) = \frac{K_i^2 \mathbb{E}[\frac{1}{\overline{\delta}^2(\beta_1 X_j + \beta_2 X_k)^2}] - \mu_s^2}{\sigma_s^2} \quad (7)$$

Proof: Given that the SIR realizations are identically distributed but not independent (i.e. correlated) across time, the temporal autocorrelation of the SIR at the time slots s and t ($s < t$) is:

$$R_\gamma(\tau) = \frac{\mathbb{E}[\gamma(s)\gamma(t)] - \mu_s^2}{\sigma_s^2}$$

where

$$\begin{aligned}
\mathbb{E}[\gamma(s)\gamma(t)] &= K_i^2 \mathbb{E}[\frac{1}{I(t)I(s)}] \\
&= K_i^2 \mathbb{E}[\int_0^{+\infty} \int_0^{+\infty} e^{-(xI(s)+yI(t))} \, dx \, dy] \\
&= K_i^2 \int_0^{+\infty} \int_0^{+\infty} \mathbb{E}[e^{-(xI(s)+yI(t))}] \, dx \, dy
\end{aligned}$$

By further decomposing the interference into two interference terms induced by the neighboring FUs and MUs as in (1), it follows that

$$\mathbb{E}[\gamma(s)\gamma(t)] = K_i^2 \left(\int_0^{+\infty} \int_0^{+\infty} \mathbb{E}[e^{-(xI_1(s)+yI_1(t))}] \, \mathbb{E}[e^{-(xI_2(s)+yI_2(t))}] \, dx \, dy \right) \quad (8)$$

When considering mobile interferers, we have

$$r_j(t) = \sqrt{r_j(s)^2 + (\overline{v_1}\tau)^2 + 2\overline{v_1}\tau r_j(s)\cos(\theta)} \, \forall j \in Z_{F1} \quad (9)$$

$$r_k(t) = \sqrt{r_k(s)^2 + (\overline{v_2}\tau)^2 + 2\overline{v_2}\tau r_k(s)\cos(\theta)} \, \forall k \in Z_{M2} \quad (10)$$

On the other hand for any point process ϕ, its Laplace functional is defined as

$$\boldsymbol{L}_\phi(f) \triangleq \mathbb{E}[e^{-\int_Z f(x)\,\phi(dx)}] = \mathbb{E}[e^{-\sum_{x \in Z} f(x)}] \quad (11)$$

Using (9) and (10), and applying (11) yield

$$\mathbb{E}[e^{-(xI_1(s)+yI_1(t))}] = e^{-\overline{\delta}X_j(\beta_1 x + \beta_3 y)}$$

where

$$\beta_1 = \int_R^{R_1} r^{-\alpha_1} 2\Pi\lambda_1 r \, dr = \frac{2\Pi\lambda_1}{\alpha_1 - 2}(\frac{1}{R^{\alpha_1 - 2}} - \frac{1}{R_1^{\alpha_1 - 2}})$$

$$\beta_3 = \int_R^{R_1} \int_0^{2\Pi} \frac{\lambda_1 r}{(r^2 + (\overline{v_1}\tau)^2 + 2\overline{v_1}\tau r \cos(\theta))^{\frac{\alpha_1}{2}}} \, dr \, d\theta$$

Likewise,

$$\mathbb{E}[e^{-(xI_2(s)+yI_2(t))}] = e^{-\overline{\delta}X_k(\beta_2 x + \beta_4 y)}$$

where

$$\beta_2 = \int_R^{R_2} r^{-\alpha_2} 2\Pi\lambda_2 r \, dr = \frac{2\Pi\lambda_2}{\alpha_2 - 2}(\frac{1}{R^{\alpha_2 - 2}} - \frac{1}{R_2^{\alpha_2 - 2}})$$

$$\beta_4 = \int_R^{R_2} \int_0^{2\Pi} \frac{\lambda_2 r}{(r^2 + (\overline{v_2}\tau)^2 + 2\overline{v_2}\tau r \cos(\theta))^{\frac{\alpha_2}{2}}} \, dr \, d\theta$$

Hence, it follows that

$$\mathbb{E}[\gamma(s)\gamma(t)] = K_i^2 \mathbb{E}[\left(\int_0^{+\infty} \int_0^{+\infty} e^{-\overline{\delta}X_j(\beta_1 x + \beta_3 y)} e^{-\overline{\delta}X_k(\beta_2 x + \beta_4 y)} \, dx \, dy \right)]$$

$$\mathbb{E}[\gamma(s)\gamma(t)] = K_i^2 \mathbb{E}[\frac{1}{\overline{\delta}^2(\beta_1 X_j + \beta_2 X_k)(\beta_3 X_j + \beta_4 X_k)}]$$

■

The characterization of the temporal auto-correlation of the SIR in FCs is important. In fact, it helps characterize the correlation of transmission failures over time. Thus, it provides useful information for the design of retransmission strategies, or power control schemes for efficient reliable FC networks.

V. Numerical Results

Using the physical model discussed in Section II, we apply Monte Carlo numerical techniques to simulate the co-channel interference observed at the FAP for 10^6 samples. At each sample instant, the locations of the active MU and FU interferers are generated as a realization of their corresponding PPPs, and their shadowing coefficients as realizations of their related log-normal distributions. In our simulation, we use the same PHY propagation parameters as in [14] and [10]. Moreover, unless otherwise stated, we fix the PPP intensities to $\lambda_1 = 0.15$ and $\lambda_2 = 0.02$. In Fig. 1, we plot a snapshot of our two-tier FC/MC network. In Fig. 2, we plot the theoretic outage probability derived in (5) and compare it with the Monte

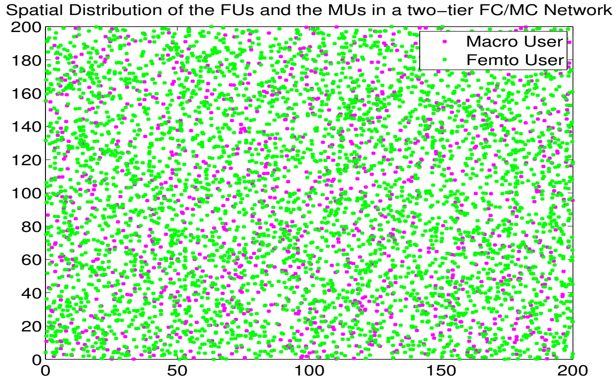

Fig. 1. FC/MC Network Snapshot

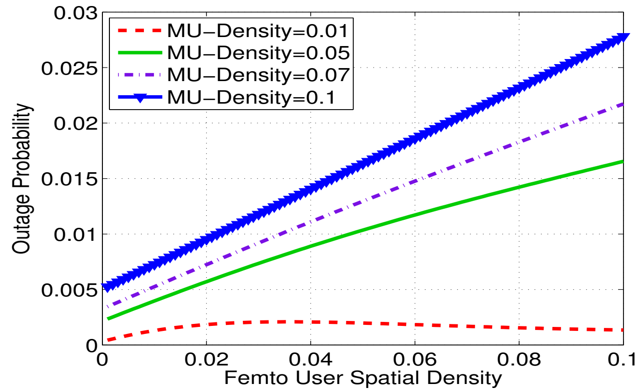

Fig. 3. The Outage Probability as a function of the FU density

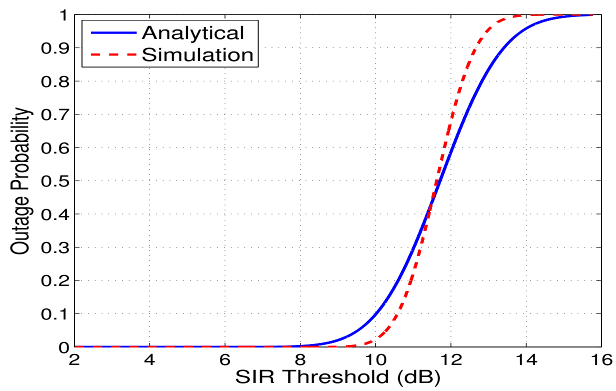

Fig. 2. The Physical Outage Probability

Carlo simulation. Note that at the low SIR regime and the high SIR regime, the analytical outage probability and the simulated one match perfectly. Finally, in Fig. 3, we illustrate the evolution of the outage probability as a function of the FU density for different MC loads (i.e. load in MUs). This curve is of a paramount importance since it constrains the density and consequently the number of active FUs that could be accepted in the underlying MC to meet a desired value of the outage probability. Hence, it would be useful for the design of admission control mechanisms. For instance, in order to maintain the outage probability at the FAP $P_o \leq 0.02$, the density of active FUs in the MC should not exceed 0.06 for a MU density $\lambda_2 \approx 0.1$.

VI. Conclusion

In this paper, we derived statistical characterizations of the UL interference, SIR, and the outage probability in FC networks. This characterization helped us determine the dominant factors affecting the physical interference and the SIR at FCs. One of the main assets of our work is that it provides insights on the interaction of the system design parameters (spatial density, wireless propagation, user activity) and their impact on the two-tier FC/MC system performance. It also extends the calculations to describe the temporal auto-correlation of the SIR with respect to the CU mobility, thereby providing a key parameter that could help in the design of power control or admission control schemes in such networks.

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