AirMAP: Scalable Spectrum Occupancy Recovery Using Local Low-Rank Matrix Approximation

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Abstract—We propose AirMAP, a framework for enabling scalable database-driven dynamic spectrum access and sharing. We bring together the merits of compressive sensing and collaborative filtering to provide accurate radio occupancy map while reducing the network overhead cost and overcome the scalability issue with conventional approaches. We start from an observation that close-by users have a highly correlated spectrum observation and we propose to recover the spectrum occupancy matrix in the borough of each sensing node by minimizing the rank of local sub-matrices. Then, we combine the recovered matrix entries using a similarity criterion to get the global spectrum occupancy map. Through simulations, we show that the proposed framework minimizes the error while reducing the network overhead. We also show that the proposed framework is scalable when considering high frequencies.

Index Terms—Wideband spectrum sensing; compressive sampling; local low rank matrix completion; collaborative filtering.

I. INTRODUCTION

Opportunistic spectrum access has great potential for overcoming radio resource shortage challenges that wireless systems are currently facing [1]. Broadly speaking, spectrum sensing techniques that have been proposed for spectrum awareness can be categorized into two classes: sensing-based approaches [2–5] and database-driven approaches [6–12]. While the former class allows users to identify unused spectrum portions on their own via local measurements, the latter provides users with radio occupancy databases, which users can query to acquire spectrum occupancy information in their vicinity. These databases can, for example, be constructed by relying on observations collected from sensing nodes (SNs) that are deployed specifically for this sensing task. Databasedriven approaches are more attractive due to their practical appeal [6], and as a result, have recently been adopted and embraced by industries (e.g., Google [10], Spectrum Bridge [11], RadioSoft [12]), standard organizations (e.g., 5G), and government agencies (e.g., FCC [13]).

However, current spectrum database-driven approaches suffer from several shortcomings. For instance, they are primarily designed for TV white spaces [6], which represent only a small portion of the wideband spectrum that can potentially be shared. In addition, TV carrier frequencies are mostly below 1 GHz, and hence, these signals can propagate long distances, requiring only a small number of SNs to get the spectrum occupancy in a relatively wide region. Therefore, to extend spectrum databases to cover wider spectrum ranges, say 10 GHz bandwidth or more, a higher number of SNs must be deployed to be able to obtain a complete radio occupancy map covering the entire wideband spectrum, as well as to overcome the hidden terminal problem, where due to, for example, fading, different SNs may observe different primary signals, thereby leading to different occupancy decisions. Fortunately, by exploiting spectrum occupancy sparsity that is inherent to spectrum usage, compressive sensing theory [2] has been leveraged to sense widebands (e.g., 1 GHz bandwidth) at lower sensing overheads (e.g. [14]). Now given that within the same region, the spectrum occupancy seen by the different SNs can roughly be the same for some set of bands, the *occupancy matrix*¹ has a low-rank property. The aim of this work is to exploit this low rank property to construct the occupancy matrix from smaller numbers of observations/sensors [4, 5].

Let us illustrate this further with a simple example. Consider the spectrum occupancy matrix whose columns again represent the occupancy decisions taken by SNs for each band of the wideband spectrum. If the SNs are close to each other, then they roughly observe the same wideband spectrum occupancy, resulting in a low-rank spectrum occupancy matrix. Therefore, one can estimate all the entries of the spectrum matrix by only taking and relying on a small number of measurements [5, 15]. This can be done by means of the low-rank matrix theory which consists of formulating an optimization problem whose objective is to minimize the rank of the matrix, as will be detailed later. This approach is often referred to as collaborative filtering in the machine learning community [16].

Although collaborative filtering reduces the network overhead, it fails to scale well with the number of bands. This limitation comes from the propagation nature of signals at different spectrum frequencies, and especially at high frequencies (e.g. millimeter waves) that is being adopted in 5G systems [17]. Note that *SNs* at different locations tend to observe a completely different spectrum occupancy, which can result in losing the low-rank property of the spectrum occupancy matrix. However, *close-by SNs do observe a similar occupancy*, which means if close-by *SNs* are re-arranged in the spectrum occupancy matrix based on their neighborhoods, then the low-rank property is preserved but only locally; i.e.,

 $^{^{1}}$ It is the matrix whose columns each corresponds to the occupancies of the different bands as seen by the corresponding *SN*.

although the entire occupancy matrix may not be low-rank, sub-matrices preserve their low-rank property. We will refer to this as *local low-rank property*. Hence, to maintain the merits of collaborative filtering in reducing network overhead while taking advantage of the cooperation, this local lowrank property can be exploited to design efficient sensing techniques suitable for database-driven wideband spectrum access.

In summary, compressive sensing and collaborative filtering are found to be useful theories for enabling cooperative wideband spectrum sensing at reduced sensing overhead. However, they suffer from a scalability issue when it comes to considering wideband spectrum (a few GHz). In this work, we propose a sensing framework that exploits the local lowrank property mentioned above to enable scalable occupancy matrix construction suitable for wideband spectrum.

Methodology and Contributions. Our key motivation is to build a radio occupancy map for the wideband spectrum (e.g. 10 GHz or wider) to enable spectrum sharing through a framework termed as AirMAP. We propose to combine the merits of compressive sensing and low-rank matrix theories to reduce the sensing and network overhead while accurately acquiring the spectrum occupancy in the borough of each SN. Unlike previous works, our work relies on *local low-rank matrix approximation* to get the complete spectrum occupancy in the neighborhood of each SN. That is, instead of completing the spectrum occupancy matrix such that it has a low-rank property, we propose to focus on exploiting the local lowrank property. This stems from the fact that neighboring SNs tend to observe the same spectrum occupancies.

The main contributions of this work are:

- We propose an efficient sensing framework that enables scalable construction of the spectrum occupancy matrix for wideband spectrum access and sharing.
- To the best of our knowledge, we are the first to use and combine local low-rank matrix approximation theory with compressive sampling to enable scalable wideband spectrum occupancy recovery at low overhead.
- We construct the spectrum sub-matrices using propagation models suitable for wideband spectrum. This allows to improve the estimation of the observations reported in edges of the regions, thereby enhancing the accuracy of the proposed recovery approach.

The rest of this paper is organized as follows. Section II describes the proposed framework and the intuition behind it. Section III discusses the proposed local low rank based spectrum occupancy matrix recovery as well as its performance. Section IV presents the numerical evaluation. Section V reviews the related works. This work is concluded in Section VI.

II. WIDEBAND SPECTRUM OCCUPANCY RECOVERY FRAMEWORK

A. Framework Overview

We propose AirMAP, a scalable sensing framework that is suitable for database-driven wideband spectrum access and sharing. AirMAP relies on a set of J SNs deployed on a region of interest to construct and update the database (DB) with accurate occupancy information of I bands in the borough of these SNs. Here, we assume that the entire wideband spectrum is composed of I bands. The different components of AirMAP are illustrated in Fig. 1. First, it is important to mention that our focus in this work is on a DB covering very wideband spectrum, e.g. more than 10 GHz. We assume that the SNs leverage compressive sampling theory that exploits spectrum occupancy sparsity to enable sub-Nyquist spectrum sampling rates (e.g., [18]). However, even with sub-Nyquist sampling rates, each SN is assumed not to able to sense the entire spectrum of interest due to its wideness, but rather senses a portion of it; say 1 GHz bandwidth as done in [2]. To reduce the computation and the reporting overheads, the compressed measurements are reported to the DB which recovers the spectrum occupancy of the portion sensed by each SN by exploiting prior information about the spectrum occupancy. Typically, different spectrum portions are assigned to different types of applications, each with a different occupancy statistics [19]. The DB exploits this occupancy heterogeneity across the different spectrum portions, as proposed in [3], to recover occupancies of the entire spectrum.

A major problem in spectrum sensing is the hidden terminal problem, which we address in this framework by relying on multiple SNs deployed across the entire region of interest to provide redundant sensing of each portion of the spectrum. Now since having each SN sense all portions of the entire spectrum is impractical, we propose to use *local low-rank approximation* to efficiently recover the spectrum occupancy in the borough of each of the J SNs.

To sum up, AirMAP consists of: (i) having each SN sense a small portion of the wideband spectrum of interest, (ii) recovery of band occupancies of all the portions at the DB by exploiting a priori information about the spectrum occupancy statistics, and (iii) completion of the occupancy matrix by using low-rank approximation theory to recover the missing band occupancies. Next, we will detail each of these phases.

B. Sub-Nyquist Wideband Spectrum Sensing and Recovery

1) Spectrum Occupancy Model: We consider a practical scenario where a wideband spectrum is allocated to multiple applications; e.g., aviation, satellite communications and maritime, wireless communications, TV broadcasting, ISM, etc. [19]. Applications of the same type are typically allocated bands within the same block. Hence, the spectrum is considered to have a block-like occupancy structure, where each block (accommodating applications of a similar type) has different occupancy behavioral characteristics. The wideband spectrum can then be grouped into g disjoint contiguous blocks, G_i , $i = 1, \dots, g$, with $G_i \cap G_j = \emptyset$ for $i \neq j$. Each block, G_i , is a set of n_i contiguous bands such that $I = \sum_{i=1}^{g} n_i$. Now provided that the actual spectrum occupancy has been observed to be under-utilized; i.e, the total number of occupied bands is small, wideband spectrum sensing can



Fig. 1. An overview of the different components of AirMAP.

be enabled at sub-Nyquist sampling rates [4, 14]. However, even with sub-Nyquist sampling rates, given that the spectrum of interest is wideband, it is unpractical to assume that each SN can sense the entire spectrum. Therefore, in this work, we assume that each SN can only sense few, g_b contiguous spectrum blocks out of the g blocks.

2) Compressed Wideband Spectrum Sensing: Exploiting the fact that the spectrum is under-utilized, compressive sampling theory allows to sense and recover the n bands using m < n branches [2]. After tuning to the block of bands of interest, each branch uses an independent pseudorandom sequence mixed with the received signal to yield a measurement vector [14]

$$\boldsymbol{y} = \boldsymbol{\Psi} \boldsymbol{F}^{-1}(\boldsymbol{x} + \boldsymbol{w}_f) = \boldsymbol{A} \boldsymbol{x} + \boldsymbol{\eta}, \quad (1)$$

where $\boldsymbol{y} \in \mathbb{R}^m$ is the measurement vector taken by each SN, F^{-1} is the inverse discrete Fourier transform, and Ψ is the sensing matrix assumed to have a full rank, i.e. $\operatorname{rank}(\Psi) = m$. Here, Ψ contains the *m* PN sequences generated at the mixer.

After collecting the compressed measurements, and in order to reduce the reporting overhead as well as the computation complexity at the SNs, the compressed measurements are sent to the DB to recover the different bands' occupancy as observed by each SN.

3) Heterogeneous Spectrum Occupancy Recovery: In AirMAP, the DB exploits the occupancy variability across the different blocks to recover the spectrum occupancy information [3]. In essence, this approach encourages the search of the occupied bands in the blocks that have higher average sparsity levels. Such a variability in the block sparsity levels can be incorporated in the formulation through carefully designed weights and formulated as the following weighted ℓ_1 -minimization recovery scheme

$$\mathscr{P}_1: \min_{\boldsymbol{x}} \sum_{i=1}^{\mathbf{g}_b} \omega_i \|\boldsymbol{x}_i\|_{\ell_1} \text{ s.t. } \|\mathbf{A}\boldsymbol{x} - \boldsymbol{y}\|_{\ell_2} \le \epsilon$$
 (2)

where $\boldsymbol{x} = [\boldsymbol{x}_1^T, \cdots, \boldsymbol{x}_{g_b}^T]^T$, \boldsymbol{x}_l^T is a $n_l \times 1$ vector, and ω_i , the weight assigned to block i for $i \in \{1, \cdots, g_b\}$, can be

expressed as [3] $\omega_i = \frac{1/\bar{k}_i}{\sum_{j=1}^{\underline{\kappa}_b} 1/\bar{k}_j}$ and ϵ is a predefined desired error.

C. Global Spectrum Occupancy Matrix Completion

Having recovered the vector \boldsymbol{x} from the compressed measurement \boldsymbol{y} using (2), the energy in each band is compared to a threshold to decide on the occupancy of each band, i.e., $= \frac{1}{T} \sum_{t=1}^{T} |x_i[t]|^2 \leq \lambda$ where T is the number of samples and λ is a predefined threshold that depends on the noise floor. Then, these spectrum decisions are updated to the spectrum occupancy matrix R. Since each SN is sensing only a small portion of the wideband spectrum, most entries of the spectrum occupancy matrix are missing. Conventionally, collaborative filtering is used to recover these missing entries of R as long as the number of observed decisions in R is at least $\xi = O(\alpha^{5/4} r \log \alpha)$ with r is the rank of R and $\alpha = \max(\mathbf{I}, \mathbf{J})$ [15, Theorem 1.1]. That is, the recovery can be formulated as a convex optimization

$$\mathscr{P}_2: \min_{\mathbf{X}} \operatorname{rank}(\mathbf{X}) \text{ s.t. } \sum_{(i,j)\in\Omega} \left(\mathbf{R}_{ij} - X_{ij}\right)^2 \le \epsilon$$
 (3)

or

$$\mathscr{P}_3: \min_{\mathbf{X}} \|\mathbf{X}\|_* \text{ s.t. } \sum_{(i,j)\in\Omega} (\mathbf{R}_{ij} - X_{ij})^2 \le \epsilon$$
 (4)

where $\|\cdot\|_*$ is the nuclear norm and Ω is the the observed part of R. Note that the main difference between both approaches is that \mathscr{P}_3 does not require any knowledge about the rank of R.

When considering high frequencies, this approach fails as the low-rank property is not preserved. This stems from the fact that SNs in different locations observe a completely different spectrum occupancy when the frequencies of interest are relatively high. In this work, we overcome this limitation by proposing an approach that relies on the fact that lowrank property (though is not preserved in the global matrix) is still preserved at the sub-matrix levels, and can therefore be used to complete the global occupancy matrix. This proposed approach is described next.

III. Airmap: Proposed Local Low-Rank Approximation Approach

The distance between SNs is an important metric for our proposed framework. We start from the following observation: the portion (sub-matrix) of the spectrum occupancy matrix that contains close-by SNs possesses a low rank property, though the global matrix does not. Therefore, each submatrix of the global occupancy matrix can be efficiently completed/constructed using \mathcal{P}_3 , as described next.

A. Spectrum Sub-matrices Construction

The spectrum occupancy matrix can be seen as a rating matrix containing zeros and ones, where zeros denote that bands are unoccupied and ones denote that the bands are occupied. First, we scale the values to make the mean equal zero by subtracting 0.5 from each entry of the observed entries in the matrix. This is to distinguish between the observed occupancies (part of Ω) and the ones that need to be recovered (containing zeros). Then, based on their locations, each SN is associated to one or more sub-regions among the q sub-regions. The width of each sub-region is decided based on how far the highest carrier frequency can be detected, which can be computed using practical propagation models for high frequencies [17]. Overlapping between the sub-regions is desired to help decide on the accupancy of the SNs in the sub-regions boundaries. The number of subregions, q, depends only on the highest carrier frequency and the desired accuracy recovery. After deciding on the number of sub-regions, q, and their anchor points (centers), $\{c_k\}_{k=1}^{q}$, the SNs are associated with the anchor points in their range. Note that unlike the approaches proposed for recommendation systems, which use local low-rank matrix approximation such as LLORMA [16] and SLOMA [20], the anchor points are constructed independently from the SNs.

B. Local Low-rank Spectrum Sub-matrices Recovery

The occupancy of each spectrum sub-matrix M^k for k = 1, ..., q is the solution to the optimization problem

$$\mathscr{P}_4: \min_{\mathbf{X}} \|\mathbf{X}\|_* \text{ s.t. } \sum_{(i,j)\in\Omega^k} \left(\mathbf{R}_{ij}^k - X_{ij}\right)^2 \le \epsilon$$
 (5)

where Ω^k is a subset of Ω containing observations used to complete the matrix \mathbb{M}^k . Note that \mathscr{P}_4 is similar to \mathscr{P}_3 except that it only considers a subset of the observed spectrum occupancies Ω .

C. Global Recovery via Weighted Decisions

Having recovered the spectrum occupancy in each submatrix, a global decision, combining these sub-matrices, is made. This is illustrated in Fig. 2. To decide on the observations of the SNs located close to the edges of the regions covered by each of the sub-matrices, we account for the decision of the neighboring SNs. The elements of the global spectrum matrix \hat{R} is then expressed as

$$\hat{R}_{ij} = \sum_{k=1}^{q} \frac{K_{ij}^{k}}{\sum_{s=1}^{q} K_{ij}^{s}} M_{ij}^{k}$$
(6)

where K_{ij}^k is a kernel function applied to the distance, d_{ik} , between SNi and an anchor point c_k . As the distance increases, K_{ij}^k converges to zero. We opted for the following kernel (similarity function)

$$\mathbf{K}_{ij}^{k}(d_{ik}) = \begin{cases} 1, & \text{if } x < d^{th} \\ e^{-\beta d_{ik}}, & \text{otherwise} \end{cases}$$
(7)

with d^{th} is a distance threshold and β is a decay parameter. This similarity function tends to give constant weight within a given neighborhood. As we get further, the similarity decays and goes exponentially to zero.

Finally, the final binary matrix is obtained by checking the sign of each element of the matrix R.



Fig. 2. The different steps of the local low rank matrix based recovery. (1) Spectrum sub-matrices construction, (2) Local low rank matrix completion of each sub-matrix, (3) and (4) Global matrix completion.

D. Computational and Communication Overhead

The merit of the proposed framework is that it builds an accurate radio occupancy map to enable database-driven wideband spectrum sharing. The proposed framework does so while ensuring scalability, in terms of network overhead. Conventionally, making occupancy decisions of spectrum in vicinity of a SN incurs a communication overhead that is linear in T, I, and J. When using compressive sensing without collaborative filtering, the incurred communication overhead is linear in T, J, the number of compressed samples m, and |I/n|. AirMAP incurs a communication overhead cost that is linear in T, J, and the number of compressed samples m. Therefore, network overhead reduction is achieved with our proposed scheme, which also results in lesser reporting energy. In terms of computational complexity, the weighted recovery results in $O(m^2n^3)$ per SN measurement. Hence, the total computation complexity for spectrum recovery is $O(Jm^2n^3T)$. The complexity of the recovery of the global occupancy is equivalent to q times the recovery of \mathscr{P}_4 , or even lesser since this can be excused in parallel.

IV. SIMULATION RESULTS

We consider synthetic data to assess the efficiency of AirMAP using Matlab where simulations is made as follows. We assume the presence of multiple primary users operating in some of I = 250 bands (this can be in the 5–15 GHz range with 20 MHz bandwidth each) unless specified otherwise. The deployment of the active users follows a Poisson point process (PPP) with density $2/Km^2$ deployed in the 2D plane. To mimic real-world scenario, we assume high-frequency bands are reused more frequently than low-frequency bands. Each SN senses only one fifth of the total bands, using sub-Nyquist sampling [3]. To define the sub-matrices, we first compute how far a signal sent over a frequency f_c with a power P = 10W can go. We adopted the 3GPP TR 38.901 UMa LOS path loss model [17] given by

$$PL_{dB} = 32.4 + 20\log_{10}\left(d(m)\right) + 30\log_{10}\left(f_c(GHz)\right)$$
(8)

for $0.5 < f_c < 100GHz$ and the shadow fading standard deviation equal to 7.8 dB. We consider the sensitivity to be -120 dBm, bellow which a signal a considered absent. This allows defining the radii of the circles centered at the anchor points as illustrated in Fig 3. The sensing nodes are deployed according to a uniform PPP with density $10/Km^2$ deployed in the 2D plane. The *SN*s are linked to the closest anchor point forming the sub-matrices. To assess the performance of



Fig. 3. Example of deployment of the sensing nodes.

AirMAP, we generate the entire spectrum occupancy matrix to compare the final recovery matrix with it. Since our focus is on the spectrum occupancy matrix completion, we consider the wideband spectrum recovery of the observed portion from each SN to be error free. The spectrum matrix completion is done using [21]. First, we observed from the generated spectrum occupancy matrix that the low-rank property for the sub-matrices is confirmed while the global matrix has no low-rank property (rank > 50 for the case of having 250 bands).



Fig. 4. Error: proposed approach vs traditional approach.

Fig. 4 shows the recovery error (computed as the Frobenius norm) as a function of the number of frequency bands. First,

observe that our proposed framework allows to achieve a high reduction gain in the error (about 10 times) compared to classical approach. This is thanks to the observation of the local rank property (confirmed through simulations). Second, we observe that as the number of bands increases, the error decreases for both classical and proposed approaches. This is because as the number of bands increases, the global low rank property tends to hold more, and hence, a lower matrix recovery error is achieved.



Fig. 5. Error: proposed approach vs traditional approach as a function of d^{th} .

In Fig. 5, we study the effect of the proposed similarity function used for the global recovery. When d^{th} is small, the user observation is given more weight with respect to the closest anchor point decision and lesser weight as the *SN* gets further. This helps mainly build an accurate decision for users located at the edges of the sub-matrices. As this parameter increases beyond a certain distance threshold, the performance drops and becomes similar to that of the classical recovery, as we no longer favor the decision with respect to the closest anchor point.



Fig. 6. Effect of the number of submatrices.

Fig. 6 studies the effect of the number of anchor points. Overall, we observe that as the number of anchor points increases, a reduction in the error is achieved which confirms the same observation made in Fig. 4.

V. RELATED WORKS

Spectrum awareness. The proposed framework combines advances in both wideband spectrum sensing [2–6, 22–24] and recommendation systems [16, 20]. Authors in [6] proposed SenseLess, a trustworthy database to provide the spectrum availability of TV wideband spectrum. However, this database is only restricted to TV bands. To be able to get the occupancy of wider bandwidth, authors in [2] made a proof of concept for a 1 GHz wide bandwidth scanner. There are also some efforts towards applying machine learning and compressive sampling theories for spectrum sensing [4, 23, 25, 26]. Authors in [25] proposed Rxminer which uses a mixed Gaussian and Rayleigh models to identify spectrum occupancy. Authors in [4] proposed to exploit the low-rank property of the measurement matrix to recover the unreported measurements. The proposed approach assumes that all sensing nodes use the same sensing matrix, which makes the approach unpractical. Moreover, the proposed modeling fails to capture frequency reuse, which is crucial in high-frequency bands. This has also been extended to detect malicious users in [22].

Collaborative filtering. Collaborative filtering was introduced in recommendation systems to handle the information overhead. The main approach uses matrix factorization, which is shown to achieve great performance while being scalable [27]. This is based on the fact that the users' preference for a particular item is only controlled by a small number of latent factors, which translates to a low-rank rating matrix. This assumption does not hold true in realworld applications as shown by [16]. Authors in [16] showed through experiments that when considering the global matrix having a number of low-rank matrices, better performance is achieved, making this approach, LLROMA, attractive to other fields such as multi-label classification, documents, etc. The main concern with LLROMA is the construction of the submatrices which is done by first randomly selecting a number of anchor points, and then, using distance metrics, points are connected to the closest anchor point. Besides, it suffers from high computation and storage cost. Recently, SLOMA [20] has been proposed to overcome LLORMA weaknesses by incorporating the social connections among users. However, the chosen number of anchor points was not justified for both approaches.

VI. CONCLUSIONS

We proposed AirMAP, a framework that builds an accurate spectrum occupancy map for wideband spectrum sharing. AirMAP exploits the under-utilization of the wideband spectrum, the heterogeneity in the spectrum occupancy, and the spatial correlation between sensing nodes to achieve scalable decisions for the spectrum occupancy while incurring small network communication overhead. While this work investigates the performance under randomly deployed SNs, better performance could be achieved when these SNs are carefully placed. Moreover, the proposed framework can be extended to other applications, such as spectrum enforcement and monitoring, which can help recognize the type of signals occupying the wideband spectrum.

ACKNOWLEDGMENT

This work was supported in part by the US National Science Foundation under NSF award CNS-1162296.

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