Nonlinear Expectation Maximization Estimator for TDOA Localization

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***Abstract*—Time-difference-of-arrival (TDOA) techniques are widely used in high-accuracy positioning systems. The weighted nonlinear least squares (NLLS) algorithm can be used in such systems to estimate the target position. If the range measurement errors can be modeled as additive white Gaussian noise (AWGN), the performance of the weighted NLLS algorithm approaches the Cramér-Rao lower bound (CRLB). However, TDOA with weighted NLLS is complex to implement because the covariance matrix of the range measurements is non-diagonal, which requires matrix multiplication and inversion. In this paper, we develop a low-complexity nonlinear expectation maximization localization algorithm. The proposed algorithm is much simpler to implement than the weighted NLLS method since no matrix manipulation is required, while their performances are similar, both approaching the CRLB.**

***Index Terms*—Location estimation, time-difference-of-arrival (TDOA), expectation maximization (EM).**

I. Introduction

The nonlinear least squares (NLLS) algorithm [1] is widely used in time-difference-of-arrival (TDOA) localization systems [2]–[8] because of its excellent performance. If the range measurement errors can be modeled as an additive white Gaussian noise, the accuracy of NLLS approaches the Cramér-Rao lower bound (CRLB). However, the covariance matrix of the range measurements in such systems is non-diagonal, resulting in a high computational complexity. Scaling by MAjorizing a COmplicated Function (SMACOF) strategy [3], [9] can also be applied for position estimation. Compared with NLLS, SMACOF is much more stable but it converges significantly slower. When the number of anchors is large, both the NLLS and SMACOF methods become very complex to implement. Method of moments (MOM) schemes [10]–[13] can be applied for such cases since it does not need iteration, but it performs much worse than the NLLS method.

In this paper, we develop an expectation maximization (EM) [14] based method for TDOA localization. EM is widely applied in estimation to transform a high-dimensional estimation problem into multiple 1-dimensional (1-D) problems. The proposed scheme is easy to implement since it does not require any matrix operation while achieving a similar performance as the NLLS method.

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II. System Model

Consider a TDOA system with M anchors whose positions $\boldsymbol{v}_m = [x_m, y_m, z_m]^T$, $m = 1, 2, \cdots, M$, are known, and one target $\boldsymbol{\theta} = [x, y, z]^T$. The goal is to estimate $\boldsymbol{\theta}$ with N sets of measured relative ranges from the target to the anchors. Without loss of generality, let anchor $\boldsymbol{v}_1$ be the reference anchor. The TDOA values are written as

$$r_{m,1}(n) = d_m(\boldsymbol{\theta}) - d_1(\boldsymbol{\theta}) + b_m - b_1 + n_m(n) - n_1(n), \quad n = 1, \cdots, N; \quad m = 2, \cdots, M \tag{1}$$

where $d_m(\boldsymbol{\theta}) = \|\boldsymbol{\theta} - \boldsymbol{v}_m\|$ is the true distance between the mth anchor and the target, b_m is a positive offset caused by non-line-of-sight (NLOS) propagation, and $n_m(n) \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_m^2)$ is the range measurement error. Note that since b_m is a constant and independent of the target position for each estimation, it does not affect the CRLB.

In practice, NLOS propagation often exists between the target and anchors [15]. Generally, there are two ways to deal with NLOS offsets: NLOS detection and NLOS mitigation. During the range measurement estimation phase, NLOS links may be detected [16], [17] and severe NLOS links that do not contain much useful range information are discarded. Otherwise, NLOS mitigation methods [1], [18], [19] can be applied to improve the performance. After NLOS detection and mitigation, it is reasonable to assume that the residual NLOS offset is sufficiently small, allowing it to be merged with the Gaussian range error term. Thus the TDOA measurement in (1) is simplified as

$$r_{m,1}(n) = d_m(\boldsymbol{\theta}) - d_1(\boldsymbol{\theta}) + n_m(n) - n_1(n). \tag{2}$$

III. Nonlinear Expectation Maximization Method

A. Non-Linear Least Squares Method

The sample mean of $r_{m,1}(n)$ in (2) is expressed as

$$\bar{r}_{m,1} = d_m(\boldsymbol{\theta}) - d_1(\boldsymbol{\theta}) + \bar{n}_m - \bar{n}_1, \tag{3}$$

where $\bar{r}_{m,1} = \frac{1}{N}\sum_{n=1}^{N} r_{m,1}(n)$ and $\bar{n}_m = \frac{1}{N}\sum_{n=1}^{N} n_m(n)$. Note that function $d_m(\boldsymbol{\theta})$ above is neither linear nor quadratic. Taylor series expansion is applied to linearize the equation:

$$d_m(\boldsymbol{\theta}) \approx d_m(\boldsymbol{\theta}_0) + \dot{\boldsymbol{d}}_m(\boldsymbol{\theta}_0)(\boldsymbol{\theta} - \boldsymbol{\theta}_0), \tag{4}$$

where $\boldsymbol{\theta}_0$ is the known position and

$$\dot{\boldsymbol{d}}_m(\boldsymbol{\theta}_0) = \left[\frac{\partial \boldsymbol{d}_m(\boldsymbol{\theta})}{\partial x}\ \frac{\partial \boldsymbol{d}_m(\boldsymbol{\theta})}{\partial y}\ \frac{\partial \boldsymbol{d}_m(\boldsymbol{\theta})}{\partial z}\right]_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}.$$

The approximate sample mean of $r_{m,1}(n)$ is expressed as

$$\tilde{r}_{m,1} \approx \boldsymbol{h}_{m,1}\boldsymbol{\theta} + \bar{n}_m - \bar{n}_1, \tag{5}$$

where

$$\tilde{r}_{m,1} = \bar{r}_{m,1} - d_m(\boldsymbol{\theta}_0) + d_1(\boldsymbol{\theta}_0) + \boldsymbol{h}_{m,1}\boldsymbol{\theta}_0$$
$$\boldsymbol{h}_{m,1} = \dot{\boldsymbol{d}}_m(\boldsymbol{\theta}_0) - \dot{\boldsymbol{d}}_1(\boldsymbol{\theta}_0).$$

In vector-matrix form, the $M-1$ TDOA values are written as

$$\tilde{\boldsymbol{r}} = [\tilde{r}_{2,1}, \tilde{r}_{3,1}, \cdots, \tilde{r}_{M,1}]^T = \boldsymbol{H}\boldsymbol{\theta} + \bar{\boldsymbol{n}}, \tag{6}$$

where

$$\boldsymbol{H} = [\boldsymbol{h}_{2,1}; \boldsymbol{h}_{3,1}; \cdots; \boldsymbol{h}_{M,1}],$$
$$\bar{\boldsymbol{n}} = [\bar{n}_2 - \bar{n}_1, \bar{n}_3 - \bar{n}_1, \cdots, \bar{n}_M - \bar{n}_1]^T.$$

The covariance matrix of the noise vector is expressed as

$$\boldsymbol{C} = \frac{1}{N}\left(\boldsymbol{A} + \sigma_1^2 \times \boldsymbol{1} \times \boldsymbol{1}^T\right), \tag{7}$$

where $\boldsymbol{A} = \mathrm{diag}([\sigma_2^2, \sigma_3^2, \cdots, \sigma_M^2])$ is a diagonal matrix and $\boldsymbol{1}$ is the $(M-1) \times 1$ vector whose elements are all equal to 1.

The least squares estimator is expressed as

$$\hat{\boldsymbol{\theta}} = \left(\boldsymbol{H}^T\boldsymbol{C}^{-1}\boldsymbol{H}\right)^{-1}\boldsymbol{H}^T\boldsymbol{C}^{-1}\tilde{\boldsymbol{r}}. \tag{8}$$

An iterative method is applied to minimize the error due to Taylor series expansion:

- Initialize $\boldsymbol{\theta}_0$
- Iteration
  1) Estimate $\hat{\boldsymbol{\theta}}^k$ with (8)
  2) If $\| \hat{\boldsymbol{\theta}}^k - \hat{\boldsymbol{\theta}}^{k-1} \| < \varepsilon_{\boldsymbol{\theta}}$, stop
  3) Set $\boldsymbol{\theta}_0 = \hat{\boldsymbol{\theta}}^k$, go to step 1
- Until a preset number of iterations is reached or until convergence

As shown in (8), for 3-D localization, each iteration requires the multiplication of a $(3 \times (M-1))$ matrix and an $((M-1) \times (M-1))$ matrix, and the inversion of the covariance matrix and a (3 × 3) matrix. When the number of anchors M is large, the NLLS method will be very expensive to implement. Furthermore, since NLLS is sensitive to the initial position ($\boldsymbol{\theta}_0$), it needs other methods to provide a coarse position estimate (e.g., SMACOF [3]) as its initial position, which will also increase the overall computational complexity.

B. Proposed Algorithm

Here we develop a much simpler method based on the EM algorithm. Re-arranging (5) results in:

$$\begin{aligned}\tilde{r}_{m,1} &\approx \boldsymbol{h}_{m,1}\boldsymbol{\theta} + \bar{n}_m - \bar{n}_1 \\ &= [\boldsymbol{h}_{m,1},\ -1][\boldsymbol{\theta}^T, \bar{n}_1]^T + \bar{n}_m \\ &= \boldsymbol{h}c_{m,1}\boldsymbol{\theta}_E + \bar{n}_m \\ &= \hat{r}_{m,1} + \bar{n}_m,\end{aligned} \tag{9}$$

where $\boldsymbol{h}c_{m,1} = [\boldsymbol{h}_{m,1}, -1]$, $\boldsymbol{\theta}_E = [\boldsymbol{\theta}^T, \bar{n}_1]^T$ and $\hat{r}_{m,1} = \boldsymbol{h}c_{m,1}\boldsymbol{\theta}_E$. Let us define

$$rc_m(l) = \mu_m(l) + nc_m(l),\ l = 1, 2, 3, 4, \tag{10}$$

where $\mu_m(l) = hc_{m,1}(l)\theta_E(l)$ and $\{nc_m(l)\}$ are assumed to be i.i.d. [14]; that is, $nc_m(l) \sim \mathcal{N}(0, \beta_l\sigma_m^2)$, which satisfies $\sum_{l=1}^4 nc_m(l) = \bar{n}_m$. The parameter β_l can be any number that satisfies the conditions $\sum_{l=1}^4 \beta_l = 1$ and $\beta_l > 0$. However, β_l affects the rate of convergence [14]. For example, since the CRLB depends on the anchor layout [20], for some specific layout, one direction (e.g., z-axis) may have a lower accuracy than the other directions [3]. In this case, β_l should be chosen to accelerate the convergence rate.

Thus the range measurement can be rewritten as

$$\tilde{r}_{m,1} = \sum_{l=1}^{4} rc_m(l). \tag{11}$$

Note that $\tilde{r}_{m,1}, m = 2, \cdots, M$ are i.i.d. observation sets and $rc_m(l),\ l = 1, \cdots, 4$ are the complete data set. Applying a similar procedure as described in [14], we can approach the optimal value with the EM iteration, which maximizes the likelihood function $f(\tilde{\boldsymbol{r}}|\boldsymbol{\theta})$:

- Expectation step:

$$U\left(\boldsymbol{\theta}_E, \hat{\boldsymbol{\theta}}_E^{(k)}\right) = c' - \frac{1}{2}\sum_{m=2}^{M}\sum_{l=1}^{4}\frac{1}{\beta_l\sigma_m^2}\left(\mu_{m|\tilde{r}_{m,1},\hat{\boldsymbol{\theta}}^{(k)}}(l) - \mu_m(l)\right)^2,$$

where c' is a constant that does not affect the solution, and

$$\mu_{m|\tilde{r}_{m,1},\hat{\boldsymbol{\theta}}^{(k)}}(1) = \boldsymbol{h}_{m,1}(1)\hat{x}^{(k)} + \beta_1\left(\tilde{r}_{m,1} - \hat{r}_{m,1}^{(k)}\right), \tag{12a}$$

$$\mu_{m|\tilde{r}_{m,1},\hat{\boldsymbol{\theta}}^{(k)}}(2) = \boldsymbol{h}_{m,1}(2)\hat{y}^{(k)} + \beta_2\left(\tilde{r}_{m,1} - \hat{r}_{m,1}^{(k)}\right), \tag{12b}$$

$$\mu_{m|\tilde{r}_{m,1},\hat{\boldsymbol{\theta}}^{(k)}}(3) = \boldsymbol{h}_{m,1}(3)\hat{z}^{(k)} + \beta_3\left(\tilde{r}_{m,1} - \hat{r}_{m,1}^{(k)}\right), \tag{12c}$$

$$\mu_{m|\tilde{r}_{m,1},\hat{\boldsymbol{\theta}}^{(k)}}(4) = -\hat{\bar{n}}_1^{(k)} + \beta_4\left(\tilde{r}_{m,1} - \hat{r}_{m,1}^{(k)}\right). \tag{12d}$$

- Maximization step:

$$\hat{x}^{(k+1)} = \arg\min_x\left[\sum_{m=2}^{M}\frac{1}{\sigma_m^2}\left(\mu_{m|\tilde{r}_{m,1},\hat{\boldsymbol{\theta}}^{(k)}}(1) - \boldsymbol{h}_{m,1}(1)x\right)^2\right], \tag{13a}$$

$$\hat{y}^{(k+1)} = \arg\min_y\left[\sum_{m=2}^{M}\frac{1}{\sigma_m^2}\left(\mu_{m|\tilde{r}_{m,1},\hat{\boldsymbol{\theta}}^{(k)}}(2) - \boldsymbol{h}_{m,1}(2)y\right)^2\right], \tag{13b}$$

$$\hat{z}^{(k+1)} = \arg\min_z\left[\sum_{m=2}^{M}\frac{1}{\sigma_m^2}\left(\mu_{m|\tilde{r}_{m,1},\hat{\boldsymbol{\theta}}^{(k)}}(3) - \boldsymbol{h}_{m,1}(3)z\right)^2\right], \tag{13c}$$

$$\hat{\bar{n}}_1^{(k+1)} = \arg\min_{\bar{n}_1}\left[\sum_{m=2}^{M}\frac{1}{\sigma_m^2}\left(\mu_{m|\tilde{r}_{m,1},\hat{\boldsymbol{\theta}}^{(k)}}(4) + \bar{n}_1\right)^2\right] \tag{13d}$$

These quantities can be easily estimated by using the least squares estimator with one variable, which does not require any matrix operation.

The proposed algorithm is summarized as follows.

- Initialize $\boldsymbol{\theta}_0$
- Outer iteration
    1) Perform Taylor series expansion as in (5)
    2) Inner iteration to estimate $\boldsymbol{\theta}^{k+1}$
        a) Expectation step with (12)
        b) Maximization step with (13)
    3) Set $\boldsymbol{\theta}_0 = \boldsymbol{\theta}^{k+1}$
- Until a preset number of iterations is reached or until convergence.

The proposed method transforms the original 3-D estimation problem into four 1-D problems, eliminating all matrix multiplications and inversions.

C. Computational Complexity

We compare the computational complexity in terms of the number of multiplications of SMACOF [3], NLLS and the proposed algorithm. Since estimating the noise variance is common for all these algorithms, this process is not considered in the comparison.

The SMACOF method needs $M-1$ multiplications to initialize, and $42(M-1)$ multiplications per iteration. However, it converges much slower than the NLLS method [3]. In a typical scenario with 30 iterations, it requires $\approx 1261(M-1)$ multiplications and the inversion of 30 (3×3) matrices.

The NLLS method needs $(M-1)^2+3(M-1)$ multiplications to initialize. Here we assume that the Woodbury matrix identity is used to invert the covariance matrix. For each iteration, it needs $3(M-1)^2+19(M-1)$ multiplications and inversion of one (3×3) matrix. In a typical scenario with 10 iterations, it requires $31(M-1)^2+193(M-1)$ multiplications and inversion of 10 (3×3) matrices. Furthermore, as the NLLS method is sensitive to the initial position, it needs other methods (SMACOF [3], for example) to provide a coarse estimate. Typically with SMACOF coarse estimation (10 iterations), the NLLS method requires a total of $\approx 31(M-1)^2+614(M-1)$ multiplications and inversion of 20 (3×3) matrices.

The proposed method needs $M-1$ multiplications to initialize, and $7M$ multiplications to perform Taylor expansion. For each inner iteration, it needs $14(M-1)$ multiplications but no matrix inversion. In a typical scenario with 10 outer iterations and 5 inner iterations, it requires $\approx 771(M-1)$ multiplications.

The computational complexities of NLLS, SMACOF, and the proposed method are shown in Fig. 1; the proposed method has the lowest complexity among the three. Also, as the number of anchors increases, the computational complexity of the proposed scheme increases at a much slower rate than the other two schemes.

D. Simulation

Localization performance of the proposed scheme is simulated in this section. Fig. 2 shows the simulation layout: 12 anchors are located evenly on a circle with a radius of $\sqrt{2}$ and 25 targets are placed on the grid $x, y = \{0, \pm 0.3, \pm 0.6\}$. The total number of range measurements is set at $N = 50$, and $\beta_l = \frac{1}{4}$.

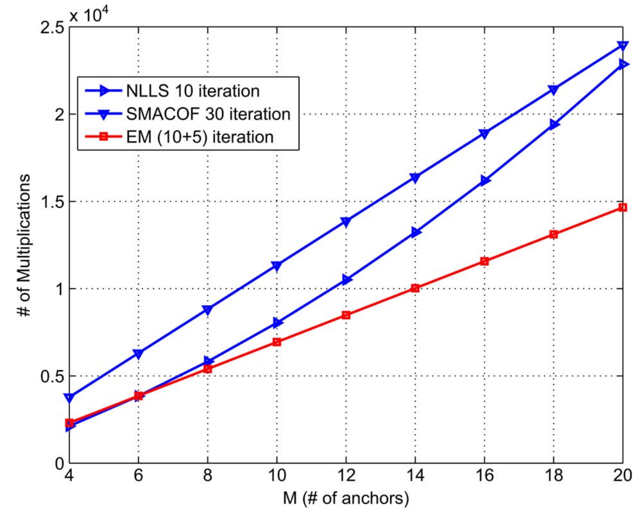

Fig. 1. Computational complexities of NLLS (10 iterations), SMACOF (30 iterations), and the proposed algorithm (10 outer & 5 inner iterations).

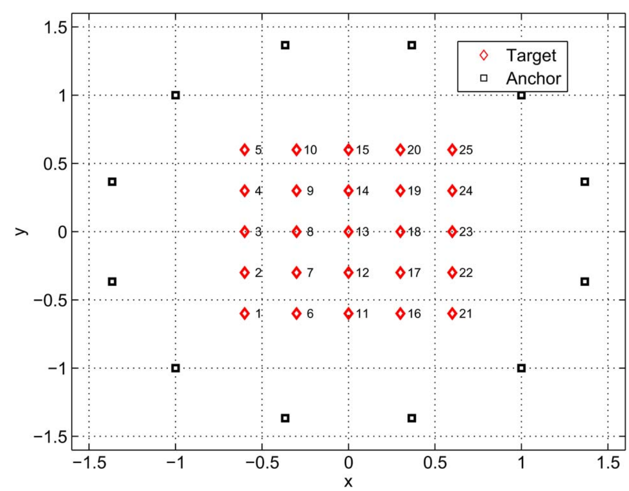

Fig. 2. Layout of the anchors and the targets for simulation.

The range measurement error definitions described in [18] (Type I to IV errors) are adopted here. When there is a severe NLOS offset (Type III and IV errors), we assume that NLOS detection and mitigation [1], [16]–[19] are applied, leaving small residual NLOS errors in the range measurements. This effectively results in Type I and II errors [see the model described by (2)].

Fig. 3 shows the convergence of the proposed method (10 outer iterations). In all cases, the proposed method converges after 5 inner EM iterations. So for the following simulations, the maximum number of inner iterations is set to be 5.

Fig. 4 compares the performances of NLLS-LS, NLLS-WLS, SMACOF and the proposed method assuming Type I range error. The CRLB is also shown for reference. It is observed that the performances of the proposed method and NLLS-WLS are very close, both approaching the CRLB, but are much better than that of NLLS-LS and SMACOF.

Fig. 5 compares the performances of NLLS-LS, NLLS-WLS, SMACOF and the proposed method assuming Type II range error (small NLOS error), where $n_m \sim \mathcal{N}(0, (0.05d_m)^2)$. The NLOS offset $(a\%)$ defined in [18] is adopted here, which is randomly chosen from the uniform distribution $b_m \sim \mathcal{U}(0, a\% \times d_m)$. The performance of the proposed method is very close to that of the NLLS-WLS method for all NLOS offset cases.

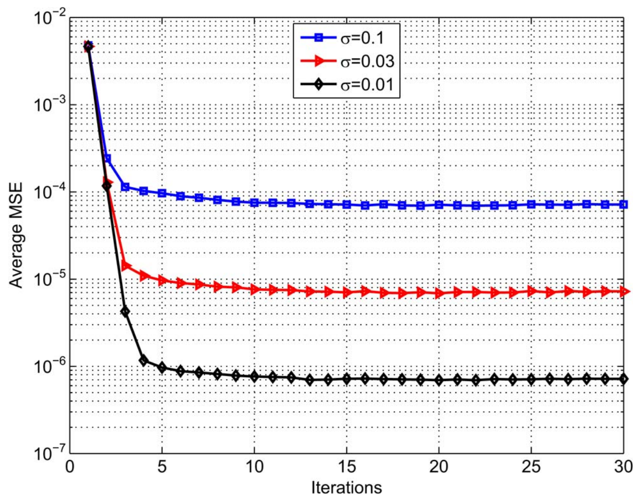

Fig. 3. Convergence of the proposed method assuming AWGN errors (10 outer iterations).

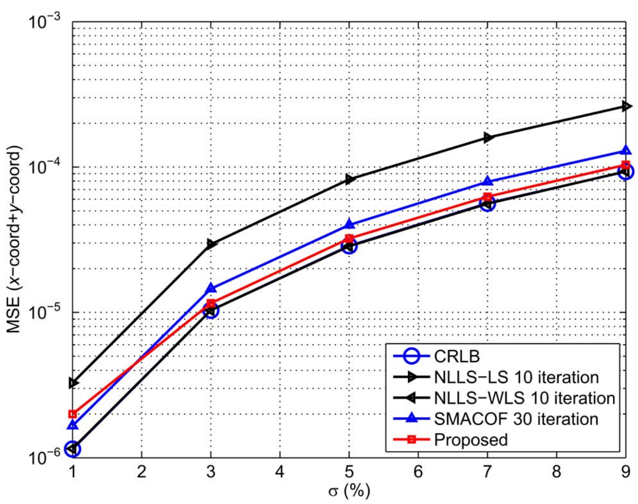

Fig. 4. MSE of NLLS-LS, NLLS-WLS, SMACOF and the proposed method assuming Type I range error, where $(\sigma(a\%))$ denotes the AWGN error with distribution $n_m(n) \sim \mathcal{N}(0, (a\% \times d_m)^2)$.

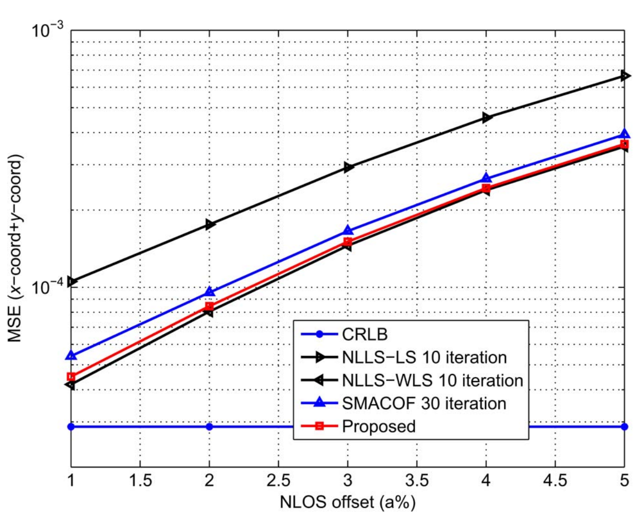

Fig. 5. MSE of NLLS-LS, NLLS-WLS, SMACOF and the proposed method assuming Type II range error, where $n_m \sim \mathcal{N}(0, (0.05d_m)^2)$ and $N = 50$. The NLOS offset $(a\%)$ denotes the NLOS error with distribution $b_m \sim \mathcal{U}(0, a\% \times d_m)$.

IV. Conclusion

We have developed a nonlinear expectation maximization estimator for TDOA localization. This method transforms the 3-D estimation problem into four 1-dimensional problems. The performances of the proposed method and the NLLS method are similar, both approaching the CRLB, but the proposed method has a much lower computational complexity and does not require matrix operations.

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