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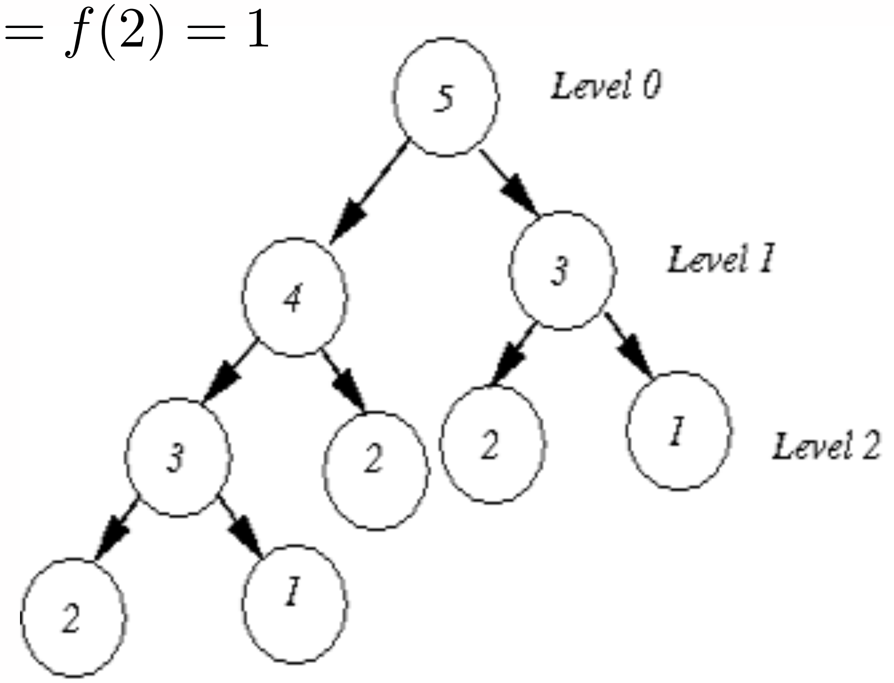

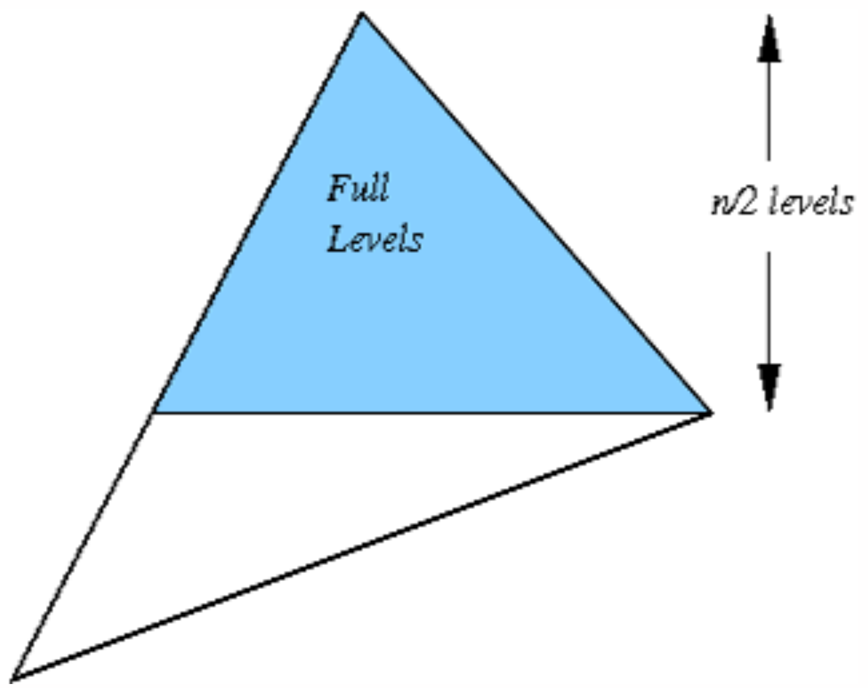

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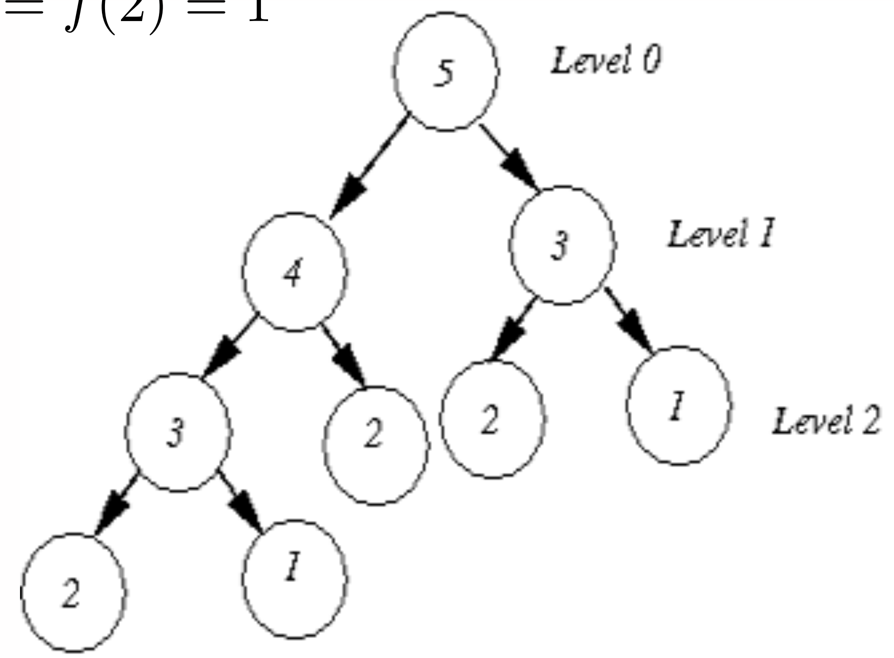

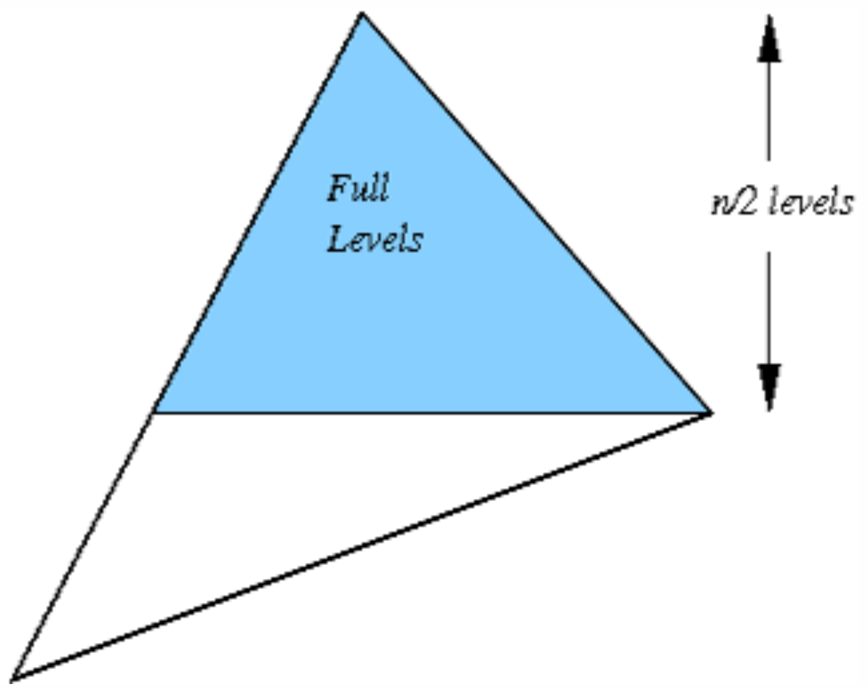

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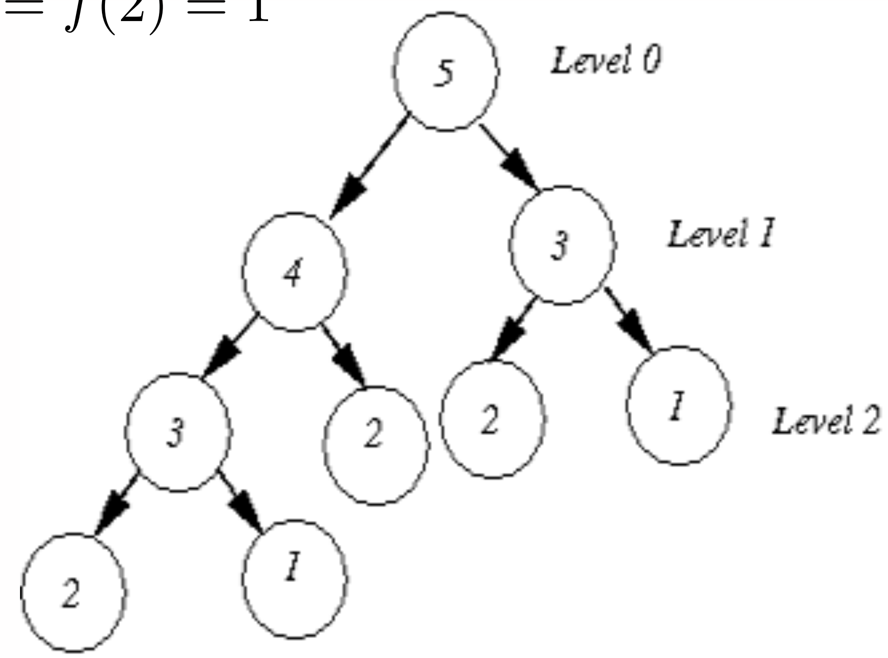

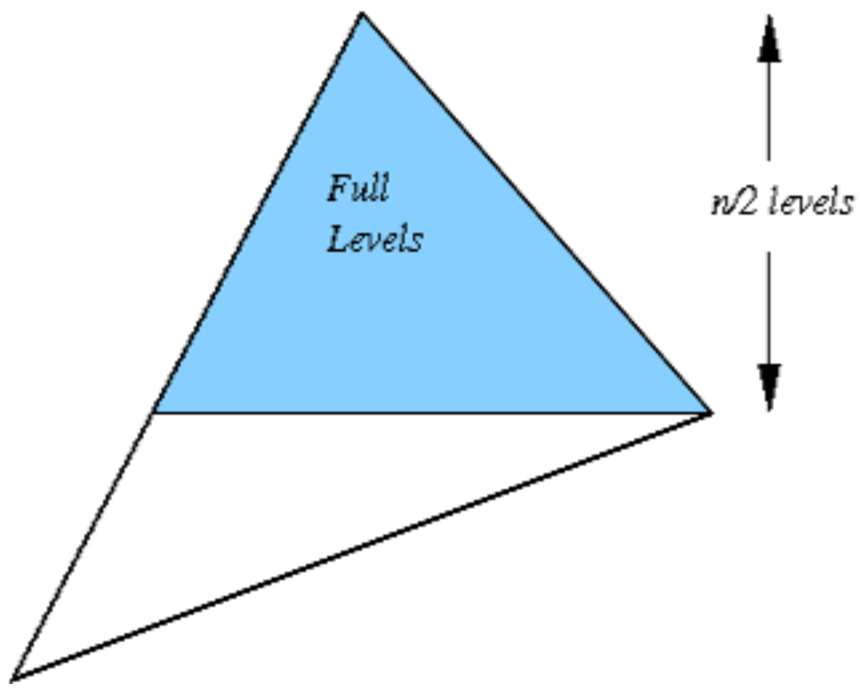

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DP1: top-down with memoization: $O(n)$

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fibs={1:1, 2:1}
def fib1(n):
    if n not in fibs:
        fibs[n] = fib1(n-1) + fib1(n-2)
    return fibs[n]
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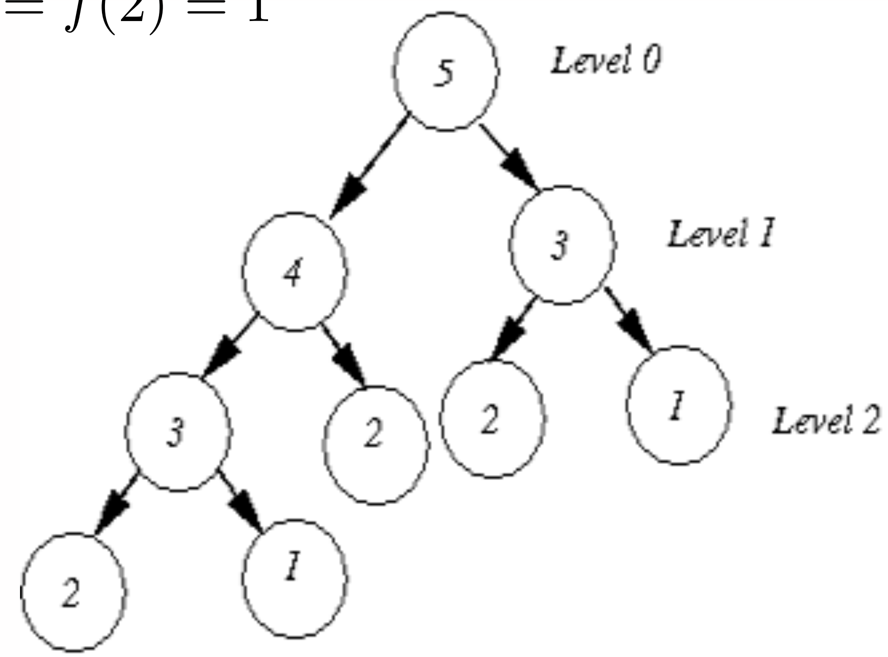

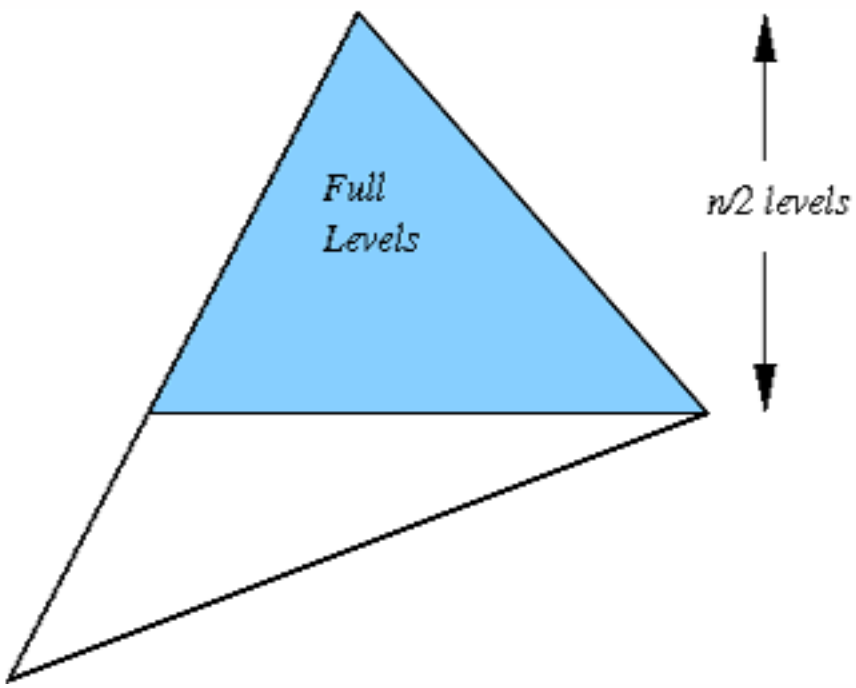

naive recursion without memoization: $O(1.618\ldots^n)$

DP2: bottom-up: $O(n)$

```
def fib0(n):
    a, b = 1, 1
    for i in range(3, n+1):
        a, b = a+b, a
    return a
```

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def fib1(n):
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$f(1)=2$, $f(0)=1$

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$$f(n) = \max\{f(n-1),\;\; f(n-2) + a[n]\}$$

$f(0)=0; f(1)=a[1]$?   better: $f(0)=0; f(-1)=0$

Summary

- Dynamic Programming = divide-n-conquer + overlapping
  - "distributivity" of work: a*c+b*c+a*d+b*d = (a+b)*(c+d)
- two implementation styles
  - 1. recursive top-down + memoization
  - 2. bottom-up
  - also need backtracking for recovering best solution
- three steps in solving a DP problem
  - define the subproblem
  - recursive formula
  - base cases