ENGR 203 – Rec4
Audio Sample Fourier Series using MATLAB

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Contents

1 Introduction

1.1 Overview

Fourier series analysis is a method of representing period waveforms as a series of sinusoidal functions with an increasing number of harmonics. The goal of this recitation is to collect real audio samples from two musical instruments—a viola and a ukulele—and produce the Fourier series representations of those audio samples.

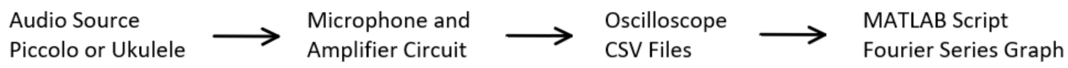

Figure 1: Process Flowchart

1.2 Procedure

Audio samples will be collected from an amplified signal of an electret microphone device and measured using an oscilloscope. The oscilloscope will translate the audio sample to a CSV file containing the waveform which will be interpreted by a MATLAB script. The MATLAB script will parse one full period of the audio sample and represent the waveform as a Fourier series.

2 Microphone and Amplifier Circuit

2.1 Materials

The circuit which collects the data requires:

- a supply voltage
- 1 electret microphone
- 2 operational amplifiers
- resistors

The output voltage will be measured using an oscilloscope which has a feature for exporting waveforms as CSV files. This feature is necessary for importing data into MATLAB.

2.2 Circuit Diagram

Figure 2 is a diagram of the circuit which collects the audio samples. A 5 volt source supplies the microphone and op-amps. The oscope is requires a 12 volt source. The microphone is used in conjunction with a pull-up resistor to output a signal according to the audio it receives. That signal is amplified by two negative feedback op-amps which hold 2.5 volts on the positive input. The amplified signal is read by the oscope which records the data and creates the CSV file. Figure 3 and 4 are the oscilloscope readings of the viola and ukulele.

3 Fourier Series using MATLAB

3.1 Excel Analysis

Two CSV files which represent snapshots of the viola and ukulele audio signals were exported from the oscilloscope and initially analyzed as a spreadsheet in Microsoft Excel. A subset from each CSV file was graphed in Excel to visualize a smaller set of periods. Figures 5 and 6 are the Excel graphs which plot the voltage of the signal as a function of time relative to the subset of data.

3.2 MATLAB Script

MATLAB is the scripting software which was used to plot the Fourier series representations of the data. The following script was used in MATLAB to import the CSV file and graph the Fourier series of one period. The

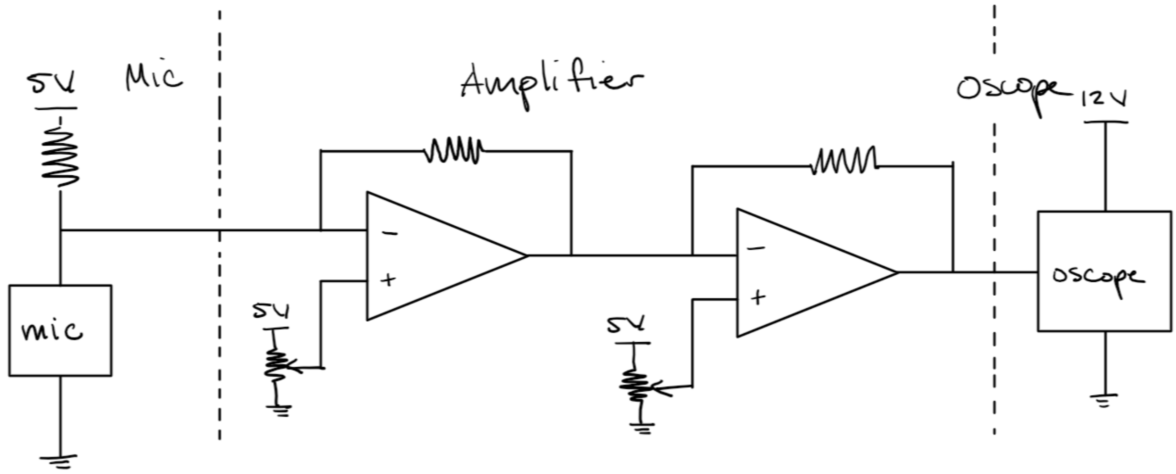

Figure 2: Circuit Diagram

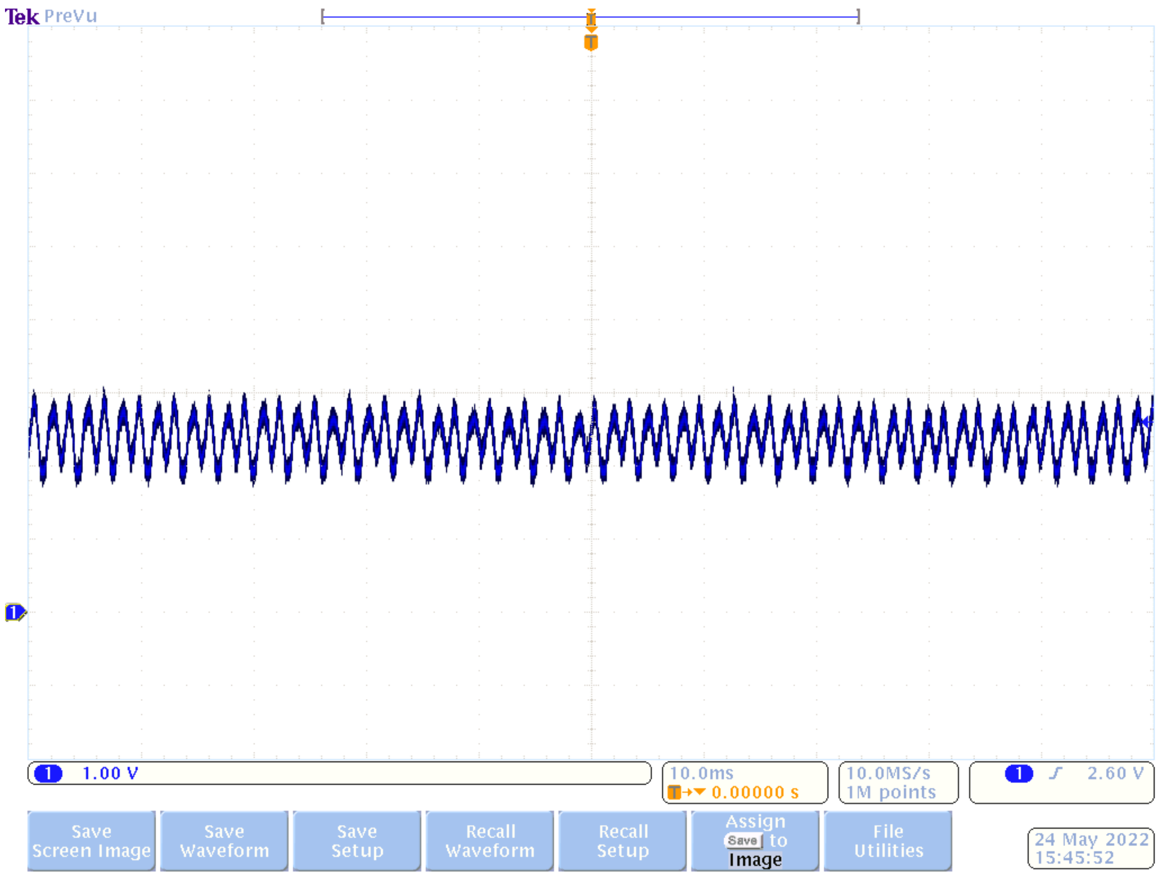

Figure 3: Oscilloscope Reading of Viola

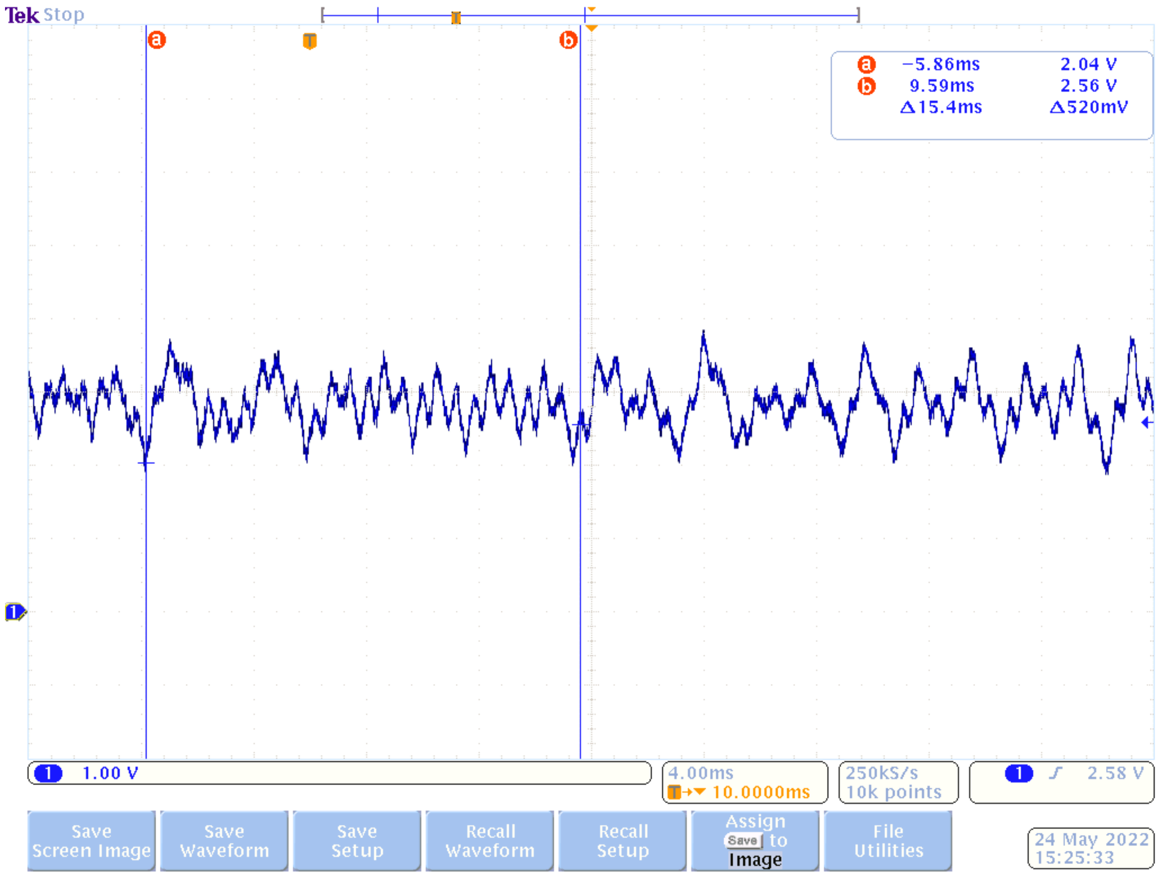

Figure 4: Oscilloscope Reading of Ukulele

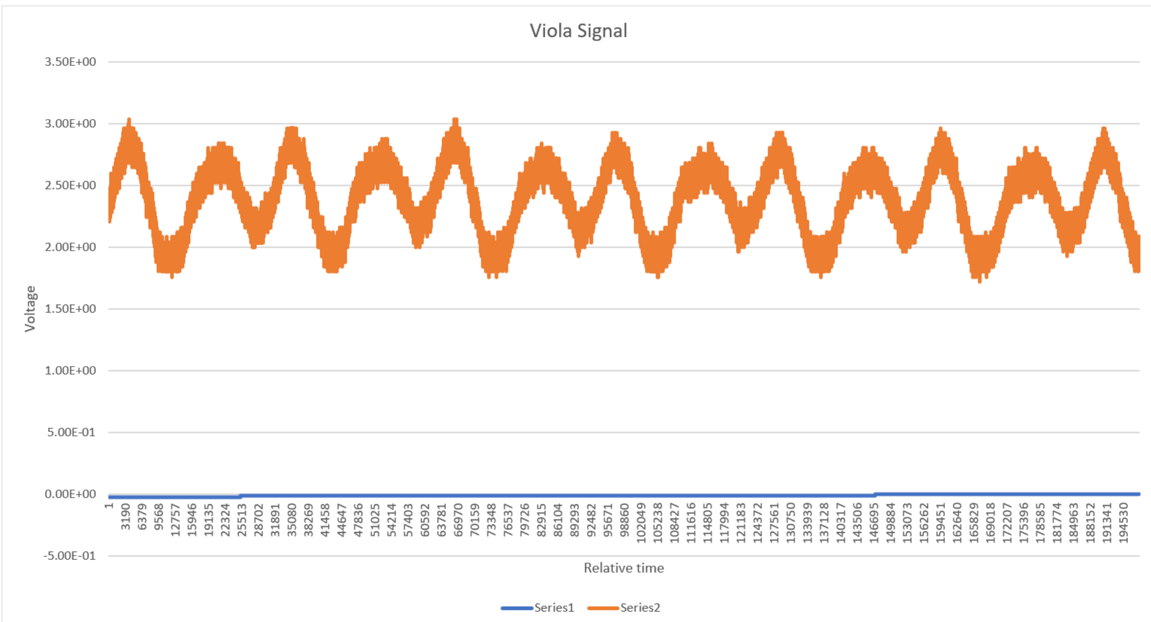

Figure 5: Excel Graph of Viola Data

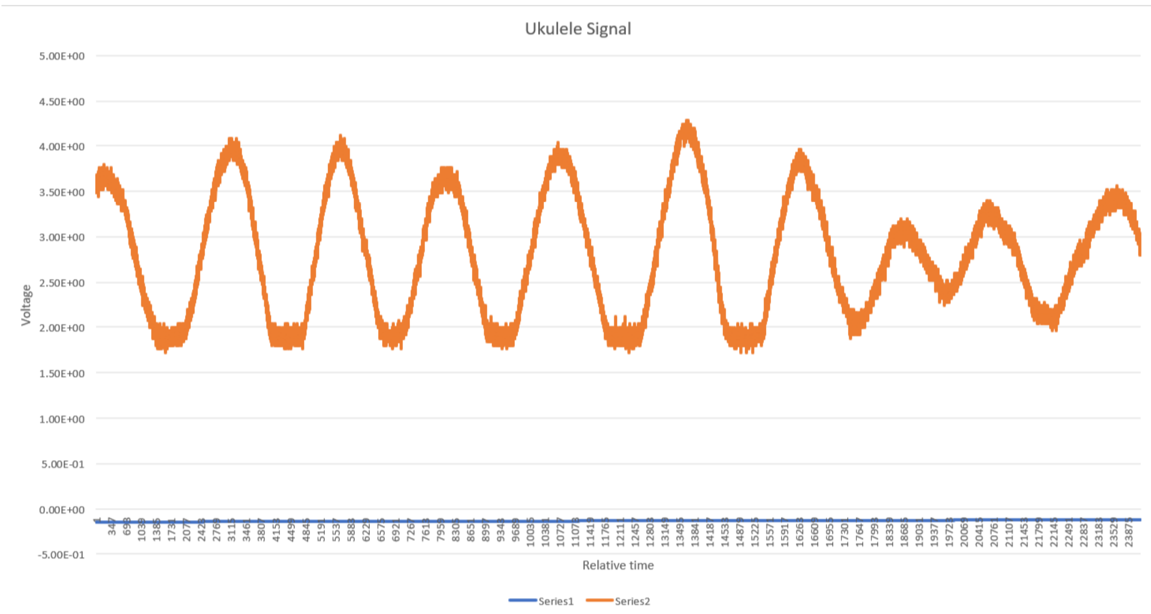

Figure 6: Excel Graph of Ukulele Data

MATLAB script used in this recitation was sourced from Section 2.1 Fourier Series MATLAB Databook[1] from Data-Driven Science and Engineering[2].

```
clear all, close all, clc

% Define domain
dx = 0.001;
L = 2;
x = (-1+dx:dx:1)*L;
n = length(x);   nquart = floor(n/4);

% Define ukulele function
gg=readmatrix('uke_long.csv');
Signal=gg(:,2);
SignalOnePeriod=Signal(366500:369300);
ff=SignalOnePeriod.';
f=imresize(ff,[1,2000]);

% Define viola function
gg=readmatrix('viola_long.csv');
Signal=gg(:,2);
SignalOnePeriod=Signal(326000:357200);
ff=SignalOnePeriod.';
f=imresize(ff,[1,2000]);

plot(x,f,'-k','LineWidth',1.5), hold on

% Compute Fourier series
CC = jet(20);
A0 = sum(f.*ones(size(x)))*dx;
fFS = A0/2;
for k=1:20
    A(k) = sum(f.*cos(pi*k*x/L))*dx; % Inner product
    B(k) = sum(f.*sin(pi*k*x/L))*dx;
    fFS = fFS + A(k)*cos(k*pi*x/L) + B(k)*sin(k*pi*x/L);
    plot(x,fFS,'-','Color',CC(k,:),'LineWidth',1.2)
```

[1] https://github.com/dynamicslab/databook_matlab/blob/master/CH02/CH02_SEC01_1_FourierSines.m

[2] http://databookuw.com/page-2/page-21/

```
end

%% Plot amplitudes
figure
clear ERR
clear A
fFS = A0/2;
A(1) = A0/2;
ERR(1) = norm(f-fFS);
kmax = 20;
for k=1:kmax
    A(k+1) = sum(f.*cos(pi*k*x/L))*dx;
    B(k+1) = sum(f.*sin(pi*k*x/L))*dx;
     plot(x,B(k)*sin(2*k*pi*x/L),'k-','LineWidth',1.2);
    fFS = fFS + A(k+1)*cos(k*pi*x/L) + B(k+1)*sin(k*pi*x/L);
    ERR(k+1) = norm(f-fFS)/norm(f);
end
Mag=sqrt(A.*A+B.*B);
thresh = median(ERR)*sqrt(kmax)*4/sqrt(3);
r = max(find(ERR>thresh));
r = 7;
subplot(2,1,1)
semilogy(0:1:kmax,Mag,'k','LineWidth',1.5)
hold on
semilogy(r,A(r+1),'bo','LineWidth',1.5)
xlim([0 kmax])
ylim([10^(-7) 1])
%subplot(2,1,2)
semilogy(0:1:kmax,Mag,'k','LineWidth',1.5)
hold on
semilogy(r,Mag(r+1),'bo','LineWidth',1.5)
```

The `readmatrix` command was used to import the data from the CSV files into workspace arrays titled `gg`. The voltage output column was extracted from `gg` into the `Signal` array. The data for one period of the waveform, represented by `SignalOnePeriod`, was extracted from `Signal` by locating approximate points indicating the start and end of a periodic wave. Those approximate points were found in Excel by close examination of the Excel graphs previously shown and by adding the relevant time to the start time of the data subset. The `ff` array transposes the `SignalOnePeriod` array, and the final command `imresize` saves the periodic data to the `f` array resized for 2000 elements. The maximum `k` values in the `for` loops in the 'Compute Fouier series' and 'Plot amplitude' sections were adjusted between 5 and 20 for the signal reconstruction and amplitude graphs, respectively.

Figures 7 and 9 are the Fourier series reconstructions of the viola and ukulele data using the first 5 values of `k`. The original CSV data is labeled as `data1` in black. The Fourier series reconstruction using 5 values of `k` is shown as the orange line, labeled `data6` in the legend. The lesser accurate reconstructions are labeled from `data2` to `data5`.

Figures 8 and 10 are the magnitudes of the amplitudes of the Fourier series representations from values 1 to 20. As can be seen for the viola signal, the second harmonic has a high contribution to the periodic waveform which resembles that of two sinusoidal periods. The first harmonic for the ukulele signal has a high contribution as it represents one sinusoidal period.

4 Conclusion

This recitation was thorough exploration of Fourier series analysis from making oscilloscope measurements of real audio samples on a microphone circuit to reconstructing those signals as Fourier series using scripting software. There were several layers to the full procedure which covered a breadth of design skills. Taking the signal data step-by-step from raw oscilloscope readins to Excel plots to reconstructions in MATLAB supplemented by amplitude plots provided required a robust understanding of Fourier series concepts as well as technical experience which could guide future projects.

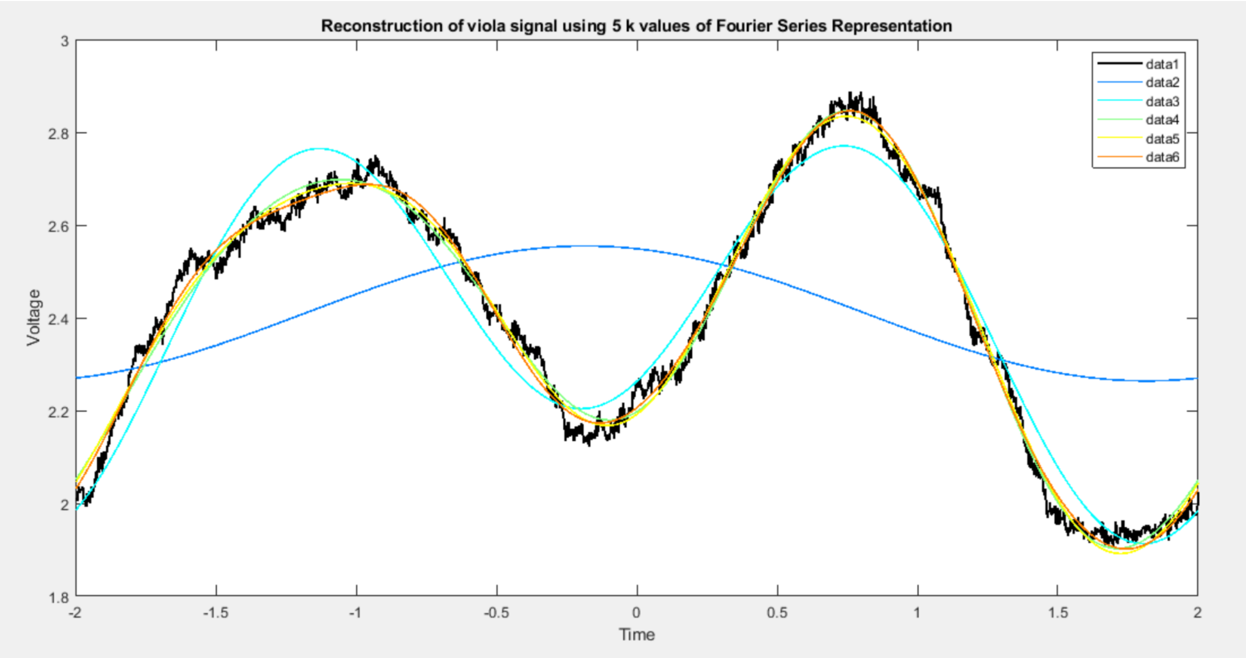

Figure 7: MATLAB Fourier series reconstruction of viola signal

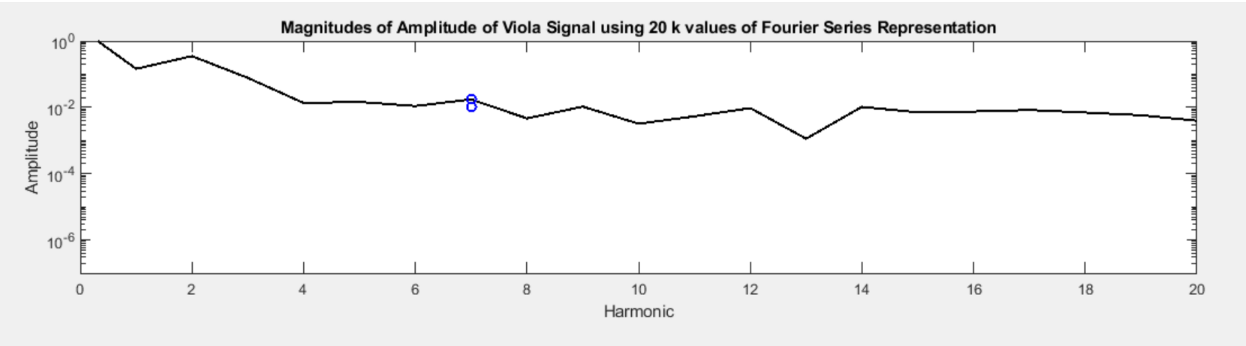

Figure 8: MATLAB amplitude plot of viola Fourier series

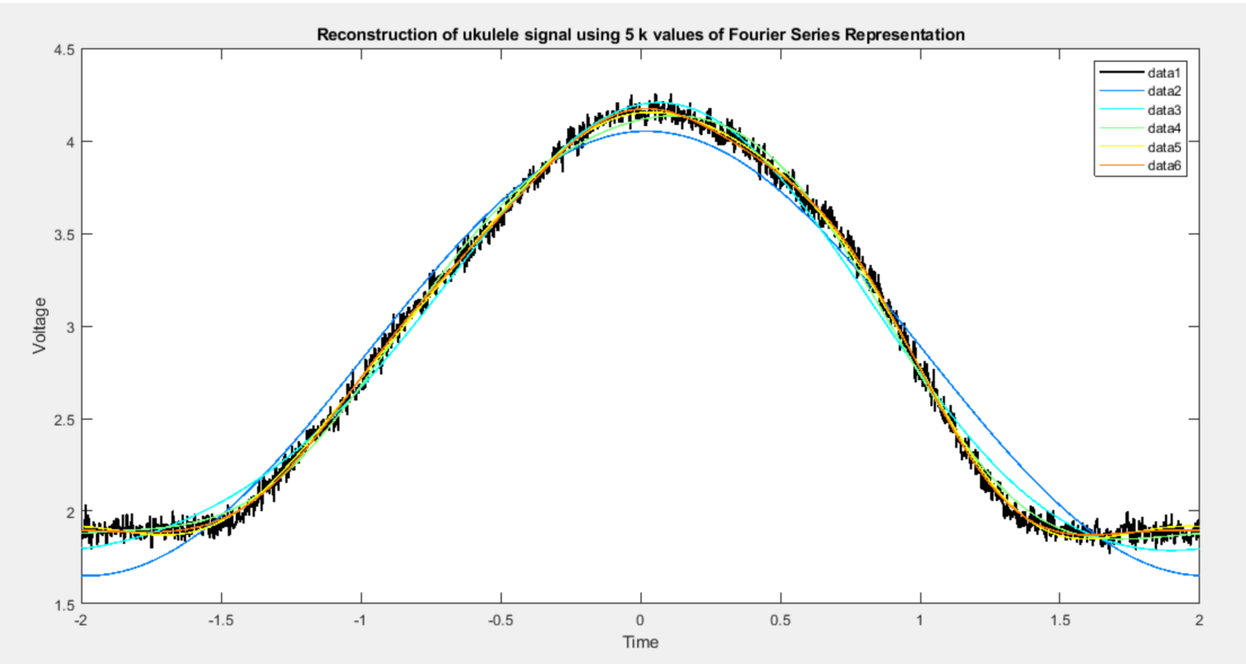

Figure 9: MATLAB Fourier series reconstruction of ukulele signal

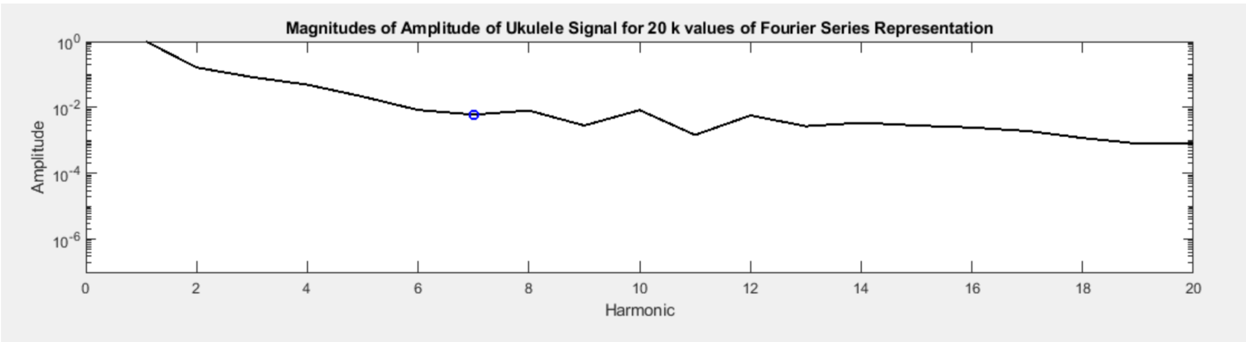

Figure 10: MATLAB amplitude plot of ukulele Fourier series