# Chapter 5

# Compression Members

270 Park Avenue, New York, NY
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## 5.1 COMPRESSION MEMBERS IN STRUCTURES

*Compression members* are structural elements subjected to axial forces that tend to push the ends of the members toward each other. The most common compression member in a building structure is a *column*. Columns are vertical members that support the horizontal elements of a roof or floor system. Several columns can be seen in Figure 5.1 as part of a building structure. They are the primary elements that provide the vertical space to form an occupiable volume. Other compression members are found in trusses as chord and web members and as bracing members in floors and walls. Other names often used to identify compression members are *struts* and *posts*. Throughout this chapter the terms *compression member* and *column* will be used interchangeably.

The compression members discussed in this chapter experience only axial forces. In real structures, additional load effects are often exerted on a compression member that would tend to combine bending with the axial force. These combined force members are called beam-columns and are discussed in Chapter 8. The majority of the provisions that apply to compression members are located in Chapter E of the *Specification*.

Table 5.1 lists the sections of the *Specification* and parts of the *Manual* discussed in this chapter.

## 5.2 CROSS-SECTIONAL SHAPES FOR COMPRESSION MEMBERS

Compression members carry axial forces, so the primary cross-sectional property of interest is the area. Thus, the simple relationship between force and stress,

$$f = \frac{P}{A} \tag{5.1}$$

**Figure 5.1** Columns in a Multistory Building
Photo courtesy of Greg Grieco

is applicable. As long as this relationship dictates compression member strength, all cross sections with the same area will perform in the same way. In real structures, however, other factors influence the strength of the compression member, and the distribution of the area becomes important.

In building structures, the typical compression member is a column and the typical column is a rolled wide-flange member. Later discussions of compression member strength will show that the W-shape does not have the most efficient distribution of material for compression members. It does, however, provide a compression member that can easily be connected to other members of the system such as beams and other columns. This feature significantly influences its selection as an appropriate column cross section.

Figure 5.2 shows examples of rolled and built-up shapes that are used as compression members. Many of these are the same shapes used for the tension members discussed in Chapter 4. This is reasonable because the forces being considered in these two cases are both axial, although they act in the opposite direction. However, other factors that influence the strength of compression members will dictate additional criteria for the selection of the most efficient shapes for these members.

The tee and angle shown in Figure 5.2c and d are commonly used as chords and webs of trusses. In these applications, the geometry of the shapes helps simplify the connections between members. Angles are also used in pairs as built-up compression members, with the connecting element between the two angles as shown in Figure 5.2h. The channel can be found in trusses as a single element or combined with another channel as shown in Figure 5.2b, i, l, and m. Built-up columns can also be found using channels. The hollow structural sections (HSS) shown in Figure 5.2e, f, and g are commonly found as columns in buildings, particularly one-story structures where the connections to the shape can be simplified by carrying beams over the columns. The distribution of the material in these shapes is the most efficient for columns.

**Table 5.1** Sections of *Specification* and Parts of *Manual* Covered in this Chapter

| | |
|---|---|
| | *Specification* |
| B3 | Design Basis |
| B4 | Classification of Sections for Local Buckling |
| E1 | General Provisions |
| E2 | Effective Length |
| E3 | Flexural Buckling of Members without Slender Elements |
| E4 | Torsional and Flexural-Torsional Buckling of Single Angles and Members without Slender Elements |
| E5 | Single-Angle Compression Members |
| E6 | Built-up Members |
| E7 | Members with Slender Elements |
| | *Manual* |
| Part 1 | Dimensions and Properties |
| Part 4 | Design of Compression Members |
| Part 6 | Design of Members Subject to Combined Loading |

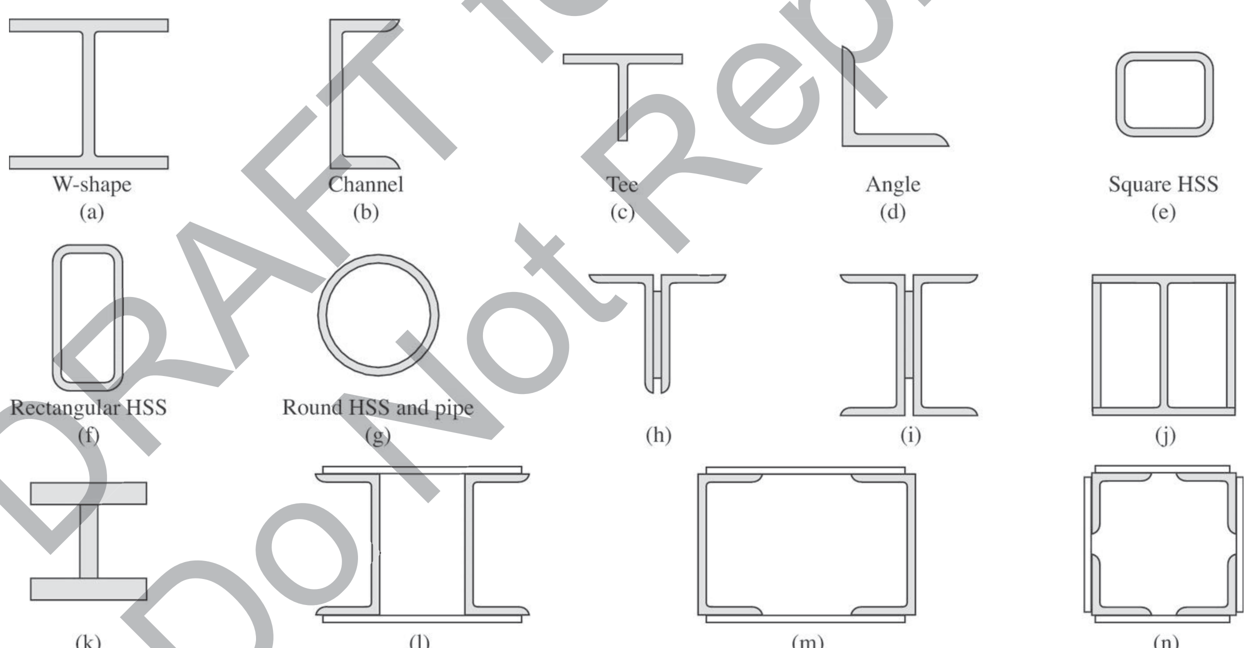



**Figure 5.2** Rolled Shapes and Built-up Shapes for Compression Members

## 5.3 COMPRESSION MEMBER STRENGTH

If no other factors were to impact the strength of a compression member, the simple axial stress relationship given in Equation 5.1 could be used to describe member strength. Thus, the maximum force that a compression member could resist at yield would be

$$P_y = F_y A_g \tag{5.2}$$

where $P_y$ is the yield load, sometimes called the squash load; $F_y$ is the yield stress; and $A_g$ is the gross area. This is the response that would be expected if a very short specimen, one whose length approximates its other two dimensions, were to be tested in compression. This type of column test specimen, shown in Figure 5.3a, is called a *stub column*. Because most compression members will have a length that greatly exceeds its other dimensions, length effects cannot be ignored. A more realistic column is shown in a test frame in Figure 5.3b.

### 5.3.1 Euler Column

To address the impact of length on compression member behavior, a simple model, as shown in Figure 5.4, is used. The Swiss mathematician Leonard Euler first presented this analysis in 1759. A number of assumptions are made in this column model: (1) the column ends are frictionless pins, (2) the column is perfectly straight, (3) the load is applied along the centroidal axis, and (4) the material behaves elastically. Based on these assumptions, this column model is usually called the *perfect column* or the *pure column*.

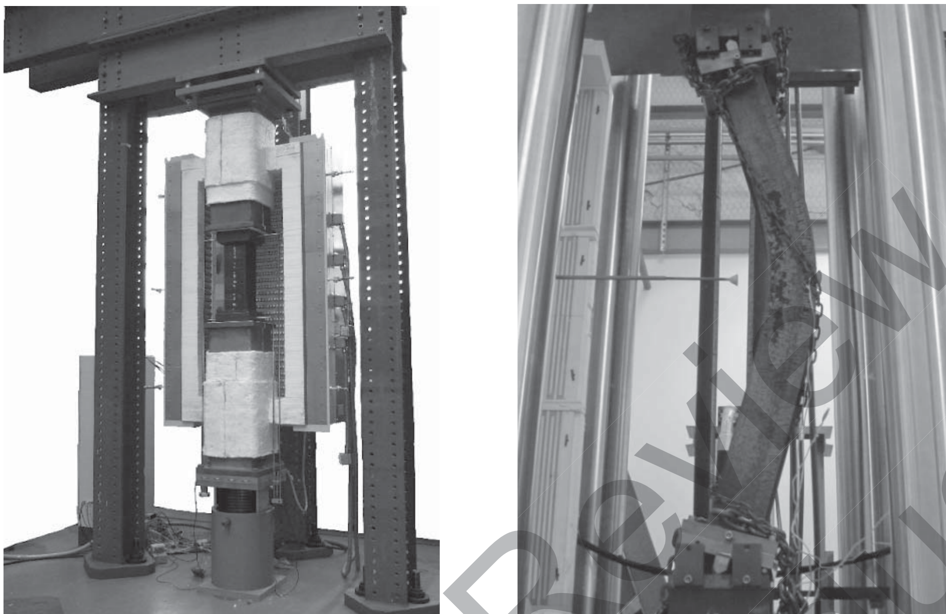
Figure 5.4a shows the perfect column with an applied load that will not cause any lateral displacement or yielding. In this arrangement, the load can be increased with no lateral displacement of the column. However, at a particular load, defined as the critical load or the buckling load, $P_{cr}$, the column will displace laterally as shown in Figure 5.4b. In this configuration, the dashed line represents the original position of the member, and the solid line represents the displaced position. Note that an axis system is presented in the figure, with the *z*-axis along the member length and the *y*-axis transverse to the member length. This places the *x*-axis perpendicular to the plane of the figure. The *x*- and *y*-axes correspond to the centroidal axes of the cross section.

A free body diagram of the lower portion of the column in its displaced position is shown in Figure 5.4c. If moments are taken about point C, equilibrium requires

$$M_z = P_{cr} y$$

From the principles of mechanics and using small displacement theory, the differential equation relating moment to curvature of the deflected member is given as

$$\frac{d^2 y}{dz^2} = -\frac{M_z}{EI_x}$$



(a) (b)

**Figure 5.3** Column Testing. (a) Stub column. (b) Long column
(a) Photo courtesy Prof. Dr. Mario Fontana, (b) Photo courtesy Mohammed Ali Morovat and Michael Engelhardt, University of Texas at Austin

Combining these two equations and rearranging the terms yields the differential equation of equilibrium,

$$\frac{d^2y}{dz^2}+\frac{P_{cr}}{EI_x}y=0$$

If the coefficient of the second term is taken as $k^2 = P_{cr}/EI_x$ , the differential equation for the column becomes

$$\frac{d^2y}{dz^2}+k^2y=0$$

which is a standard second-order linear ordinary differential equation. The solution to this equation is given by

$$y = A\sin kz + B\cos kz \tag{5.3}$$

where $A$ and $B$ are constants of integration. To further evaluate this equation, the boundary conditions must be applied. Because at $z = 0$, $y = 0$ and at $z = L$, $y = 0$, we find that

$$B = 0$$

and

$$A\sin kL = 0$$

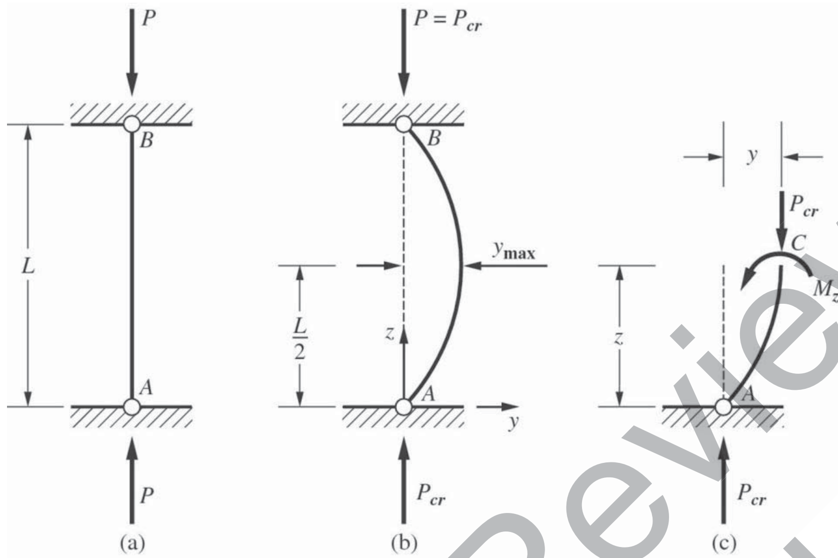



**Figure 5.4** Stability Conditions for Elastic Columns

For Equation 5.3 to have a nontrivial solution, (sin *kL*) must equal zero. This requires that $kL = n\pi$, where *n* is any integer. Substituting for *k* and rearranging yields

$$P_{cr} = \frac{n^2\pi^2 EI_x}{L^2} \tag{5.4}$$

Because *n* can be taken as any integer, Equation 5.4 has a minimum when $n = 1$. This is called the *Euler buckling load* or the *critical buckling load* and is given as

$$P_{cr} = \frac{\pi^2 EI_x}{L^2} \tag{5.5}$$

If values for *B* and *kL* are substituted into Equation 5.3, the shape of the buckled column can be determined from

$$y = A\sin\left(n\pi\frac{z}{L}\right) \tag{5.6}$$

Because any value for *A* will satisfy Equation 5.6, a unique magnitude for the displacement cannot be determined; however, it is clear that the shape of the buckled column is a half sine curve when $n = 1$. This is shown in Figure 5.5a. For other values of *n*, different buckled shapes will result along with the higher critical buckling load. When $n > 1$, these shapes are referred to as higher mode shapes. Several cases are shown in Figure 5.5b, c, and d. In all cases, the basic shape is the sine curve. In order for these higher modes to occur, some type of physical restraint against buckling is required at the point where the buckled shape crosses the original, undeflected shape. This can be accomplished with the addition of braces, which is discussed later.

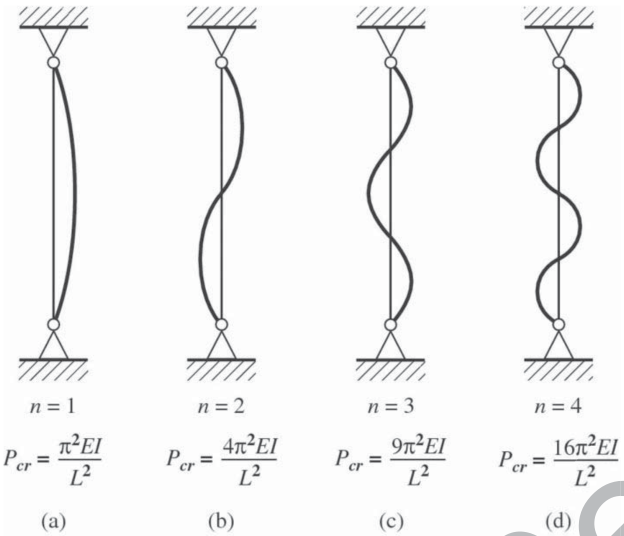



**Figure 5.5** Shape of Buckled Columns

We now have two equations to predict the column strength: Equation 5.2, which does not address length; and Equation 5.5, which does. These two equations are plotted in Figure 5.6. Because the derivation of the Euler equation was based on elastic behavior and the column cannot carry more load than the yield load, there is an upper limit to the column strength.

If the length at which this limit occurs is taken as $L_y$, it can be determined by setting Equation 5.2 equal to Equation 5.5 and solving for length, giving

$$L_y = \pi\sqrt{\frac{EI_x}{F_y A_g}}$$

To simplify this equation, the radius of gyration, $r$, will be used, where

$$r = \sqrt{\frac{I}{A}}$$

Because the moment of inertia depends on the axis being considered, and $A$ is the gross area of the section, which is independent of axis, $r$ will depend on the buckling axis. In the derivation just developed, the axis of buckling for the column of Figure 5.4 was taken as the $x$-axis; thus,

$$L_y = \pi r_x\sqrt{\frac{E}{F_y}}$$

For this theoretical development, a column whose length is less than $L_y$ would fail by yielding and could be called a short column, whereas a column with a length greater than $L_y$ would fail by buckling and be called a long column.

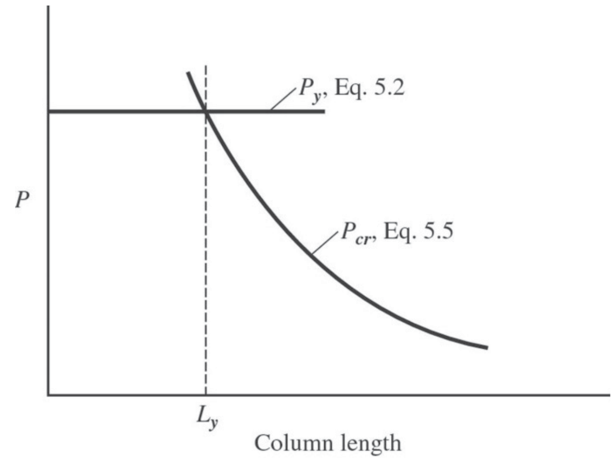



**Figure 5.6** Column Strength Based on Length

It is also helpful to write Equation 5.5 in terms of stress. Dividing both sides by the area and substituting again for the radius of gyration yields

$$F_{cr} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2} \tag{5.7}$$

In this equation, the radius of gyration is left unsubscripted so that it can be applied to whichever axis is determined to be the critical axis. A plot of stress versus $L/r$ would be of the same shape as the plot of force versus $L$ in Figure 5.6.

### 5.3.2 Other Boundary Conditions

Derivation of the buckling equations presented as Equations 5.5 and 5.7 included the boundary condition of frictionless pins at both ends. For perfect columns with other boundary conditions, the moment will not be zero at the ends, and this will result in a nonhomogeneous differential equation. Solving the resulting differential equation and applying the appropriate boundary conditions will lead to a buckling equation of a form similar to the previous equations. To generalize the buckling equation for other end conditions, the column length, $L$, is replaced by the column effective length, $KL$, where $K$ is the effective length factor. Thus, the general buckling equations become

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \tag{5.8}$$

and

$$F_{cr} = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \tag{5.9}$$

Figure 5.7 depicts the original pin-ended column with several examples of columns showing the influence of different end conditions. All columns are shown with the lower support fixed against lateral translation. Three of the columns have upper ends that are also restrained from lateral translation, and three others have upper ends that are free to translate. The effective length can be visualized as the length between inflection points, where the curvature reverses. This result is similar to the original derivation when *n* was taken as some integer other than 1. It is most easily seen in Figure 5.7b and c but can also be seen in Figure 5.7d by visualizing the extended buckled shape above the column as shown in Figure 5.8. In all cases, the buckled curve is a segment of the sine curve. The most important thing to observe is that the column with fixed ends in Figure 5.7b has an effective length of 0.5*L*, whereas the column in Figure 5.7a has an effective length of *L*. Thus, the fixed-end column will have four times the strength of the pin-ended column.

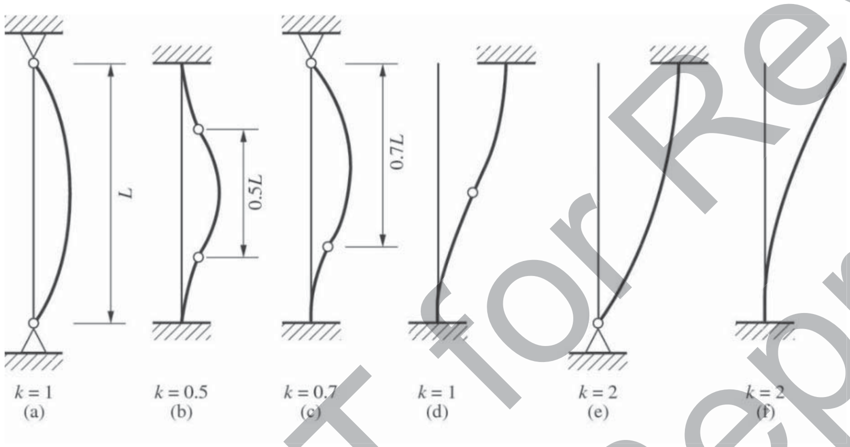



**Figure 5.7** Column Buckled Shape for Different End Conditions

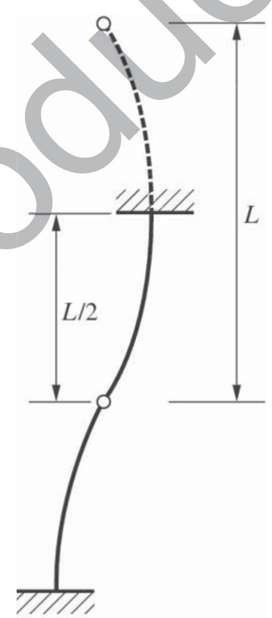



**Figure 5.8** Extended Shape of Buckled Column from Figure 5.7d

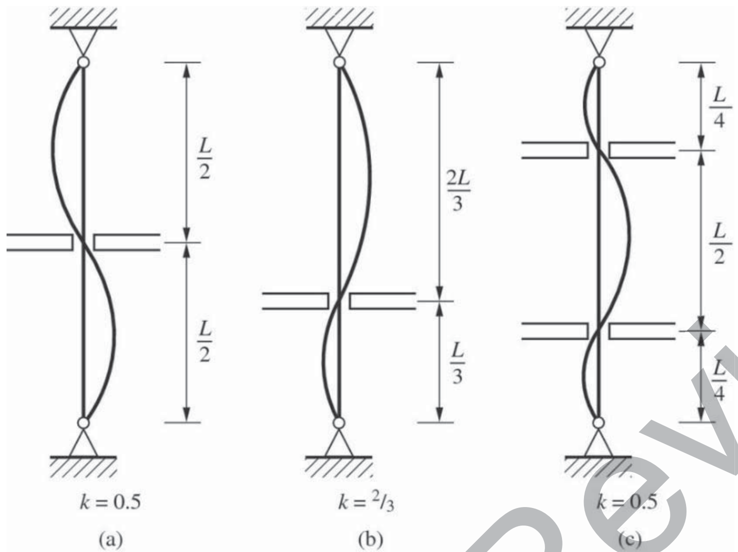



**Figure 5.9** Buckled Shape for Columns with Intermediate Braces

### 5.3.3 Combination of Bracing and End Conditions

The influence of intermediate bracing on the effective length was touched upon in the discussion of the higher modes of buckling. In those cases, the buckling resulted in equal-length segments that reflected the mode number. Thus, a column with $n = 2$ had two equal segments, whereas a column with $n = 3$ buckled with three equal segments. If physical braces are used to provide buckling resistance to the column, the effective length will depend on the location of the braces. Figure 5.9 shows three columns with pinned ends and intermediate supports. The column in Figure 5.9a is the same as the column in Figure 5.5b. The effective length is $0.5L$, so $K = 0.5$. The column in Figure 5.9b shows lateral braces in an unsymmetrical arrangement with one segment equal to $L/3$ and the other to $2L/3$. Although the exact location of the inflection point would be slightly into the longer segment, normal practice is to take the longest unbraced length as the effective length; thus $KL = 2L/3$, so $K = 2/3$ . The column in Figure 5.9c is braced at two locations. The longest unbraced length for this case gives an effective length $KL = 0.5L$ and a corresponding $K = 0.5$. A general rule can be stated that, when the column ends are pinned, the longest unbraced length is the effective length for buckling in that direction.

When other end conditions are present, these two influences must be combined. The columns of Figure 5.10 illustrate the influence of combinations of end supports and bracing on the column effective length. The end conditions would influence only the effective length of the end segment of the column. For the column in Figure 5.10a, the lower segment has $L = a$, and that segment would buckle with an effective length $KL = a$. The upper segment has $L = b$ but also has a fixed end. Thus, it would buckle with an effective length $KL = 0.7b$, obtained by combining the end conditions of Figure 5.7c with the length, $b$. Thus, the relationship between lengths $a$ and $b$ determine which end of the column dictates the overall column effective length. As an example, the column in Figure 5.10b shows that the lowest segment would set the column effective length at $0.35L$.

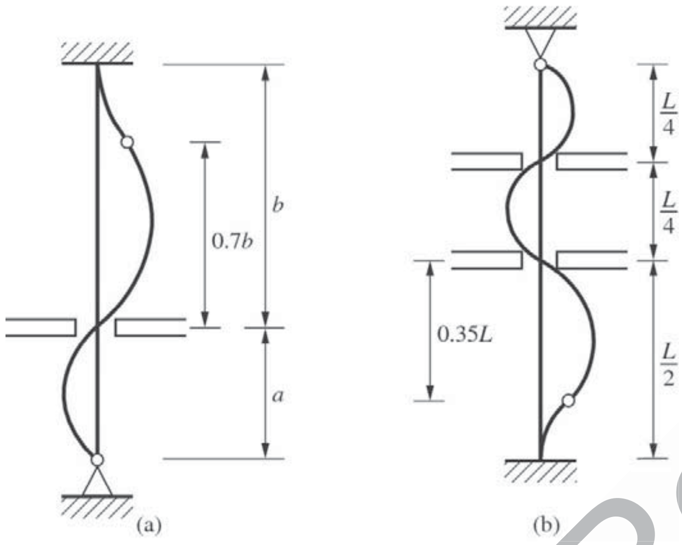



**Figure 5.10** Buckled Shape for Columns with Different End Conditions and Intermediate Braces

**EXAMPLE 5.1**
***Theoretical Column Strength***

**Goal:** Determine the theoretical strength for a pin-ended column and whether it will first buckle or yield.

**Given:** A W10×33, A992, column with a length of 20 ft.

***SOLUTION***

**Step 1:** Determine the load that would cause buckling.

With no other information, it must be assumed that this column will buckle about its weak axis, if it buckles at all, because the effective length, $KL$ = 20 ft for both axes.

From *Manual* Table 1-1, $I_y$ = 36.6 in.$^4$ and $A_g$ = 9.71 in.$^2$. The load that would cause it to buckle is

$$P_{cr} = \frac{\pi^2 EI_y}{(KL)^2} - \frac{\pi^2 (29{,}000)(36.6)}{\left(20(12)\right)^2} = 182 \text{ kips}$$

**Step 2:** Determine the load that would cause yielding.

$$P_y = F_y A_g = 50(9.71) = 486 \text{ kips}$$

**Step 3:** Conclusion: Because $P_{cr} < P_y$, the theoretical column strength is

$$P = 182 \text{ kips}$$

and the column would buckle before it could reach its yield stress.

**EXAMPLE 5.2**
***Critical Buckling Load***

**Goal:** Determine the overall column length that, if exceeded, would theoretically cause the column to buckle elastically before yielding.

**Given:** A W8×31 column with fixed supports. Use steels with (a) $F_y$ = 40 ksi and (b) $F_y$ = 100 ksi.

***SOLUTION***

**Step 1:** From *Manual* Table 1-1, $I_y$ = 37.1 in.$^4$ and $A_g$ = 9.13 in.$^2$.

**Part a**

**Step 2:** Determine the force that would cause the column to yield when $F_y$ = 40 ksi.

$$P_y = F_y A_g = 40(9.13) = 365 \text{ kips}$$

**Step 3:** To determine the length that would cause this same load to be the buckling load for the pinned-pinned case, set this force equal to the buckling load equation and determine the effective length from

$$365 \text{ kips} = \frac{\pi^2 E I_y}{(KL)^2} = \frac{\pi^2 (29{,}000)(37.1)}{(KL)^2}$$

which gives

$$KL = \sqrt{\frac{\pi^2 (29{,}000)(37.1)}{365}} = 171 \text{ in.}$$

So the effective length is

$$KL = \frac{171}{12} = 14.3 \text{ ft}$$

**Step 4:** From Figure 5.7b, a fixed-end column has an effective length equal to one-half the actual length, buckling will not occur if the actual length is less than or equal to:

$$L = KL/0.5 = 2(14.3) = 28.6 \text{ ft} \quad \text{for a column with } F_y = 40 \text{ ksi}$$

**Part b**

**Step 5:** Determine the force that would cause the column to yield when $F_y$ = 100 ksi.

$$P_y = F_y A_g = 100(9.13) = 913 \text{ kips}$$

**Part 6:** To determine the effective length that would cause this same load to be the buckling load for the pinned-pinned case, set this force equal to the buckling force and determine the length from

$$913 \text{ kips} = \frac{\pi^2 EI_y}{(KL)^2} = \frac{\pi^2 (29{,}000)(37.1)}{(KL)^2}$$

which gives

$$KL = \sqrt{\frac{\pi^2 (29{,}000)(37.1)}{913}} = 108 \text{ in.}$$

So the effective length is

$$KL = \frac{108}{12} = 9.0 \text{ ft}$$

**Step 7:** From Figure 5.7b, a fixed-end column has an effective length equal to one-half the actual length, buckling will not occur if the actual length is less than or equal to:

$$L = KL/0.5 = 2(9.0) = 18.0 \text{ ft} \text{ for a column with } F_y = 100 \text{ ksi}$$

### 5.3.4 Real Column

Physical testing of specimens that effectively model columns found in real building structures, like that seen in Figure 5.3b, has shown that column strength was not as great as either the buckling load predicted by the Euler buckling equation or the squash load predicted by material yielding. This inability of the theory to predict actual behavior was recognized early, and numerous factors were found to be the cause. Three main factors influence column strength: material inelasticity, column initial out-of-straightness, and end conditions. The influence of column end conditions has already been discussed with respect to effective length determination. Material inelasticity and initial out-of-straightness, which also significantly impact real column strength, are discussed here.

Inelastic behavior of a column directly results from built-in or residual stresses in the cross section. These residual stresses are, in turn, the direct result of the manufacturing process. Steel is produced with heat, and heat is also necessary to form the steel into the shapes used in construction. Once the shape is fully formed, it is cooled. During this cooling process residual stresses are developed. Figure 5.11 shows a wide-flange cross section in various stages of cooling. Initially, as shown in Figure 5.11a, the tips of the flanges with the most surface area to give off heat begin to cool. This material contracts as it cools, eventually reaching the ambient temperature. At this point, the fibers in this part of the section reach what is expected to be their final length.

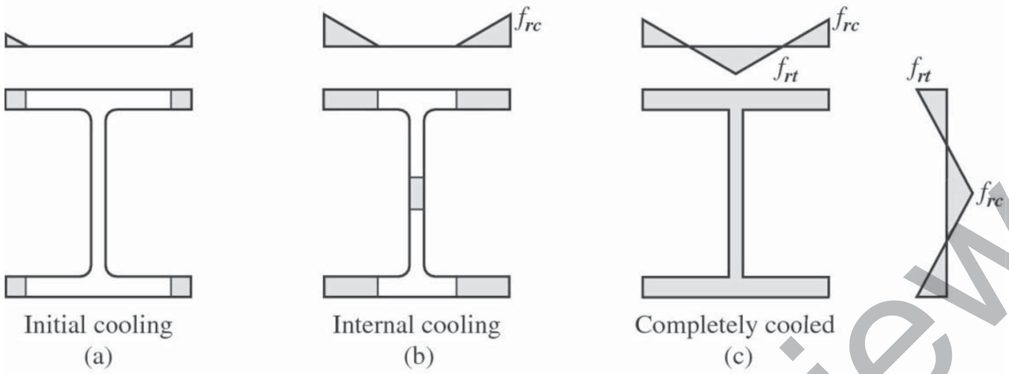



**Figure 5.11** Distribution of Residual Stresses

As adjacent fibers cool, they too contract. In the process of contracting, these subsequently cooling fibers pull on the previously cooled fibers, placing the latter under some amount of compressive stress. Figure 5.11b shows a cross section with additional flange elements cooled. When the previously cooled portion of the cross section provides enough stiffness to restrain the contraction of the subsequently cooling material, a tensile stress is developed in the now-cooling material because it cannot contract as it would without this restraint. When completely cooled, as shown in Figure 5.11c, the tips of the flanges and the middle of the web are put into compression, and the flange-web juncture is put into tension. Thus, the first fibers to cool are in compression, whereas the last to cool are in tension.

Several different representations of the residual stress distribution have been suggested. One distribution is shown in Figure 5.11c. The magnitude of the maximum residual stress does not depend on the material yield strength but is a function of material thickness. In addition, the compressive residual stress is of critical interest in the consideration of compression members. The magnitude of this residual stress varies from 10 ksi to about 30 ksi, depending on the shape. The higher values are found in wide flanges with the thickest flange elements.

To understand the overall impact of these residual stresses on column behavior, a stub column will again be investigated. Figure 5.12 shows the stress-strain relation for a short column, one that will not buckle but exhibits the influence of residual stresses. As the column is loaded with an axial load, the member shortens and the corresponding strain and stress are developed, as if this were a perfectly elastic specimen. The response of a perfectly elastic, perfectly plastic column is shown by the dashed line in Figure 5.12. When the applied stress is added to a member with residual compressive stress, the stub column begins to shorten at a greater rate as the tips of the flange become stressed beyond the yield stress. This point is identified in Figure 5.12 as $F_p$, the proportional limit Thus, the stress-strain curve moves off the straight dashed line and follows the curved solid line. Continuing to add load to the column results in greater strain for a given stress, and the column eventually reaches the yield stress of the perfectly elastic material. Thus, the only difference between the behavior of the actual column and the usual test specimen used to determine the stress-strain relationship is that the real column behaves inelastically as those portions of its cross section with compressive residual stresses reach the material yield stress.

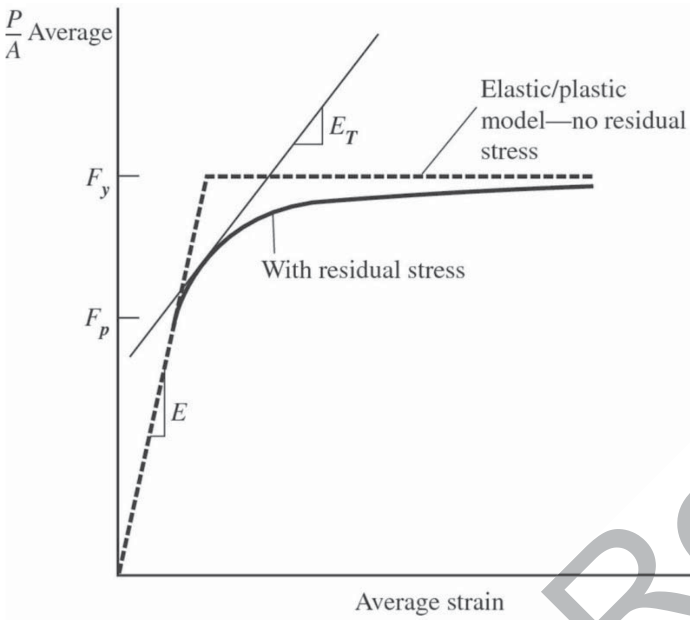



**Figure 5.12** Stub Column Stress-Strain Diagrams with and without Residual Stress

If a new term, the *tangent modulus*, $E_T$, is defined as the slope of a tangent to the actual stress-strain curve at any point shown in Figure 5.12, an improved prediction of column buckling strength can be obtained by modifying the Euler buckling equation so that

$$P_{cr} = \frac{\pi^2 E_T I_x}{(KL)^2}$$

Thus, as the column is loaded beyond its elastic limit, $E_T$ decreases, and the buckling strength does also. This partially accounts for the inability of the Euler buckling equation to accurately predict column strength.

Another factor to significantly impact column strength is the column initial out-of-straightness. Once again, the manufacturing process for steel shapes impacts the ability of the column to carry the predicted load. In this case, the problem is related to the fact that no structural steel member comes out of the production process perfectly straight. In the past, the AISC *Code of Standard Practice* had limited the initial out-of-straightness to 1/1000 of the length between points with lateral support. Although this appears to be a small variation from straightness, it still impacts column strength.

Figure 5.13a shows a perfectly elastic, pin-ended column with an initial out-of-straightness, δ. A comparison of this column diagram with that used to derive the Euler column, Figure 5.4, shows that the moment along the column length will be greater for this initially crooked column in its buckled position than it would have been for an initially straight column. Thus, the solution to the differential equation would be different. In addition, because the applied load works at an eccentricity from the column along its length, even before buckling, a moment is applied to the column that has not yet been accounted for. Figure 5.13b shows the load versus lateral displacement diagram for this initially crooked column compared to that of the initially straight column. This column not only exhibits greater lateral displacement, it also has a lower maximum strength.

When these two factors are combined, the Euler equation cannot properly describe column behavior on its own. Thus, the development of curves to predict column behavior has historically been a matter of curve-fitting the test data in an attempt to present a simple representation of column behavior.

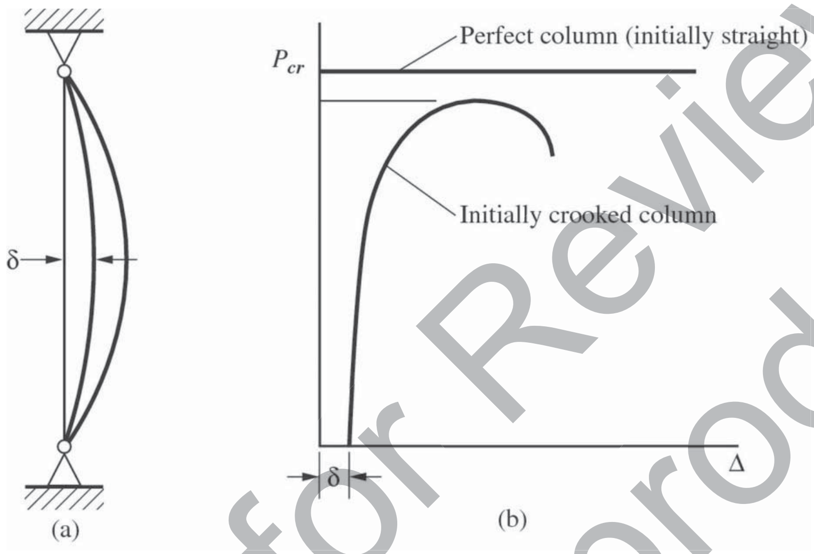



**Figure 5.13** Influence of Initial Out-of-Straightness on Column Strength



### 5.3.5 AISC Provisions

The compression members discussed thus far have either yielding or overall column buckling as the controlling limit state. Figure 5.14 plots sample column test data compared to the Euler equation and the squash load. The Structural Stability Research Council proposed three equations to predict column behavior. To simplify column design, AISC selected a single curve described using two segments as their representation of column strength.

The design basis for ASD and LRFD were presented in Sections 1.9 and 1.10, respectively. The strength equations are repeated here in order to reinforce the relationship between the nominal strength, resistance factor, and safety factor presented throughout the *Specification*.

The requirement for ASD is

$$R_a \leq \frac{R_n}{\Omega} \qquad \text{(AISC B3.2)}$$

The requirement for LRFD is

$$R_u \leq \phi R_n \qquad \text{(AISC B3.1)}$$

As indicated earlier, the *Specification* provides the relationship to determine nominal strength and the corresponding resistance factor and safety factor for each limit state to be considered. The provisions for compression members with nonslender elements, i.e., no local buckling, are given in *Specification* Section E3. The nominal column strength for the limit state of flexural buckling of members with nonslender elements is

$$P_n = F_n A_g \qquad \text{(AISC E3-1)}$$

and

$$\phi_c = 0.9\left(\text{LRFD}\right) \qquad \Omega_c = 1.67\left(\text{ASD}\right)$$

where $A_g$ is the gross area of the section and $F_n$ is the nominal flexural buckling stress. (The Euler column derivation in Section 5.3.1 addressed the limit state of flexural buckling.)

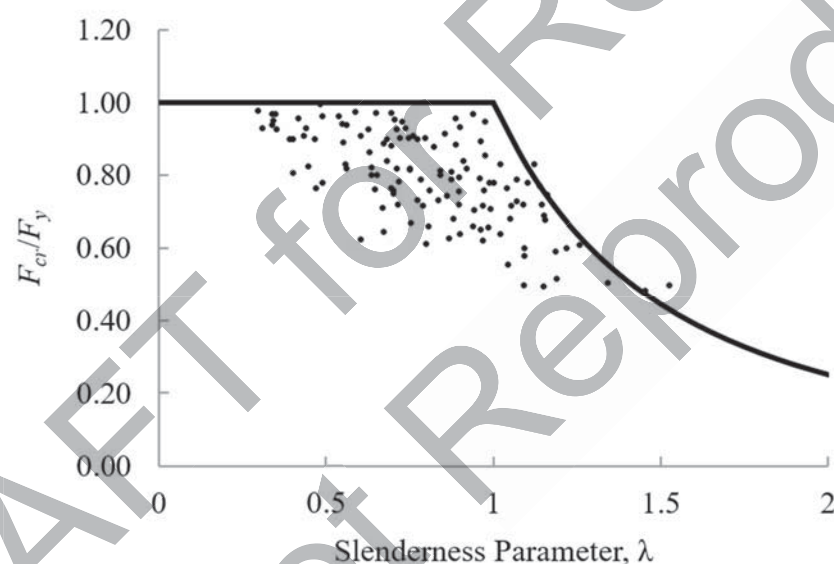



**Figure 5.14** Sample Column Test Data Compared to Theoretical Column Strength

The *Specification* defines $L_c$ as the effective length and shows it equal to *KL*. This is the same effective length factor, *K*, discussed earlier. To capture column behavior when inelastic buckling dominates column strength, that is, where residual stresses become important, the *Specification* provides that when

$$L_c / r \le 4.71\sqrt{E / F_y} \text{ or } \frac{F_y}{F_e} \le 2.25$$

$$F_n = \left[0.658^{\frac{F_y}{F_e}}\right] F_y \qquad \text{(AISC E3-2)}$$

To capture behavior when inelastic buckling is not a factor and initial crookedness is dominant, that is when

$$L_c / r > 4.71\sqrt{E / F_y} \text{ or } \frac{F_y}{F_e} > 2.25$$

$$F_n = 0.877F_e \qquad \text{(AISC E3-3)}$$

where $F_e$ is the elastic buckling stress; the Euler buckling stress previously presented as Equation 5.9 and restated here is

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} \qquad \text{(AISC E3-4)}$$

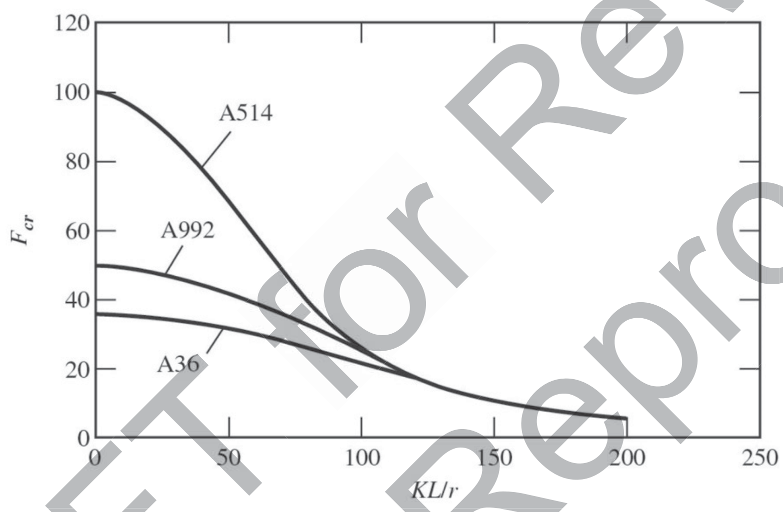



**Figure 5.15** $L_c/r$ versus Critical Strength

The nominal flexural buckling stresses for three different steels, A36, A992, and A514, versus the slenderness ratio, $L_c/r$, are shown in Figure 5.15. For very slender columns, the buckling stress is independent of the material yield. The division between elastic and inelastic behavior, Equations E3-2 and E3-3, corresponds to $L_c/r$ values of 134, 113, and 80.2 for steels with a yield of 36, 50, and 100 ksi, respectively.

Early editions of the LRFD *Specification* defined the exponent of Equation E3-2 in a slightly different form that makes the presentation a bit simpler. If a new term is defined such that

$$\lambda_c^2 = \frac{F_y}{F_e} = \left(\frac{L_c}{\pi r}\right)^2 \frac{F_y}{E}$$

then the dividing point between elastic and inelastic behavior, where

$$\frac{L_c}{r} = 4.71\sqrt{\frac{E}{F_y}}$$

becomes

$$\lambda_c = \frac{L_c}{\pi r}\sqrt{\frac{F_y}{E}} = \frac{4.71}{\pi} = 1.5$$

By substituting $\lambda_c^2 = F_y/F_e$, the nominal flexural buckling stress for $\lambda_c \leq 1.5$ becomes

$$F_n = \left(0.658^{\lambda_c^2}\right)F_y \tag{5.10}$$

and for $\lambda_c > 1.5$,

$$F_n = \frac{0.877}{\lambda_c^2}F_y \tag{5.11}$$

A plot of the ratio of nominal flexural buckling stress to yield stress as a function of the slenderness parameter, $\lambda_c$, is given in Figure 5.16. Using this formulation, it is evident that regardless of the steel yield stress, the ratio of nominal flexural buckling stress to yield stress is the same when plotted against the slenderness parameter, $\lambda_c$. Table 5.2 provides these numerical values in a convenient, usable form.

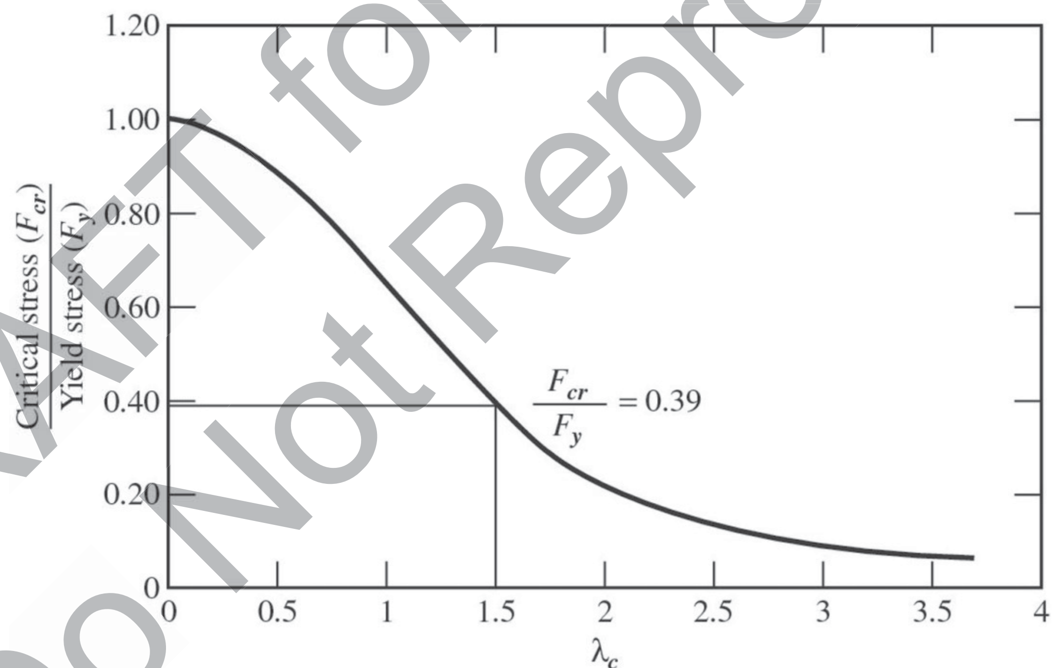



**Figure 5.16** $\lambda_c$ versus Stress Ratio

Earlier editions of the ASD and LRFD *Specifications* indicated that there should be an upper limit on the magnitude of the slenderness ratio at $L_c/r$ = 200. The intent for this limit was to have the engineer recognize that for very slender columns, the nominal flexural buckling stress was so low as to make the column very inefficient. This limit has been removed in recent editions of the *Specification* because there are many factors influencing column strength that indicate that a very slender column might actually be acceptable. Section E2 simply informs the designer, through a User Note, that column slenderness should preferably be kept to something less than 200. It also points to the same recommendation that had been given for tension members, that the slenderness of the member, as fabricated, preferably should not exceed 300. *Manual* Table 4-14 gives the nominal flexural buckling stress for values of slenderness ratio, $L_c/r$, from 0 to 200 in increments of 1.0 for steels with six different yield stresses.

**Table 5.2** Ratio of Nominal Stress to Yield Stress

| $\lambda_c$ | $F_n/F_y$ | $\lambda_c$ | $F_n/F_y$ | $\lambda_c$ | $F_n/F_y$ | $\lambda_c$ | $F_n/F_y$ |
|---|---|---|---|---|---|---|---|
| 0.00 | 1.000 | 0.95 | 0.685 | 1.90 | 0.243 | 2.85 | 0.108 |
| 0.05 | 0.999 | 1.00 | 0.658 | 1.95 | 0.231 | 2.90 | 0.104 |
| 0.10 | 0.996 | 1.05 | 0.630 | 2.00 | 0.219 | 2.95 | 0.101 |
| 0.15 | 0.991 | 1.10 | 0.603 | 2.05 | 0.209 | 3.00 | 0.0974 |
| 0.20 | 0.983 | 1.15 | 0.575 | 2.10 | 0.199 | 3.05 | 0.0943 |
| 0.25 | 0.974 | 1.20 | 0.547 | 2.15 | 0.190 | 3.10 | 0.0913 |
| 0.30 | 0.963 | 1.25 | 0.520 | 2.20 | 0.181 | 3.15 | 0.0884 |
| 0.35 | 0.950 | 1.30 | 0.493 | 2.25 | 0.173 | 3.20 | 0.0856 |
| 0.40 | 0.935 | 1.35 | 0.466 | 2.30 | 0.166 | 3.25 | 0.0830 |
| 0.45 | 0.919 | 1.40 | 0.440 | 2.35 | 0.159 | 3.30 | 0.0805 |
| 0.50 | 0.901 | 1.45 | 0.415 | 2.40 | 0.152 | 3.35 | 0.0781 |
| 0.55 | 0.881 | 1.50 | 0.390 | 2.45 | 0.146 | 3.40 | 0.0759 |
| 0.60 | 0.860 | 1.55 | 0.365 | 2.50 | 0.140 | 3.45 | 0.0737 |
| 0.65 | 0.838 | 1.60 | 0.343 | 2.55 | 0.135 | 3.50 | 0.0716 |
| 0.70 | 0.815 | 1.65 | 0.322 | 2.60 | 0.130 | 3.55 | 0.0696 |
| 0.75 | 0.790 | 1.70 | 0.303 | 2.65 | 0.125 | 3.60 | 0.0677 |
| 0.80 | 0.765 | 1.75 | 0.286 | 2.70 | 0.120 | 3.65 | 0.0658 |
| 0.85 | 0.739 | 1.80 | 0.271 | 2.75 | 0.116 | 3.70 | 0.0641 |
| 0.90 | 0.712 | 1.85 | 0.256 | 2.80 | 0.112 | 3.75 | 0.0624 |

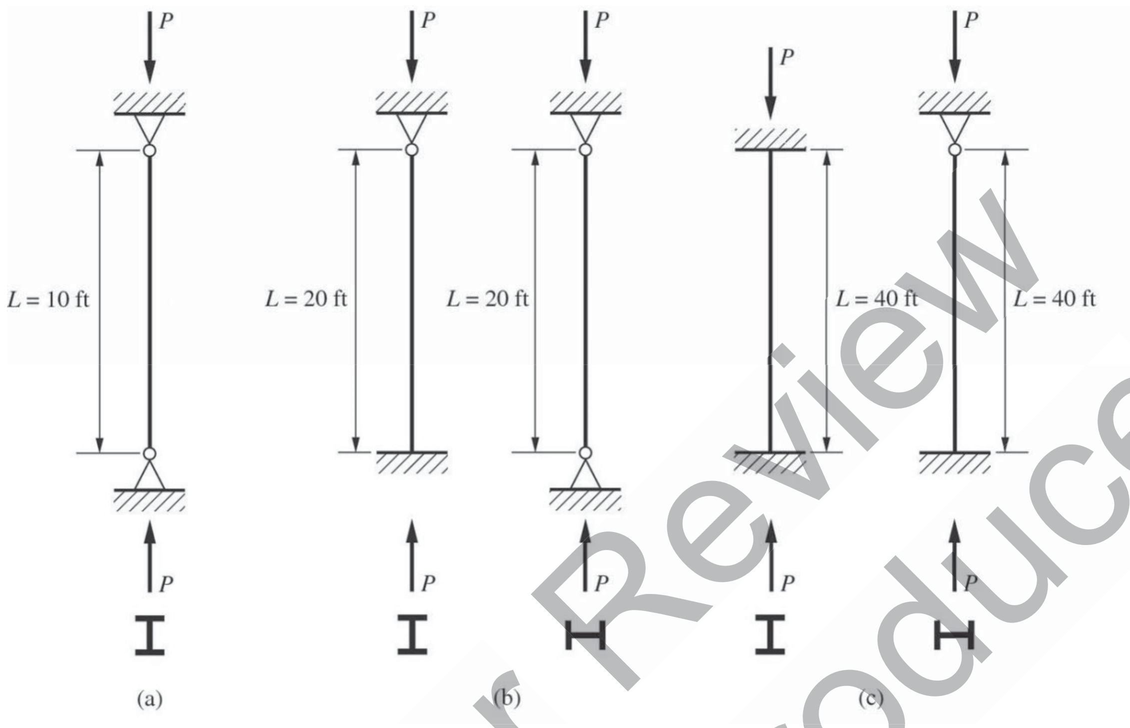



**Figure 5.17** Columns for Examples 5.3, 5.4, and 5.5.

**EXAMPLE 5.3**
***Column Strength by AISC Provisions***

**Goal:** Determine the available column strength.

**Given:** A W12×79 pin-ended column with a length of 10.0 ft. as shown in Figure 5.17a. Use A992 steel.

***SOLUTION***

**Step 1:** From *Manual* Table 1-1, $r_x = 5.34$ in., $r_y = 3.05$ in., and $A = 23.2$ in.$^2$.

**Step 2:** Determine the controlling effective slenderness ratio.

Because the length is 10.0 ft and the column has pinned ends, $L_c = KL = 10.0$ ft for both the $x$-axis and $y$-axis. Thus,

$$\frac{L_{cx}}{r_x} = \frac{10.0(12)}{5.34} = 22.5$$

and

$$\frac{L_{cy}}{r_y} = \frac{10.0(12)}{3.05} = 39.3$$

Since

$$\frac{L_{cy}}{r_y} > \frac{L_{cx}}{r_x}$$

the *y*-axis controls.

**Step 3:** Determine which column strength equation to use. Since

$$\frac{L_c}{r} = 39.3 < 4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

use Equation E3-2

**Step 4:** Determine the Euler buckling stress.

$$F_e = \frac{\pi^2(29{,}000)}{(39.3)^2} = 185 \text{ ksi}$$

**Step 5:** Determine the nominal stress from Equation E3-2.

$$F_n = 0.658^{\left(\frac{F_y}{F_e}\right)} F_y = 0.658^{\left(\frac{50}{185}\right)}(50) = 44.7 \text{ ksi}$$

**Step 6:** Determine the nominal strength.

$$P_n = 44.7(23.2) = 1040 \text{ kips}$$

**For LRFD Step 7:** Determine the design strength for LRFD.

$$\phi P_n = 0.9(1040) = 936 \text{ kips}$$

**For ASD Step 7:** Determine the allowable strength for ASD.

$$\frac{P_n}{\Omega} = \frac{1040}{1.67} = 623 \text{ kips}$$

**EXAMPLE 5.4**
***Column Strength by AISC Provisions***

**Goal:** Determine the available column strength.

**Given:** A W10×49 column with a length of 20.0 ft, one end pinned and the other end fixed for the *y*-axis, and both ends pinned for the *x*-axis, as shown in Figure 5.17b. Use A992 steel.

***SOLUTION***

**Step 1:** From *Manual* Table 1-1, $r_x = 4.35$ in., $r_y = 2.54$ in., and $A = 14.4$ in.$^2$.

**Step 2:** Determine the effective length factors from Figure 5.7.

Comparing the columns shown in Figure 5.17b with those shown in Figure 5.7, the effective length factors are $K_y = 0.7$ and $K_x = 1.0$.

**Step 3:** Determine the *x*- and *y*-axis slenderness ratios.

$$\frac{L_{cx}}{r_x} = \frac{K_x L}{r_x} = \frac{1.0(20.0)(12)}{4.35} = 55.2$$

$$\frac{L_{cy}}{r_y} = \frac{K_y L}{r_y} = \frac{0.7(20.0)(12)}{2.54} = 66.1$$

**Step 4:** Using the larger slenderness ratio, determine which column strength equation to use. Since

$$\frac{L_c}{r} = 66.1 < 4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113,$$

use Equation E3-2

**Step 5:** Determine the Euler buckling stress.

$$F_e = \frac{\pi^2(29{,}000)}{(66.1)^2} = 65.5 \text{ ksi}$$

**Step 6:** Determine the nominal stress from Equation E3-2.

$$F_n = 0.658^{\left(\frac{F_y}{F_e}\right)} F_y = 0.658^{\left(\frac{50}{65.5}\right)}(50) = 36.3 \text{ ksi}$$

**Step 7:** Determine the nominal strength.

$$P_n = 36.3(14.4) = 523 \text{ kips}$$

**For LRFD Step 8:** Determine the design strength for LRFD.

$$\phi P_n = 0.9(523) = 471 \text{ kips}$$

**For ASD Step 8:** Determine the allowable strength for ASD.

$$P_n/\Omega = 523/1.67 = 313 \text{ kips}$$

**EXAMPLE 5.5**
***Column Strength by AISC Provisions***

**Goal:** Determine the available column strength.

**Given:** A W14×53 column with a length of 40.0 ft, both ends fixed for the $y$-axis, and one end pinned and one end fixed for the $x$-axis, as shown in Figure 5.17c. Use A992 steel.

***SOLUTION***

**Step 1:** From *Manual* Table 1-1, $r_x = 5.89$ in., $r_y = 1.92$ in., and $A = 15.6$ in.$^2$.

**Step 2:** Determine the effective length factors from Figure 5.7.

Comparing the columns shown in Figure 5.17c with those shown in Figure 5.7, the effective length factors are $K_y = 0.5$ and $K_x = 0.7$.

**Step 3:** Determine the $x$- and $y$-axis slenderness ratios.

$$\frac{L_{cx}}{r_x} = \frac{K_x L}{r_x} = \frac{0.7(40.0)(12)}{5.89} = 57.0$$

$$\frac{L_{cy}}{r_y} = \frac{K_y L}{r_y} = \frac{0.5(40.0)(12)}{1.92} = 125$$

**Step 4:** Using the larger slenderness ratio, determine which column strength equation to use. Since

$$\frac{L_c}{r} = 125 > 4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

use Equation E3-3

**Step 5:** Determine the Euler buckling stress.

$$F_e = \frac{\pi^2(29{,}000)}{(125)^2} = 18.3 \text{ ksi}$$

**Step 6:** Determine the nominal stress from Equation E3-3.

$$F_n = 0.877F_e = 0.877(18.3) = 16.0 \text{ ksi}$$

**Step 7:** Determine the nominal strength.

$$P_n = 16.0(15.6) = 250 \text{ kips}$$

**For LRFD Step 8:** Determine the design strength for LRFD.

$$\phi P_n = 0.9(250) = 225 \text{ kips}$$

**For ASD Step 8** Determine the allowable strength for ASD.

$$P_n/\Omega = 250/1.67 = 150 \text{ kips}$$

## 5.4 ADDITIONAL LIMIT STATES FOR COMPRESSION

Two limit states for compression members were discussed in Section 5.3, yielding and flexural buckling. The strength equations provided in *Specification* Section E3 clearly show that the upper limit for column strength, $F_yA_g$, is reached only for the zero-length column. Thus, the provisions are presented in the *Specification* as applying to the limit state of flexural buckling only, even though they do consider yielding.

Singly symmetric, unsymmetric, and certain doubly symmetric members may also be limited by torsional buckling or flexural-torsional buckling. The strength provisions for these limit states are given in Section E4 of the *Specification* and are discussed here in Section 5.8.

For some column profiles, another limit state may actually control overall column strength. The individual elements of a column cross section may buckle locally at a stress below the stress that would cause the overall column to buckle. If this is the case, the column is said to be a *column with slender elements*. The impact of these slender elements on column strength is determined through the use of an effective area which is smaller than the actual area of the member. The additional provisions for these types of members are presented in Section 5.6.

## 5.5 LENGTH EFFECTS

The effective lengths that have been discussed were all related to fairly simple columns with easily defined end conditions and bracing locations. Once a column is recognized as being a part of a real structure, determining the effective length becomes more involved. Moreover, for more complex structures, it might be simpler to determine the buckling strength of the structure through analysis than through the use of the effective length factor, $K$. Using that analysis, the elastic buckling stress of the individual columns, $F_e$, can be determined. This can then be used directly in the column strength equations. However, for this book, column elastic buckling is determined through a calculation of effective length. This approach may incorporate some simplifications that would not be made in an actual buckling analysis and, depending on the approach used to determine $K$, may include assumptions of behavior that the actual structure may not satisfy.

A first attempt at incorporating some realistic aspects of structures is shown in Table C-A-7.1 of the Commentary and here in Figure 5.18. The columns shown in this figure are the same as those shown in Figure 5.7, and the same $K$-factors are shown and identified here as the theoretical $K$-values. What is new here is the presentation of recommended design values when ideal conditions are approximated. Most of these recommended values are based on the fact that perfectly rigid connections are difficult to obtain. Thus, for example, a fixed-end column (case a of Figure 5.18) would have a theoretical $K = 0.5$, but if the end connections were to actually rotate, even just a small amount, the effective length would increase. As the end rotation increases toward what would occur for a pin-ended column, $K$ would approach 1.0. Thus, the recommended value of $K$ is 0.65. A similar assessment of the other cases with a fixed end should lead to an understanding of the idea behind these recommended values, each being a bit higher than the theoretical value because actual column end conditions are unlikely to match the theoretical assumptions. In addition to the recommended values for $K$, this Commentary table shows the column end conditions differently than they have been shown historically and in this book. For instance, the upper support for Figure 5.18 column (c) is shown here with rollers so that it is clear that the support can transfer the load directly to the column. In Figure 5.7 column (a) this is shown with the same symbol as the lower support, with no attempt to show graphically that the load is transferred to the column. Although the distinction is critical in structural analysis, it has always been assumed for individual columns that the graphical distinction was not necessary

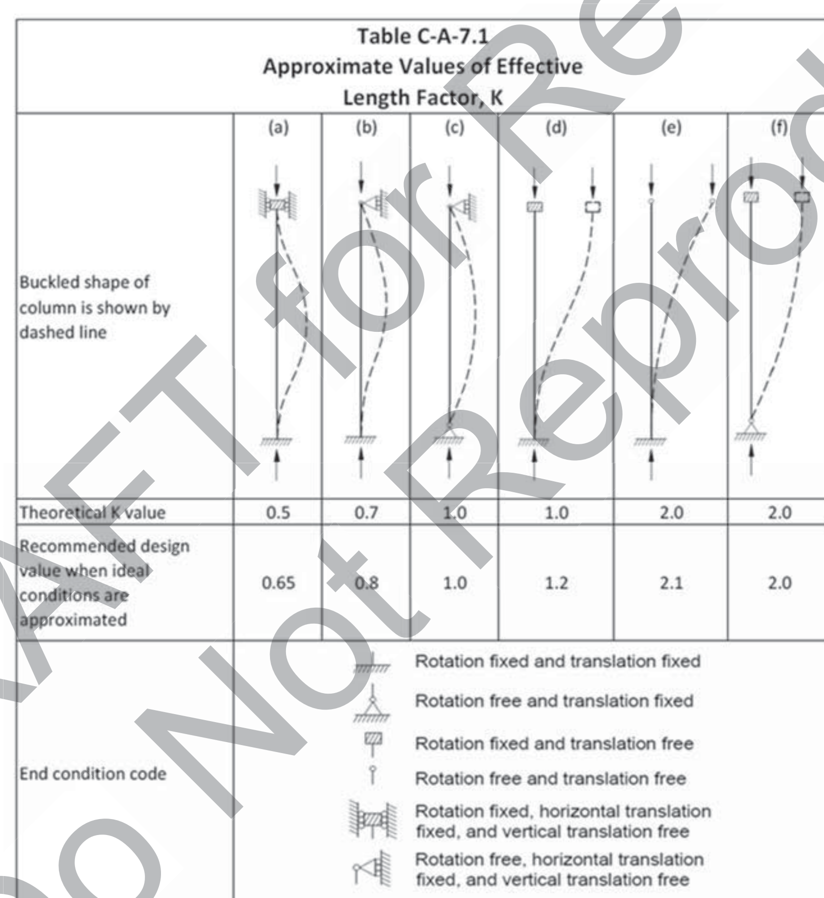


**Table C-A-7.1**
**Approximate Values of Effective Length Factor, K**

| | (a) | (b) | (c) | (d) | (e) | (f) |
|---|---|---|---|---|---|---|
| Buckled shape of column is shown by dashed line | | | | | | |
| Theoretical K value | 0.5 | 0.7 | 1.0 | 1.0 | 2.0 | 2.0 |
| Recommended design value when ideal conditions are approximated | 0.65 | 0.8 | 1.0 | 1.2 | 2.1 | 2.0 |
| End condition code | Rotation fixed and translation fixed<br>Rotation free and translation fixed<br>Rotation fixed and translation free<br>Rotation free and translation free<br>Rotation fixed, horizontal translation fixed, and vertical translation free<br>Rotation free, horizontal translation fixed, and vertical translation free | | | | | |

**Figure 5.18** Values of Effective Length Factor, *K*

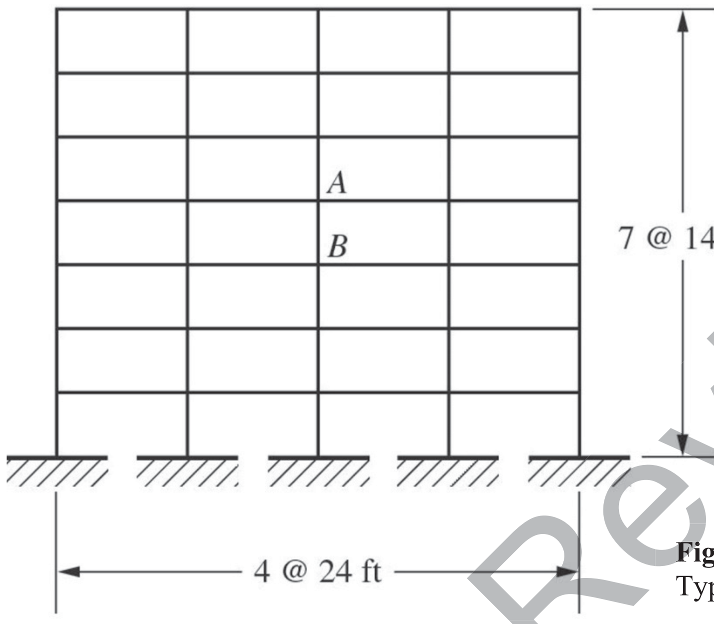



**Figure 5.19**
Typical Moment Frame

When a column is part of a frame, as shown in Figure 5.19, the stiffness of the members framing into the column impact the rotation that could occur at the column ends.

As with the rigid supports discussed for the columns in Figure 5.18, these end conditions permit the column end to rotate. The amount of this rotation is something between the zero rotation of a fixed support and the free rotation of a pin support. When the column under consideration is part of a frame where the ends of the column are not permitted to displace laterally relative to each other, the frame is called a *braced frame*, a *sidesway prevented frame*, or a *sidesway inhibited frame*—shown as cases a, b, and c in Figure 5.18. For a column in a braced frame, the possible $K$-factors range from 0.5 to 1.0. In frames of this type, $K$ is often taken as 1.0, a conservative approximation that simplifies design. In fact, *Specification* Appendix 7, Section 7.2.3(a) says that in braced frames $K$ shall be taken as 1.0 unless analysis shows that a lower value is appropriate.

When the column under consideration is in a frame in which the ends are permitted to move laterally, the frame is called a *moment frame*, an *unbraced frame*, a *sidesway permitted frame*, or a *sidesway uninhibited frame*— shown as cases d, e, and f in Figure 5.18. For the three cases shown there, the lowest value of $K$ is 1.0. The other extreme case, not shown in Figure 5.18, is a pin-ended column in an unbraced frame. The effective length of this column would theoretically be infinite. Thus, $K$-values for columns in moment frames range from 1.0 to infinity.

The determination of reliable effective length factors and thus reliable effective lengths is a critical aspect of column design. Several approaches are presented in the literature, but the most commonly used approach is through the alignment charts presented in the Commentary to Appendix 7. The development of these charts is based on a set of assumptions that are often violated in real structures; nevertheless, the alignment charts are used extensively and often modified in an attempt to account for variations from these assumptions.

These assumptions, as given in the Commentary to Appendix 7, are:

1. Behavior is purely elastic.
2. All members have a constant cross section.
3. All joints are rigid.
4. For columns in frames with sidesway inhibited, rotations at opposite ends of the restraining beams are equal in magnitude and opposite in direction, producing single curvature bending.
5. For columns in frames with sidesway uninhibited, rotations at opposite ends of the restraining beams are equal in magnitude and direction, producing reverse curvature bending.
6. The stiffness parameter $L\sqrt{P/EI}$ of all columns is equal.
7. Joint restraint is distributed to the column above and below the joint in proportion to *EI/L* for the two columns.
8. All columns buckle simultaneously.
9. No significant axial compression force exists in the girders.
10. Shear deformations are neglected

Using these assumptions, the following equation can be obtained for columns in sidesway inhibited frames.

$$\frac{G_A G_B}{4}\left(\pi/K\right)^2+\left(\frac{G_A+G_B}{2}\right)\left(1-\frac{\pi/K}{\tan\left(\pi/K\right)}\right)+\frac{2\tan\left(\pi/2K\right)}{\left(\pi/K\right)}-1=0 \qquad \text{(AISC C-A-7-1)}$$

For sidesway uninhibited frames, the following equation is obtained.

$$\frac{G_A G_B\left(\pi/K\right)^2-36}{6\left(G_A+G_B\right)}-\frac{\left(\pi/K\right)}{\tan\left(\pi/K\right)}=0 \qquad \text{(AISC C-A-7-2)}$$

In Equations C-A-7-1 and C-A-7-2, the terms $G_A$ and $G_B$ relate to the relative stiffness of the columns and beams framing into the column at ends *A* and *B*, respectively, as given by

$$G=\frac{\Sigma\left(EI/L\right)_{col}}{\Sigma\left(EI/L\right)_g} \qquad \text{(AISC C-A-7-3)}$$

If the beams and columns behave elastically, as noted in assumption 1, this reduces to

$$G=\frac{\Sigma\left(I/L\right)_{col}}{\Sigma\left(I/L\right)_g} \qquad (5.12)$$

Equations C-A-7-1 and C-A-7-2 are transcendental equations that do not have a closed-form solution. With the computer methods readily available today, iterative solutions are easily obtained. However, that was not always the case, and a graphical solution was developed in the early 1960s that has become a standard approach for obtaining solutions. Such graphical solutions are called *nomographs* or *alignment charts*. Figure 5.20 shows the nomograph for sidesway inhibited frames, and Figure 5.21 gives the chart for sidesway uninhibited frames.

Since these alignment charts are based on the assumptions given previously, Section 7.2 Commentary provides several adjustments that may be made to model the actual structure more accurately. One of those adjustments is to account for column end conditions. A column end simply supported on a footing would have $G$ theoretically equal to infinity. But, unless the connection is designed and constructed as a true pin, it is more reasonable to take $G$ = 10 for practical design. Similarly, for a column end rigidly attached to a properly designed footing, $G$ would theoretically be zero, but it is reasonable to take G = 1.0 which would account for a small amount of potential rotation.

Approximate solutions to Equations C-A-7-1 and C-A-7-2 have also been presented in design rules and the literature. The French have used the following equations in their design rules since 1966[1]. For sidesway inhibited,

$$K = \frac{3G_A G_B + 1.4(G_A + G_B) + 0.64}{3G_A G_B + 2(G_A + G_B) + 1.28} \tag{5.13}$$

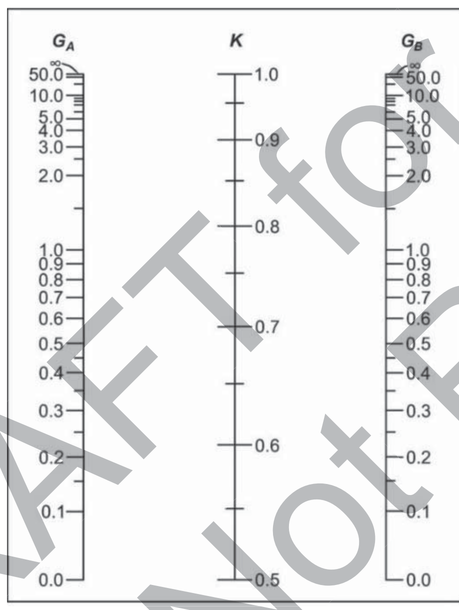



**Figure 5.20** Alignment Chart for a Braced Frame (Sidesway Inhibited) 

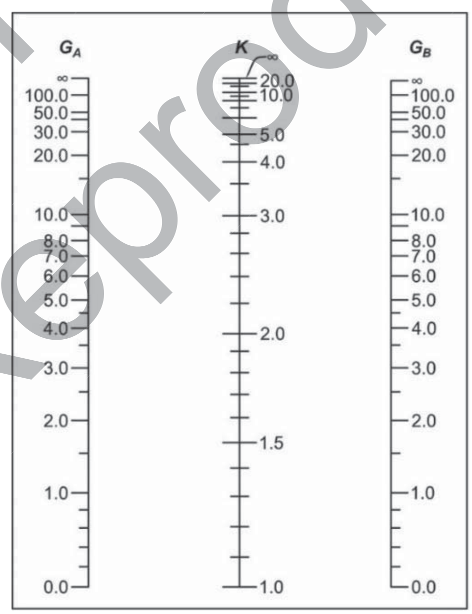



**Figure 5.21** Alignment Chart for an Unbraced Frame (Sidesway Uninhibited)




[1] Dumonteil, P. (1992), "Simple Equations for Effective Length Factors," *Engineering Journal*, American Institute of Streel Construction, Vol. 29, No. 3, pp. 111-115.

For sidesway uninhibited,

$$K = \sqrt{\frac{1.6G_A G_B + 4\left(G_A + G_B\right) + 7.5}{G_A + G_B + 7.5}} \tag{5.14}$$

These approximate equations are said to be accurate within 2 percent. For design this should easily yield results as accurate as those obtained by reading a value from the alignment charts.

For the special case where $G_A = G_B$, even simpler equations can be expressed. For sidesway inhibited,

$$K = \frac{G + 0.4}{G + 0.8} \tag{5.15}$$

For sidesway uninhibited,

$$K = \sqrt{0.8G + 1.0} \tag{5.16}$$

Equations 5.15 and 5.16 might be particularly useful for preliminary design.

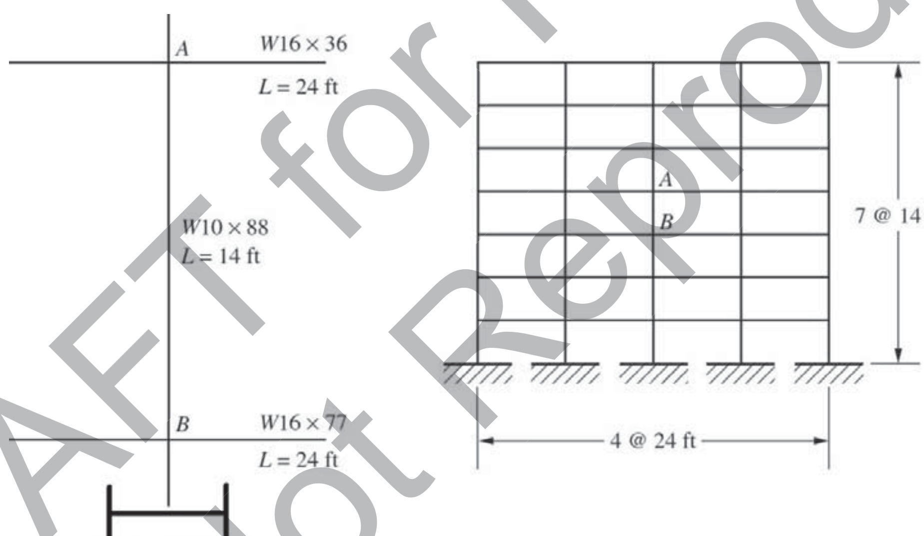



**Figure 5.22** Multi-story Frame for Examples 5.6 and 5.7

**EXAMPLE 5.6**
***Column Effective Length***

**Goal:** Determine the column effective length using (a) the alignment chart and (b) Equation 5.14.

**Given:** The column *AB* in a moment frame is shown in Figure 5.22. Assume that the column has its web in the plane of the frame. The beams also have their webs in the plane of the frame and thus beams and columns are bending about their major, *x*-axis. It would be very unusual for a beam in a moment frame to have the primary bending moments about other than the *x*-axis. However, columns may be oriented for bending about either principal axis.

***SOLUTION***

**Part a**

**Step 1:** Determine member properties from *Manual* Table 1-1.

end *A*:

$$\text{W16×36;}\quad I_{gx} = 448 \text{ in.}^4$$
$$\text{W10×88;}\quad I_{cx} = 534 \text{ in.}^4$$

end *B*:

$$\text{W16×77;}\quad I_{gx} = 1110 \text{ in.}^4$$
$$\text{W10×88;}\quad I_{cx} = 534 \text{ in.}^4$$

**Step 2:** Determine the stiffness ratio at each end using Equation 5.12

$$G_A = \frac{2\left(\dfrac{534}{14}\right)}{2\left(\dfrac{448}{24}\right)} = 2.04$$

$$G_B = \frac{2\left(\dfrac{534}{14}\right)}{2\left(\dfrac{1110}{24}\right)} = 0.825$$

**Step 3:** Use the alignment chart shown in Figure 5.21 for a sidesway uninhibited frame. Enter $G_A$ and $G_B$ on the appropriate scales and construct a straight line between them, as shown in Figure 5.23. The intersection with the scale for *K* gives the effective length factor, in this case,

$$K = 1.42$$

Thus,

$$L_c = KL = 1.42(14.0) = 19.9 \text{ ft}$$

**Part b**

**Step 4:** Determine *K* using the stiffness ratios, $G_A$ and $G_B$, determined in part (a) Step 2 and Equation 5.14.

$$K = \sqrt{\frac{1.6(2.04)(0.825) + 4(2.04 + 0.825) + 7.5}{2.04 + 0.825 + 7.5}} = 1.45$$

Thus,

$$L_c = KL = 1.45(14.0) = 20.3 \text{ ft}$$

Note that *K* determined graphically from the alignment chart and *K* calculated with Equation 5.14 are very close, as might be expected.

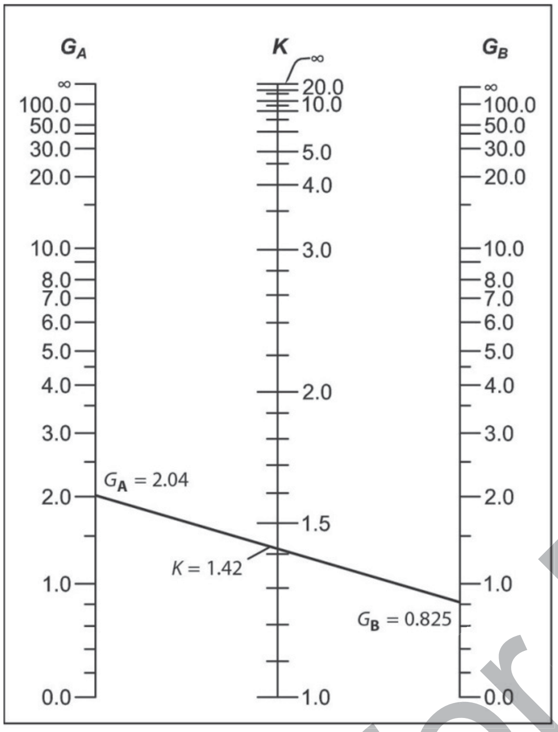



**Figure 5.23** Alignment Chart for Example 5.6


**EXAMPLE 5.7**
***Column Effective Length***

**Goal:** Determine the column effective length for the column of Example 5.6 using the alignment chart if the column is bending about its weak axis.

**Given:** The column *AB* in a moment frame is shown in Figure 5.22. However, for this example assume that the column has its web perpendicular to the plane of the frame, thus it is bending about its minor or weak axis.

***SOLUTION***

**Step 1:** Find member properties from *Manual* Table 1-1.

end A:

W16×36; $I_{gx} = 448 \text{ in.}^4$
W10×88; $I_{cy} = 179 \text{ in.}^4$

end B:

W16×77; $I_{gx} = 1110 \text{ in.}^4$
W10×88; $I_{cy} = 179 \text{ in.}^4$

**Step 2:** Determine the stiffness ratio at each end using Equation 5.12.

$$G_A = \frac{2\left(\frac{179}{14}\right)}{2\left(\frac{448}{24}\right)} = 0.685$$

$$G_B = \frac{2\left(\frac{179}{14}\right)}{2\left(\frac{1110}{24}\right)} = 0.276$$

**Step 3:** Use the alignment chart shown in Figure 5.21 for a sidesway uninhibited frame. Enter the values for $G_A$ and $G_B$ on the appropriate scales and draw a straight line between them. The line's intersection with the scale for $K$ gives the effective length factor—in this case,

$$K = 1.16$$

Thus,

$$L_c = KL = 1.16(14.0) = 16.2 \text{ ft}$$

**Step 4:** Note that the reduction in moment of inertia of the columns results in the beams providing more end restraint, reducing the effective length factor for the column and thus reducing the column effective length.

### 5.5.1 Effective Length for Inelastic Columns

The assumption of elastic behavior for all members of a frame is regularly violated. We have already seen the role that residual stresses play in determining column strength through inelastic behavior. Thus, it is useful to accommodate this inelastic behavior in the determination of $K$-factors. The assumption of elastic behavior is important in the calculation of $G$ as the simplification is made to move from Equation C-A-7-3 to Equation 5.12. Returning to Equation C-A-7-3 and assuming that all columns framing into a joint have the same modulus of elasticity—which is equal to the tangent modulus, $E_T$ shown in Figure 5.12—and that the beams behave elastically, the definition of $G$ for inelastic behavior becomes

$$G_{inelastic} = \frac{E_T(\sum(I/L)_c)}{E(\sum(I/L)_g)} \tag{5.17}$$

If $G$ for elastic behavior is taken as $G_{elastic}$, then $G_{inelastic}$ can be formulated as

$$G_{inelastic} = \left(\frac{E_T}{E}\right)G_{elastic} \tag{5.18}$$

Thus, including inelastic column behavior simply results in a modification of $G$. The ratio of tangent modulus to elastic modulus is always less than 1, so the assumption of elastic behavior for this application leads to a conservative estimate, as can be seen by entering the nomograph with lower $G$-values and determining the corresponding $K$-factor. Before a straightforward approach to including inelastic effects in the determination of effective length can be proposed, the relationship between the tangent modulus and the elastic modulus must be established.

The Commentary to Appendix 7 of the *Specification* indicates that $\tau_b = E_T/E$, as given in Chapter C for the direct analysis method, should be used to account for column inelasticity in the effective length method. Thus, if $\alpha P_r/P_{ns} \leq 0.5$

$$\tau_b = 1.0 \qquad \text{(AISC C2-2a)}$$

and if $\alpha P_r/P_{ns} > 0.5$

$$\tau_b = 4\left(\frac{\alpha P_r}{P_{ns}}\right)\left[1-\left(\frac{\alpha P_r}{P_{ns}}\right)\right] \qquad \text{(AISC C2-2b)}$$

where $\alpha$ = 1.0 for LRFD and $\alpha$ = 1.6 for ASD. $P_{ns}$ is the cross section compression strength. For members without slender elements $P_{ns} = P_y$. For compression members with slender elements, $P_{ns} = F_yA_e$ which is addressed in Section 5.6. *Manual* Table 4-13 provides values for $\tau_b$ based on the required strength, $P_r/A_g$. The use of Table 4-13 assumes that the column is loaded to its full available strength. If it is not, the table provides a conservative assessment of the inelastic stiffness reduction factor and the effective length.

**EXAMPLE 5.8**
***Inelastic Column Effective Length***

**Goal:** Determine the inelastic column effective length using the alignment chart.

**Given:** Determine the inelastic effective length for the column in Example 5.6. The column has an LRFD required strength of $P_u$ = 950 kips and an ASD required strength of $P_a$ = 633 kips. Use Equation 5.14 in place of the alignment chart. The column is A992 steel.

***SOLUTION***

**Step 1:** From *Manual* Table 1-1, for a W10×88 $A$ = 26.0 in.$^2$, and from Example 5.6, the elastic stiffness ratios are $G_A$ = 2.04 and $G_B$ = 0.825.

**For LRFD**

**Step 2:** Determine the required stress based on the required strength.

$$\frac{P_r}{A} = \frac{P_u}{A} = \frac{950}{26.0} = 36.5 \text{ ksi}$$

**Step 3:** Determine the stiffness reduction factor from *Manual* Table 4-13, interpolating between 36 and 37 ksi.

$$\tau_b = 0.788$$

**Step 4:** Determine the inelastic stiffness ratios by multiplying the elastic stiffness ratios by the stiffness reduction factor.

$$G_{iA} = 0.788(2.04) = 1.61$$

$$G_{iB} = 0.788(0.825) = 0.650$$

**Step 5:** Determine $K$ from Equation 5.14.

$$K = \sqrt{\frac{1.6(1.61)(0.650) + 4(1.61 + 0.650) + 7.5}{1.61 + 0.650 + 7.5}} = 1.37$$

Thus,

$$L_c = KL = 1.37(14.0) = 19.2 \text{ ft}$$

Note that the effective length factor and thus the effective length is less than that determined in Example 5.6, as expected.

**For ASD**

**Step 2:** Determine the required stress based on the required strength.

$$\frac{P_r}{A} = \frac{P_a}{A} = \frac{633}{26.0} = 24.3 \text{ ksi}$$

**Step 3:** Determine the stiffness reduction factor from *Manual* Table 4-13, interpolating between 24 and 25 ksi.

$$\tau_b = 0.691$$

**Step 4:** Determine the inelastic stiffness ratios by multiplying the elastic stiffness ratios by the stiffness reduction factor.

$$G_{iA} = 0.691(2.04) = 1.41$$

$$G_{iB} = 0.691(0.825) = 0.570$$

**Step 5:** Determine $K$ from Equation 5.14.

$$K = \sqrt{\frac{1.6(1.41)(0.570) + 4(1.41 + 0.570) + 7.5}{1.41 + 0.570 + 7.5}} = 1.33$$

Thus,

$$L_c = KL = 1.33(14.0) = 18.6 \text{ ft}$$

Note that the effective length factor and thus the effective length is less than that determined in Example 5.6, as expected.

### 5.5.2 Effective Length when Supporting Gravity Only Columns

Another condition that influences column buckling and thus the effective length factor is the existence of columns that carry only gravity load and contribute nothing to the lateral load resistance or stability of the structure. Figure 5.24a illustrates a simple structure of this type where the stability or lateral load resisting column is the flagpole column on the left, column A, and the gravity only column is the pin ended column on the right, column B. The load $P$ is applied to column A and the load $Q$ is applied to column B.

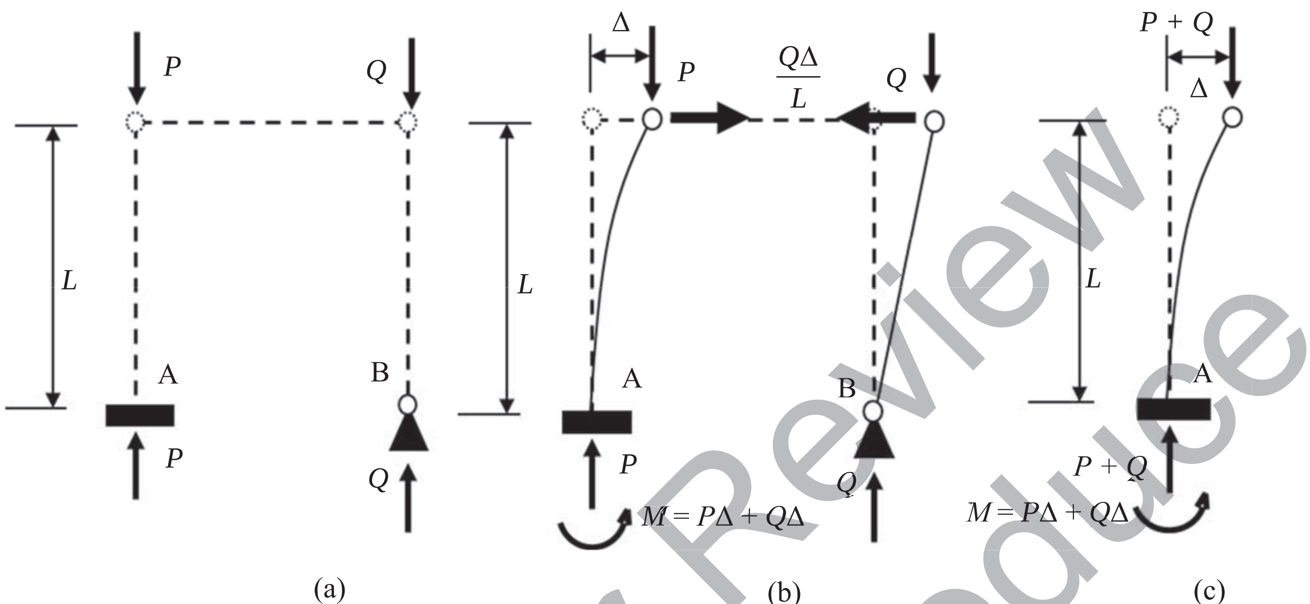



**Figure 5.24** Flagpole Column Providing Lateral Restraint for a Gravity Only Column

If $Q = 0$, column A behaves as if column B did not exist since it just goes along for the ride. However, when $Q$ is not zero, buckling of column A leads to lateral displacement, Δ, at the top of columns A and B. Thus, equilibrium requires a lateral force be exerted at the top of column B. This force must be resisted by column A as shown in Figure 5.24b. In the displaced position illustrated in Figure 5.24b, equilibrium of column A requires a resisting moment at the support of $M = P\Delta + Q\Delta$. Figure 5.24c shows column A with an applied load, $(P+Q)$, which in the displaced position produces a moment at the support of $M = P\Delta + Q\Delta$. Thus, the column in Figure 5.24c can be thought of as a representation of column A in Figure 5.24b with only slight error.

Since these two columns are considered equal, if column A can support the load $(P+Q)$, it should be adequate for column A to support its load, $P$, and the effect of the load $Q$ on column B. Considering elastic buckling this can be stated as

$$(P+Q) = \frac{\pi^2 EI}{(K_o L)^2} \tag{5.19}$$

Where $K_o$ is the $K$-factor for column A. For this example, the theoretical $K$-factor for the flagpole column is $K_o = 2$.

Another way to approach this problem would be to continue to consider that column A supports only the load $P$ but use a $K$-factor that accounts for the influence of the load on the gravity only column, $K_n$. This can be stated as

$$P = \frac{\pi^2 EI}{(K_n L)^2} \tag{5.20}$$

Since these two equations represent the same structure, they can be solved for $\pi^2 EI/L^2$. Thus, from Equation 5.19 $\pi^2 EI/L^2 = K_o^2(P+Q)$ and from Equation 5.20 $\pi^2 EI/L^2 = K_n^2 P$. Setting these equal and solving for $K_n$ yields

$$K_n = K_o\sqrt{\frac{P+Q}{P}} = K_o\sqrt{1+\frac{Q}{P}} \tag{5.21}$$

Thus, a column that supports a load $P$ and also must provide stability for load $Q$ on gravity only columns may be designed using this modified effective length factor $K_n$.

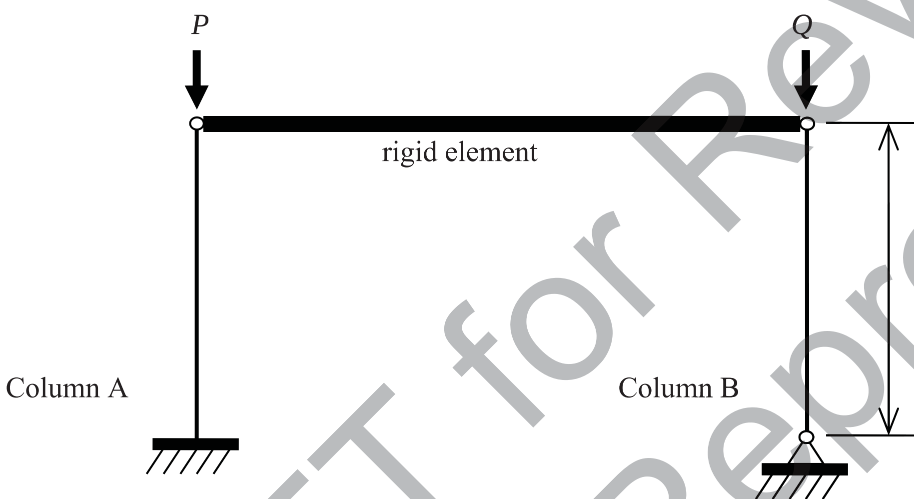



**Figure 5.25** Single Story Frame for Example 5.9

**EXAMPLE 5.9**
***Gravity Only Columns and Effective Length***

**Goal:** Determine the in-plane nominal strength of the column that is required to carry a concentrated load and provide lateral stability for gravity only columns. Also, determine the strength of the column if there is no load on the gravity only column.

**Given:** A W14×90 column shown in Figure 5.25 is to a) carry an applied load, $P$, and provide lateral restraint to a gravity only column carrying the load $2P$ and b) carry an applied load, $P$, with no load on the gravity only column. The W14×90 is oriented so the web is in the plane of the frame. Use A992 steel.

***SOLUTION***

**Step 1:** Determine the effective length factor for the W14×90 column without considering the gravity only column. Since this is a flagpole column, from Figure 5.7f, the theoretical $K$-factor is 2.0

**Part a**
**Step 2:** Using Equation 5.21, determine the modified effective length factor to account for the gravity only column load, $Q = 2P$. Thus

$$K_n = K_o\sqrt{1+\frac{Q}{P}} = 2.0\sqrt{1+\frac{2P}{P}} = 3.46$$

**Step 3:** From *Manual* Table 1-1

$$A = 26.5 \text{ in.}^2 \text{ and } r_x = 6.14$$

**Step 4:** Determine which column strength equation to use.
Since

$$\frac{K_n L}{r_x} = \frac{3.46(15(12))}{6.14} = 101 < 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

use Equation E3-2

**Step 5:** Determine the Euler buckling stress

$$F_e = \frac{\pi^2(29{,}000)}{(101)^2} = 28.1 \text{ ksi}$$

**Step 6:** Determine the nominal stress from Equation E3-2

$$F_n = \left(0.658^{\left(\frac{F_y}{F_e}\right)}\right)F_y = \left(0.658^{\left(\frac{50}{28.1}\right)}\right)50 = 23.7 \text{ ksi}$$

**Step 7:** Determine the nominal strength

$$P_n = 23.7(26.5) = 628 \text{ kips}$$

**Part b**

**Step 8:** With no load on the gravity only column, $K_n = K_o = 2.0$. Determine which column strength equation to use.
Since

$$\frac{K_n L}{r_x} = \frac{2.0(15(12))}{6.14} = 58.6 < 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

use Equation E3-2

**Step 9:** Determine the Euler buckling stress

$$F_e = \frac{\pi^2(29{,}000)}{(58.6)^2} = 83.3 \text{ ksi}$$

**Step 10:** Determine the nominal stress from Equation E3-2

$$F_n = \left(0.658^{\left(\frac{F_y}{F_e}\right)}\right)F_y = \left(0.658^{\left(\frac{50}{83.3}\right)}\right)50 = 38.9 \text{ ksi}$$

**Step 11:** Determine the nominal strength

$$P_n = 38.9(26.5) = 1030 \text{ kips}$$

## 5.6 SLENDER ELEMENTS IN COMPRESSION

As mentioned in Section 5.4, the columns discussed thus far are controlled by overall column buckling. For some shapes, another form of buckling may actually control column strength: local buckling of the elements that make up the column shape. Whether the shape is rolled or built up, it can be thought of as being composed of a group of interconnected plates. Depending on how these plates are supported by each other, they could buckle at a stress below the critical buckling stress of the overall column. This is *local buckling*, also called *plate buckling*, and is shown in Figure 5.26. Local buckling is described through a plate critical buckling equation similar to the Euler buckling equation for columns. The critical buckling stress for an axially loaded plate is

$$F_{cr} = \frac{k\pi^2 E}{12\left(1-\nu^2\right)\left(\frac{b}{t}\right)^2} \tag{5.22}$$

where $k$ is a plate buckling coefficient that depends on the plate loading, edge conditions, and length-to-width ratio; $\nu$ is Poisson's ratio; and $b/t$ is the ratio of the width perpendicular to the compression force to the thickness of the plate. The width-to-thickness ratio is called the *plate slenderness* and is similar in function to the column slenderness. This critical stress plotted as a function of width-to-thickness ratio is shown as the dashed curve in Figure 5.27.

As with overall column buckling, an inelastic transition exists between elastic buckling and element yielding. This transition is due to the existence of residual stresses and imperfections in the element, just as in the case of overall column buckling, and results in the inelastic portion of the curve shown in Figure 5.27. The point identified in the figure as $F_p$-$\lambda_p$ indicates where the elastic curve and the inelastic curve become tangent. In addition, for plates with low $b/t$ ratios, strain hardening plays a critical role in their behavior, indicated by $\lambda_o$, and plates with large $b/t$ ratios have significant post-buckling strength as shown in the figure.

**Figure 5.26** Column Tested to Failure through Local Buckling
Photo courtesy Perry Green

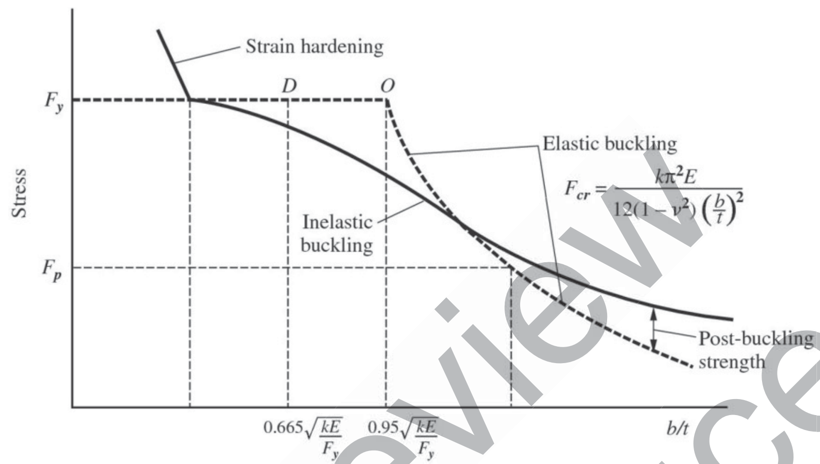



**Figure 5.27** Plate Strength in Compression

To ensure that local buckling will not control column strength, the critical plate buckling stress for local buckling is limited to the critical buckling stress for overall column buckling. This approach results in a different maximum plate slenderness value for each corresponding column slenderness value. However, to alert the designer to the need for consideration of plate buckling, an initial check on element slenderness is made assuming that the stress in the plate has reached the yield stress. The development of the *Specification* provisions starts by finding a plate slenderness that sets the plate buckling stress equal to the column yield stress. Equation 5.22 then becomes

$$\frac{b}{t} = \sqrt{\frac{k\pi^2 E}{12\left(1-\nu^2\right)F_y}} \tag{5.23}$$

Taking $\nu = 0.3$, the standard value for steel, this plate slenderness becomes

$$\frac{b}{t} = 0.95\sqrt{\frac{kE}{F_y}} \tag{5.24}$$

which is shown as point *O* in Figure 5.27. This point is well above the inelastic buckling curve. In order to obtain a *b*/*t* that would bring the inelastic buckling stress closer to the yield stress, a somewhat arbitrary slenderness limit is taken as 0.7 times the limit that corresponds to the column yield stress, which gives

$$\frac{b}{t} = 0.665\sqrt{\frac{kE}{F_y}}$$

This is indicated as point *D* in Figure 5.27.

The remaining factor to be determined is the plate buckling coefficient, *k*. This factor is a function of the stress distribution, the edge support conditions and the aspect ratio of the plate. For a plate with uniform compression on opposite ends and simply supported on all four sides, the minimum value of *k* can be shown to be 4.0. For actual columns, with the variety of potential cross section shapes available, the determination of the value of *k* becomes much more complicated as the actual edge supports, stress distribution and aspect ratio vary.

The limiting width-to-thickness ratios are given in *Specification* Table B4.1a. These limits may be given as

$$\lambda_r = c_3\sqrt{\frac{E}{F_y}} \tag{5.25}$$

where $c_3$ is given in Table 5.4 for several elements in uniform compression taken from Table B4.1a of the *Specification*. The apparent plate buckling coefficient, *k*, used to obtain these values is also given in Table 5.4. For shapes with element slenderness exceeding these $\lambda_r$ values plate buckling must be considered. As already shown, these limits are based on the assumption that the column is stressed to $F_y$. Since columns are rarely stressed to that level, it is very possible that what appears to be a slender element compression member based on Table B4.1a may not actually see its strength limited by the limit state of local buckling.

**Table 5.4** Parameters for Consideration of Compression Member Local Buckling

| Case* | | $\lambda$ | $c_3$ | $k$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|---|
| 1 | Flanges of rolled I-shaped sections, plates projecting from rolled I-shaped sections; outstanding legs of pairs of angles connected with continuous contact, flanges of channels, and flanges of tees | $b/t$ | 0.56 | 0.71 | 0.834 | 0.184 |
| 3 | Legs of single angles, legs of double angles with separators, and all other unstiffened elements | $b/t$ | 0.45 | 0.46 | 0.671 | 0.148 |
| 4 | Stems of tees | $d/t$ | 0.75 | 1.27 | 1.12 | 0.246 |
| 5 | Webs of doubly symmetric rolled and built-up I-shaped sections and channels | $h/t_w$ | 1.49 | 5.0 | 1.95 | 0.351 |
| 6 | Walls of rectangular HSS | $b/t$ | 1.40 | 4.43 | 1.93 | 0.386 |

* From Table B4.1a in the *Specification*.

For W-shapes, Case 1 in Table B4.1a, with $F_y$ = 50 ksi, the flange slenderness limit is $\lambda_{rf} = 0.56\sqrt{E/F_y} = 13.5$ , and all W-shapes have a flange slenderness less than this limit. For webs of these W-shapes, Case 5 in Table B4.1a, $\lambda_{rw} = 1.49\sqrt{E/F_y} = 35.9$, and many available W-shapes have a web slenderness that exceeds this limit and are classified as slender.

Design of slender element compression members according to the *Specification* follows the same requirements as those for compression members without slender elements, with one modification. To account for slender element behavior, the full area of the slender element cannot be used. Thus, a reduced effective area, $A_e$, is used in place of the gross area, $A_g$, to determine column strength. For columns with slender elements, Section E7 indicates that column strength is given by

$$P_n = F_n A_e \quad \text{(AISC E7-1)}$$

which is to be used in place of Equation E3-1 but with the same nominal stress, $F_n$.

Once the designer is directed to the slender element provisions of Section E7 it is apparent that the nominal stress based on the controlling limit state must first be determined. Then, using that stress, the actual plate element slenderness at the transition from elastic to inelastic buckling can be determined. For the web of a rolled W-shape, Case 5 in Table B4.1a, the limiting width-to-thickness ratio becomes

$$\lambda_r = \frac{h}{t_w} = c_3\sqrt{\frac{E}{F_n}} = 1.49\sqrt{\frac{E}{F_n}}$$

If the width-to-thickness ratio of the web does not exceed this value, the usable width of the web is the actual width and no change in area is required. If the width-to-thickness ratio of the web does exceed this value, a reduced width must be determined through

$$b_e = b\left(1 - c_1\sqrt{\frac{F_{el}}{F_n}}\right)\sqrt{\frac{F_{el}}{F_n}} \quad \text{(AISC E7-3)}$$

where the elastic plate buckling stress from Equation 5.22 is presented as

$$F_{el} = \left(c_2\frac{\lambda_r}{\lambda}\right)^2 F_y \quad \text{(AISC E7-5)}$$

The constants $c_1$ and $c_2$ are given in *Specification* Table E7.1. If Equations 5.25 and E7-5 are substituted into Equation E7-3, the effective width becomes

$$b_e = c_2 c_3 t\sqrt{\frac{E}{F_n}}\left(1 - \frac{c_1 c_2 c_3}{(b/t)}\sqrt{\frac{E}{F_n}}\right) \quad (5.26)$$

Equation 5.26 may be simplified by substituting $c_4 = c_2 c_3$ and $c_5 = c_1 c_2 c_3$ which yields

$$b_e = c_4 t\sqrt{\frac{E}{F_n}}\left(1 - \frac{c_5}{(b/t)}\sqrt{\frac{E}{F_n}}\right) \quad (5.27)$$

Values for $c_4$ and $c_5$ are given in Table 5.4.

Continuing the consideration of W-shape webs by making the appropriate substitutions into Equation 5.27 yields

$$b_e = 1.95t\sqrt{\frac{E}{F_n}}\left[1 - \frac{0.351}{(b/t)}\sqrt{\frac{E}{F_n}}\right] \tag{5.28}$$

Once the effective width of a slender element is obtained, the corresponding effective area of the member can be determined. Since hot-rolled shapes have fillets at the junction of the plate elements, the best approach for determining the effective area is to use the gross area and deduct the appropriate ineffective element area. This will be illustrated in the example.

**EXAMPLE 5.10**
***Strength of Column with Slender Elements***

**Goal:** Determine the available strength of a compression member with a slender web.

**Given:** Use an A992 W16×26 as a column with $L_{cy}$ = 5.0 ft.

**Note:** In *Manual* Table 1-1, this shape is identified with footnote c, indicating that it must be considered as a slender element member for compression. It is the most slender-web W-shape available and is not normally used as a column.

***SOLUTION***

**Step 1:** From *Manual* Table 1-1,

$$A = 7.68 \text{ in.}^2, \quad h/t_w = 56.8, \quad t_w = 0.250 \text{ in.}, r_y = 1.12 \text{ in.}$$

**Step 2:** Determine the web slenderness limit from *Specification* Table B4.1a, Case 5.

$$\lambda_{rw} = 1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29{,}000}{50}} = 35.9$$

**Step 3:** Check the slenderness of the web.

$$\frac{h}{t_w} = 56.8 > \lambda_{rw} = 35.9$$

Thus, the shape must be treated as one with a slender web. It has already been established that all W-shapes, with $F_y$ = 50 ksi, have nonslender flanges, so that check will not be made here.

**Step 4:** Determine the Euler buckling stress, $F_e$, for $L_c$ = 5.0 ft.

$$F_e = \frac{\pi^2(29{,}000)}{\left(\frac{5(12)}{1.12}\right)^2} = 99.7 \text{ ksi}$$

**Step 5:** Determine $F_n$

$$F_e = 99.7 \text{ ksi } > F_y/2.25 = 50/2.25 = 22.2 \text{ ksi}$$

Therefore, use Equation E3-2

$$F_n = 0.658^{\left(\frac{50}{99.7}\right)}(50) = 40.5 \text{ ksi}$$

**Step 6:** Check the slenderness of the web against the new limit using $F_n$ in place of $F_y$.

$$\lambda_{rw} = 1.49\sqrt{\frac{E}{F_n}} = 1.49\sqrt{\frac{29,000}{40.5}} = 39.9$$

since

$$\frac{h}{t_w} = 56.8 > \lambda_{rw} = 39.9$$

the web is slender and the effective web width must be determined.

**Step 7:** Determine the effective width of the web using Equation 5.28

$$b_e = 1.95t\sqrt{\frac{E}{F_n}}\left[1 - \frac{0.351}{(b/t)}\sqrt{\frac{E}{F_n}}\right]$$

$$= 1.95(0.250)\sqrt{\frac{29,000}{40.5}}\left[1 - \frac{0.351}{56.8}\sqrt{\frac{29,000}{40.5}}\right] = 10.9 \text{ in.}$$

**Step 8:** Determine the actual web width.

The width of the web plate is given by $h$. However, a value of $h$ is not specifically available in the *Manual*, so with $h/t_w$ = 56.8 and $t_w$ = 0.250, $h$ can be determined as

$$h = (h/t_w)t_w = 56.8(0.250) = 14.2 \text{ in.}$$

**Step 9:** Determine the effective area.

Because $b_e < h$, use $b_e$ to determine $A_e$. To properly account for the fillets at the web-flange junction, the area of the ineffective web is deducted from the gross area of the shape; thus,

$$A_e = A_g - (h - b_e)t_w = 7.68 - (14.2 - 10.9)(0.250) = 6.86 \text{ in.}^2$$

**Step 10:** Determine the nominal strength of the column.

$$P_n = F_n A_e = 40.5(6.86) = 278 \text{ kips}$$

**For LRFD**

**Step 11:** Determine the design strength for this slender web column with $L_c$ = 5.0 ft.

$$\phi P_n = 0.9(278) = 250 \text{ kips}$$

**For ASD**

**Step 11:** Determine the design strength for this slender web column with $L_c$ = 5.0 ft.

$$P_n/\Omega = 278/1.67 = 166 \text{ kips}$$

**EXAMPLE 5.11**
***Strength of Column with Slender Elements***

**Goal:** Determine the available strength of a compression member with a slender web.

**Given:** Use the W16×26 column from Example 5.10 but with $L_{cy}$ = 15.0 ft.

**Note:** This shape has already been shown to have a slender web based on Table B4.1a

***SOLUTION***

**Step 1:** From *Manual* Table 1-1,

$$A = 7.68 \text{ in.}^2, \quad h/t_w = 56.8, \quad t_w = 0.250 \text{ in.}, \; r_y = 1.12 \text{ in.}$$

**Step 2:** Determine the web slenderness limit from *Specification* Table B4.1a, Case 5.

$$\lambda_{rw} = 1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29.000}{50}} = 35.9$$

**Step 3:** Check the slenderness of the web.

$$\frac{h}{t_w} = 56.8 > \lambda_{rw} = 35.9$$

Thus, the shape has a slender web. It has already been established that all W-shapes with $F_y$ = 50 ksi have nonslender flanges.

**Step 4:** Determine the Euler buckling stress, $F_e$, for $L_c$ = 15.0 ft.

$$F_e = \frac{\pi^2(29{,}000)}{\left(\frac{15(12)}{1.12}\right)^2} = 11.1 \text{ ksi}$$

**Step 5:** Determine $F_n$.

$$F_e = 11.1 \text{ ksi} < F_y/2.25 = 50/2.25 = 22.2 \text{ ksi}$$

Therefore, use Equation E3-3

$$F_n = 0.877(11.1) = 9.73 \text{ ksi}$$

**Step 6:** Check the slenderness of the web against the new limit using $F_n$ in place of $F_y$.

$$\lambda_{rw} = 1.49\sqrt{\frac{E}{F_n}} = 1.49\sqrt{\frac{29{,}000}{9.73}} = 81.3$$

since

$$\frac{h}{t_w} = 56.8 < \lambda_{rw} = 81.3$$

the column will not be limited by local buckling of the web

**Step 7:** Determine the nominal strength of the column.

$$P_n = F_n A_g = 9.73(7.68) = 74.7 \text{ kips}$$

**For LRFD Step 8:** Determine the design strength for this slender web column with $L_c$ = 15.0 ft.

$$\phi P_n = 0.9(74.7) = 67.2 \text{ kips}$$

**For ASD Step 8:** Determine the design strength for this slender web column with $L_c$ = 15.0 ft.

$$P_n/\Omega = 74.7/1.67 = 44.7 \text{ kips}$$

Examples 5.10 and 5.11 illustrate that a W-shape that appears to be a slender element shape based on Table B4.1a may not actually be limited in strength because of that slender element, the limit state of local buckling. It can be shown that the W16×26 considered in these examples will have its strength limited by local buckling for columns with effective length, $L_c$, up to about 10.8 ft. Above that effective length, the slender elements will not impact overall column strength.

## 5.7 COLUMN DESIGN TABLES

A review of the AISC column equations, E3-2 and E3-3, shows that the only factor other than shape geometry and material strength that influences the determination of column strength is the slenderness ratio. Therefore, it is convenient to tabulate column strength as a function of slenderness. Part 4 of the *Manual* contains tables for W-shapes, HP-shapes, and HSS and several singly symmetric shapes. Figure 5.28 shows a sample of *Manual* Table 4-1a for several W14 sections with $F_y$ = 50 ksi. As with all of the available strength tables in the *Manual*, both allowable strength (ASD) and design strength (LRFD) values are given. Tables 4-1b and 4-1c are provided for selected W-shapes that are commonly available with $F_y$ = 65 and 70 ksi respectively.

The values in these column tables are based on the assumption that the column will buckle about its weak axis. For all W-shapes this is the *y*-axis, so the values in the tables are given in terms of the effective length with respect to the least radius of gyration, $r_y$. Their use is quite straightforward when the critical buckling length is about this axis. An approach that permits the use of these tables when the strong axis controls will be addressed following the example.

**Table 4-1a (continued)**
**Available Strength in Axial Compression, kips**
**W-Shapes**

$F_y$ = 50 ksi — W14

| Shape | W14× | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| lb/ft | 82 | | 74 | | 68 | | 61 | | 53 | | 48 | | 43[c] | |
| Design | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ |
| | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD |
| **Effective length, $L_c$ (ft), with respect to least radius of gyration, $r_y$** | | | | | | | | | | | | | | |
| 0 | 719 | 1080 | 653 | 981 | 599 | 900 | 536 | 805 | 467 | 702 | 422 | 634 | 374 | 562 |
| 6 | 676 | 1020 | 614 | 922 | 562 | 845 | 503 | 756 | 421 | 633 | 380 | 572 | 339 | 510 |
| 7 | 661 | 993 | 600 | 902 | 550 | 826 | 492 | 739 | 406 | 610 | 366 | 551 | 327 | 491 |
| 8 | 644 | 968 | 585 | 879 | 536 | 805 | 479 | 720 | 389 | 585 | 351 | 527 | 312 | 470 |
| 9 | 626 | 940 | 568 | 854 | 520 | 782 | 465 | 699 | 371 | 557 | 334 | 502 | 297 | 447 |
| 10 | 606 | 910 | 550 | 827 | 503 | 756 | 450 | 676 | 351 | 528 | 316 | 475 | 281 | 422 |
| 11 | 584 | 878 | 531 | 797 | 485 | 729 | 433 | 651 | 331 | 497 | 298 | 447 | 264 | 397 |
| 12 | 562 | 844 | 510 | 767 | 466 | 701 | 416 | 626 | 310 | 465 | 279 | 419 | 247 | 371 |
| 13 | 538 | 809 | 489 | 735 | 446 | 671 | 398 | 599 | 288 | 433 | 259 | 390 | 229 | 345 |
| 14 | 514 | 772 | 467 | 701 | 426 | 640 | 380 | 571 | 267 | 401 | 240 | 360 | 212 | 318 |
| 15 | 489 | 735 | 444 | 667 | 405 | 608 | 361 | 543 | 246 | 369 | 221 | 331 | 194 | 292 |
| 16 | 464 | 697 | 421 | 633 | 384 | 577 | 342 | 514 | 225 | 338 | 202 | 303 | 177 | 267 |
| 17 | 438 | 659 | 398 | 598 | 362 | 544 | 323 | 485 | 205 | 308 | 183 | 276 | 161 | 242 |
| 18 | 413 | 620 | 375 | 563 | 341 | 512 | 304 | 456 | 185 | 278 | 166 | 249 | 145 | 218 |
| 19 | 387 | 582 | 352 | 529 | 320 | 480 | 285 | 428 | 166 | 250 | 149 | 224 | 130 | 196 |
| 20 | 362 | 545 | 329 | 495 | 299 | 449 | 266 | 399 | 150 | 226 | 134 | 202 | 117 | 177 |
| 22 | 314 | 472 | 285 | 428 | 258 | 388 | 229 | 345 | 124 | 186 | 111 | 167 | 97.1 | 146 |
| 24 | 267 | 402 | 243 | 365 | 219 | 330 | 195 | 293 | 104 | 157 | 93.2 | 140 | 81.6 | 123 |
| 26 | 228 | 343 | 207 | 311 | 187 | 281 | 166 | 249 | 88.8 | 133 | 79.4 | 119 | 69.5 | 104 |
| 28 | 197 | 295 | 179 | 268 | 161 | 242 | 143 | 215 | 76.6 | 115 | 68.5 | 103 | 59.9 | 90.1 |
| 30 | 171 | 257 | 156 | 234 | 140 | 211 | 125 | 187 | 66.7 | 100 | 59.7 | 89.7 | 52.2 | 78.5 |
| 32 | 150 | 226 | 137 | 205 | 123 | 185 | 110 | 165 | 58.6 | 88.1 | | | | |
| 34 | 133 | 200 | 121 | 182 | 109 | 164 | 97.0 | 146 | | | | | | |
| 36 | 119 | 179 | 108 | 162 | 97.5 | 147 | 86.5 | 130 | | | | | | |
| 38 | 107 | 160 | 96.9 | 146 | 87.5 | 131 | 77.7 | 117 | | | | | | |
| 40 | 96.3 | 145 | 87.5 | 131 | 79.0 | 119 | 70.1 | 105 | | | | | | |
| **Available Strength Parameters for Concentrated Forces[i]** | | | | | | | | | | | | | | |
| $P_{wo}$, kips | 123 | 185 | 104 | 155 | 90.6 | 136 | 77.5 | 116 | 77.1 | 116 | 67.4 | 101 | 56.9 | 85.4 |
| $P_{wi}$, kip/in. | 17.0 | 25.5 | 15.0 | 22.5 | 13.8 | 20.8 | 12.5 | 18.8 | 12.3 | 18.5 | 11.3 | 17.0 | 10.2 | 15.3 |
| $P_{wb}$, kips | 201 | 302 | 138 | 207 | 108 | 163 | 80.1 | 120 | 76.7 | 115 | 59.5 | 89.5 | 43.0 | 64.7 |
| $P_{fb}$, kips | 137 | 206 | 115 | 173 | 97.0 | 146 | 77.8 | 117 | 81.5 | 123 | 66.2 | 99.6 | 52.6 | 79.0 |
| **Properties** | | | | | | | | | | | | | | |
| $L_p$, ft | 8.76 | | 8.76 | | 8.69 | | 8.65 | | 6.78 | | 6.75 | | 6.68 | |
| $L_r$, ft | 33.2 | | 31.0 | | 29.3 | | 27.5 | | 22.3 | | 21.1 | | 20.0 | |
| $A_g$, in.$^2$ | 24.0 | | 21.8 | | 20.0 | | 17.9 | | 15.6 | | 14.1 | | 12.6 | |
| $I_x$, in.$^4$ | 881 | | 795 | | 722 | | 640 | | 541 | | 484 | | 428 | |
| $I_y$, in.$^4$ | 148 | | 134 | | 121 | | 107 | | 57.7 | | 51.4 | | 45.2 | |
| $r_y$, in. | 2.48 | | 2.48 | | 2.46 | | 2.45 | | 1.92 | | 1.91 | | 1.89 | |
| $r_x/r_y$ | 2.44 | | 2.44 | | 2.44 | | 2.44 | | 3.07 | | 3.06 | | 3.08 | |
| $P_{ex}L_c^2/10^4$, kip-in.$^2$ | 25200 | | 22800 | | 20700 | | 18300 | | 15500 | | 13900 | | 12300 | |
| $P_{ey}L_c^2/10^4$, kip-in.$^2$ | 4240 | | 3840 | | 3460 | | 3060 | | 1650 | | 1470 | | 1290 | |

| ASD | LRFD |
|---|---|
| $\Omega_c = 1.67$ | $\phi_c = 0.90$ |

[c] Shape is slender for compression with $F_y$ = 50 ksi; tabulated values have been adjusted accordingly.
[i] Flange local buckling, web local buckling, and web compression buckling are considered. Web local crippling, web sidesway buckling, and web panel zone shear are not addressed in this table.
Note: Heavy line indicates $L_c/r_y$ equal to or greater than 200.

**Figure 5.28** Available Strength in Axial Compression 

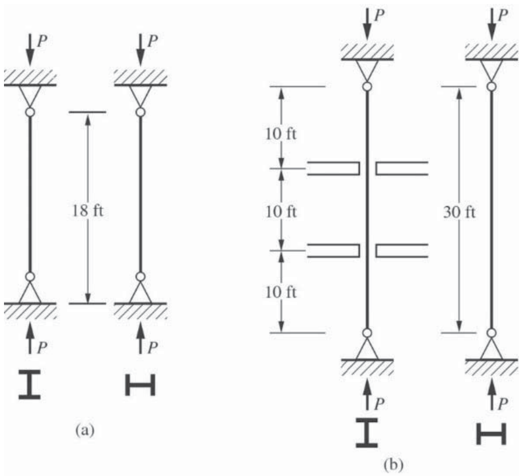



**Figure 5.29** Columns for Examples 5.12 and 5.13

**EXAMPLE 5.12a**
***Column Design by LRFD***

**Goal:** Determine the least-weight section to carry the loads given using the limited selection available in Figure 5.28.

**Given:** The column is shown in Figure 5.29a. It must resist the following loads in the appropriate combinations: $P_D = 56$ kips, $P_L = 172$ kips, and $P_W = 176$ kips. Use A992 steel. Assume the live load comes from a distributed load less than 100 psf, so that the LRFD load factor on live load may be taken as 0.5 for load combination 4.

***SOLUTION***

**Step 1:** Determine the maximum required strength using the LRFD load combinations from Section 2.4.

**1.** $1.4P_D = 1.4(56) = 78.4$ kips
**2.** $1.2P_D + 1.6P_L = 1.2(56) + 1.6(172) = 342$ kips
**4.** $1.2P_D + 0.5P_L + 1.0P_W$
$= 1.2(56) + 0.5(172) + 1.0(176) = 329$ kips
**6.** $0.9P_D + 1.0P_W = 0.9\,(56) + 1.0(176) = 226$ kips

So the column must carry $P_u = 342$ kips.

**Step 2:** The column has the same effective length about the *x*- and *y*-axes, so enter the table in Figure 5.28 with $L_c = 18$ ft. Scanning across the table at $L_c = 18$ ft and checking the LRFD values, select the least-weight shape in this portion of the table that can support this load.

Select a W14×61 with a design compression strength

$$\phi P_n = 456 \text{ kips}$$

| | | |
|---|---|---|
| **EXAMPLE 5.12b** ***Column Design by ASD*** | **Goal:** | Determine the least-weight section to carry the loads given using the limited selection available in Figure 5.28. |
| | **Given:** | The column is shown in Figure 5.29a. It must resist the following loads in the appropriate combinations: $P_D$ = 56 kips, $P_L$ = 172 kips, and $P_W$ = 176 kips. Use A992 steel. |
| ***SOLUTION*** | **Step 1:** | Determine the maximum required strength using the ASD load combinations from Section 2.4.<br>**1.** $P_D = 56$ kips<br>**2.** $P_D + P_L = 56 + 172 = 228$ kips<br>**5.** $P_D + 0.60P_W = 56 + 0.6(176) = 162$ kips<br>**6.** $P_D + 0.75\,P_L + 0.75(0.6P_W)$<br>$= 56 + 0.75(172) + 0.75(0.6(176)) = 264$ kips<br>**7.** $0.6\,P_D + 0.6\,P_W = 0.6(56) + 0.6(176) = 139$ kips<br><br>So the column must carry $P_a = 264$ kips. |
| | **Step 2:** | The column has the same effective length about the *x*- and *y*-axes, so enter the table in Figure 5.28 with $L_c$ = 18 ft. Scanning across the table at $L_c$ = 18 ft and checking the ASD values, select the least-weight shape in this portion of the table that can support this load<br><br>Select a W14×61 with an allowable compression strength<br>$P_n/\Omega = 304$ kips |

If the largest slenderness ratio for a particular column happens to be for *x*-axis buckling, the tables may not be entered directly with the *x*-axis effective length because the table effective length is intended to be used in conjunction with the least radius of gyration. However, it is possible to determine a modified effective length that, when used in the table, will result in the correct column strength.

When the *x*-axis controls column strength, the slenderness ratio used in the column equations is $L_{cx}/r_x$. To use the column tables, an effective length, $(L_c)_{eff}$, must be determined that, when combined with $r_y$, gives the same slenderness ratio. So

$$\frac{(L_c)_{eff}}{r_y} = \frac{L_{cx}}{r_x}$$

Solving this equation for $(L_c)_{eff}$ yields

$$(L_c)_{eff} = \frac{L_{cx}}{(r_x / r_y)}$$

With this modified effective length, the tables can be entered and a suitable column selected. There is one difficulty with this process, however. Until a column section is known, the value for $r_x/r_y$ cannot be determined. To account for this, a quick scan of the column tables should be made to estimate $r_x/r_y$. Then, when a section is selected, the assumption can be verified and an adjustment made if necessary.

**EXAMPLE 5.13**
***Column Design***

**Goal:** Determine the least-weight section to carry the force given using the limited selection available through Figure 5.28. Design by LRFD and ASD.

**Given:** The column is shown in Figure 5.29b. Use the loading from Example 5.12.

***SOLUTION***

**Step 1:** Determine the effective length for each axis.

Bracing of the *y*-axis, shown in Figure 5.29b, yields $L_{cy}$ = 10.0 ft. The unbraced *x*-axis has $L_{cx}$ = 30.0 ft.

**Step 2:** Determine $(L_c)_{eff}$ for the *x*-axis.

Select a representative $r_x/r_y$ from Figure 5.28. There are two general possibilities. Assume that the larger shapes might be needed to carry the load, and try $r_x/r_y$ = 2.44. Thus,

$$(L_c)_{eff} = \frac{L_{cx}}{(r_x/r_y)} = \frac{30.0}{2.44} = 12.3\text{ ft}$$

**Step 3:** Determine the controlling effective length.

Because $(L_c)_{eff}$ = 12.3 ft is greater than $L_{cy}$ = 10.0 ft, enter the table with $L_c$ = 12.3 ft and interpolate between 12 ft and 13 ft.

**For LRFD**

**Step 4:** From Example 5.12a the column must have a design strength greater than $P_u$ = 342 kips with $L_c$ = 12.3 ft. Try a W14×43, which happens to be the smallest column available with the limited selection available in Figure 5.28. This column has $r_x/r_y$ = 3.08.

**Step 5:** Determine $(L_c)_{eff}$ with this new $r_x/r_y$. Thus,

$$(L_c)_{eff} = \frac{30.0}{3.08} = 9.74\text{ ft}$$

**Step 6:** Determine the new controlling effective length.

Because $(L_c)_{eff}$ = 9.74 ft is now less than $L_{cy}$ = 10.0 ft, enter the table with 10.0 ft and note that the W14×43 has a design strength of 422 kips, which is greater than the required strength of 342 kips.

**Step 7:** Therefore, use the selected

W14×43

*Note:* The W14×43 is identified in the table by a footnote as slender for $F_y$ = 50 ksi. This is not an issue for our design because the impact of any slender element has already been taken into account in generating the table as stated in the same footnote.

Using the full complement of tables available in the *Manual* results in a smaller W12 section having the ability to carry the given load.

**For ASD**

**Step 4:** From Example 5.12b the column must have an allowable strength greater than $P_a$ = 264 kips with $L_c$ = 12.3 ft. Try a W14×48. This column has $r_x/r_y$ = 3.06.

**Step 5:** Determine $(L_c)_{eff}$ with this new $r_x/r_y$. Thus,

$$(L_c)_{eff} = \frac{30.0}{3.06} = 9.80 \text{ ft}$$

**Step 6:** Determine the new controlling effective length.

Because $(L_c)_{eff}$ = 9.80 ft is now less than $L_{cy}$ = 10.0 ft, enter the table with 10.0 ft and see that the W14×43 has an allowable strength of 281 kips, which is greater than the required strength of 268 kips and $r_x/r_y$ = 3.08 which is greater than that for the W14×48 so $L_{cy}$ = 10.0 ft will still control.

**Step 7:** Therefore, use the selected

W14×43

*Note:* The W14×43 is identified in the table by a footnote as slender for $F_y$ = 50 ksi. This is not an issue for our design because the impact of any slender element has already been taken into account in generating the table as stated in the same footnote.

Using the full complement of tables available in the *Manual* results in a smaller W12 section having the ability to carry the given load.

Table 4-1a in Part 4 of the *Manual* includes shapes from a W8×31 up to a W14×873. All of the shapes included are considered column shapes and have reasonably similar strengths about the *x*- and *y*-axes. That is, the shapes are close to being square and $r_x/r_y$ is not extremely large, ranging from 1.59 to 3.08. Any of the other available W-shapes may be used for columns if desired, but it must be recognized that the relationship between the *x*- and *y*-axes is such that the *y*-axis will control unless significant bracing is provided. These shapes are generally considered beam shapes. Since beams are intended to be used to carry flexure, the relationship between the $x$- and $y$-axes is not as critical. For example, for a W16×26 with a length of 24 ft braced at the ends only for the $x$-axis and $r_x/r_y = 5.59$, the $y$-axis will control unless it is braced at least every 4.29 ft.

The W-shape column tables in Part 4 of the *Manual* for $F_y = 50$ ksi also exclude the smallest W-shapes in an attempt to direct the design engineer toward using shapes that are more appropriate when considering connections. That does not mean that these smaller shapes are not acceptable for use as columns. The tables in Part 6 of the *Manual*, which will be discussed in Chapter 8, can be used for the design of columns and they include all of the W-shapes.

**EXAMPLE 5.14**
***Column Design***

**Goal:** Determine the least-weight section to carry the force given using the small shapes provided in the W-shape tables in *Manual* Part 6. Design by LRFD and ASD.

**Given:** The A992 column has an effective length for both axes of 10 ft and must carry a concentrated dead load of 8 kips and a concentrated live load of 24 kips.

***SOLUTION***

**For LRFD**

**Step 1:** Determine the required strength for the load combination 1.2D + 1.6L.

$$P_u = 1.2(8.0) + 1.6(24.0) = 48.0 \text{ kips}$$

**Step 2:** Using *Manual* Table 6-2, select the lightest column to support this load.

Select the W4×13.

$$\phi P_n = 60.1 \text{ kips}$$

**For ASD**

**Step 1:** Determine the required strength for the load combination D + L.

$$P_a = 8.0 + 24.0 = 32.0 \text{ kips}$$

**Step 2:** Using *Manual* Table 6-2, select the lightest column to support this load.

Select the W4×13.

$$\frac{P_n}{\Omega} = 40.0 \text{ kips}$$

## 5.8 TORSIONAL BUCKLING AND FLEXURAL-TORSIONAL BUCKLING

Up to this point, the discussion has addressed the limit states of flexural buckling and local buckling. Two additional limit states for column behavior must be addressed: torsional buckling and flexural-torsional buckling. Doubly symmetric shapes normally fail through flexural buckling, as discussed earlier in this chapter, or through torsional buckling. Singly symmetric and unsymmetric shapes can fail through flexural, torsional, or flexural-torsional buckling. Because the shapes normally used for steel members are not well suited to resist torsion, except for closed HSS, it is usually desirable to avoid any torsional limit states through proper bracing of the column or by avoiding torsional loading.

If either of the torsional limit states must be evaluated, the applicable *Specification* provisions are found in Section E4, except for the special cases associated with single angles, which are found in Section E5. For doubly symmetric, singly symmetric and unsymmetric members braced so that they buckle torsionally about their shear center, specific elastic buckling stress equations are provided. For doubly symmetric members with bracing offset from the shear center, separate elastic buckling stress equations are given depending on if the bracing is offset from the strong or weak axis. For all these shapes, once the elastic buckling stress, $F_e$, is determined, it is then used in Equations E3-2 and E3-3 to determine the nominal compressive stress, $F_n$. The equations given in Section E4 defining the elastic buckling stress for the limit states of torsional and flexural-torsional buckling are also found in several other books with varying notation, including *Buckling Strength of Metal Structures*.[2] The equations of Section E3 are used to account for such factors as inelastic buckling, initial out-of-straightness, and residual stresses.

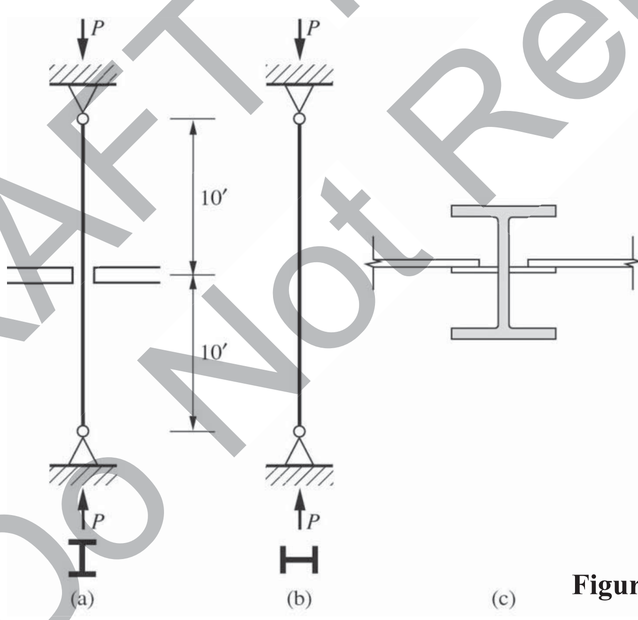



**Figure 5.30** Column for Example 5.15

[2] Bleich, F. *Buckling Strength of Metal Structures*. New York: McGraw-Hill, 1952.

Because single-angle compression members are so common, the *Specification* provides a simplified approach for those members meeting a specific set of criteria. By limiting the way that load is applied to the ends of a single-angle compression member, an effective slenderness is established, which is then used in Equations E3-2 and E3-3 to determine the nominal compressive stress, $F_n$.

The limit state of torsional buckling is not normally considered in the design of W-shape columns when the *y*-axis is the controlling axis for flexural buckling. Torsional buckling generally does not govern, and when it does, the critical load differs very little from the strength determined from flexural buckling. For other member types, such as WT or double-angle compression members often used in trusses, torsional limit states are quite important.

An additional factor in determining strength based on these limit states is the torsional effective length. The Commentary recommends that, conservatively, the torsional effective length be taken as the column length and provides several other possibilities if greater accuracy is desired.

**EXAMPLE 5.15**
***Strength of a W-Shape Column with Torsional Buckling***

**Goal:** Determine the available strength of a W-shape column and consider torsional buckling.

**Given:** A W14×48 A992 column as shown in Figure 5.30 is braced laterally and torsionally at its ends. At mid-height it is braced to resist buckling about the *y*-axis, but it cannot resist torsional buckling based on the bracing shown in Figure 5.30c.

***SOLUTION***

**Step 1:** From *Manual* Table 1-1,

$$A_g = 14.1\text{ in.}^2,\ I_x = 484\text{ in.}^4,\ I_y = 51.4\text{ in.}^4,\ r_x = 5.85\text{ in.},\ r_y = 1.91\text{ in.},$$
$$C_w = 2240\text{ in.}^6,\ J = 1.45\text{ in.}^4,\ h/t_w = 33.6$$

**Step 2:** Determine the web slenderness limit.

$$\lambda_{rw} = 1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29{,}000}{50}} = 35.9$$

**Step 3:** Check slenderness of the web and flange.

$$\frac{h}{t_w} = 33.6 < \lambda_{rw} = 35.9$$

Therefore, the web is not slender. As previously discussed, all W-shapes with $F_y = 50$ ksi have nonslender flanges.

**Step 4:** Determine the nominal stress for *y*-axis buckling.

$$\left(\frac{L_c}{r}\right)_y = \frac{10(12)}{1.91} = 62.8$$

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} = \frac{\pi^2 (29{,}000)}{(62.8)^2} = 72.6 \text{ ksi} > \frac{F_y}{2.25} = 22.2 \text{ ksi}$$

Therefore, use Equation E3-2:

$$F_n = (0.658)^{\left(\frac{50}{72.6}\right)}(50) = 37.5 \text{ ksi}$$

**Step 5:** Determine the nominal stress for *x*-axis buckling.

$$\left(\frac{L_c}{r}\right)_x = \frac{20(12)}{5.85} = 41.0$$

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} = \frac{\pi^2 (29{,}000)}{(41.0)^2} = 170 \text{ ksi} > \frac{F_y}{2.25} = 22.2 \text{ ksi}$$

Therefore, use Equation E3-2:

$$F_n = (0.658)^{\left(\frac{50}{170}\right)}(50) = 44.2 \text{ ksi}$$

**Step 6:** Determine the nominal stress for *z*-axis buckling, or twisting about the shear center, using Section E4(a) Equation E4-2.

$$F_e = \left[\frac{\pi^2 E C_w}{(L_{cz})^2} + GJ\right]\frac{1}{I_x + I_y}$$

$$= \left[\frac{\pi^2 (29{,}000)(2240)}{(20(12))^2} + 11{,}200(1.45)\right]\frac{1}{484 + 51.4}$$

$$= 51.1 \text{ ksi} > \frac{F_y}{2.25} = 22.2 \text{ ksi}$$

Therefore use Equation E3-2:

$$F_n = (0.658)^{\left(\frac{50}{51.1}\right)}(50) = 33.2 \text{ ksi}$$

**Step 7:** Select the lowest nominal stress determined in Steps 4, 5, and 6.

$$F_n = 33.2 \text{ ksi}$$

Since the controlling nominal stress comes from Step 6, the strength of the column is controlled by torsional buckling.

**Step 8:** Determine the nominal strength of the column

$$P_n = 33.2(14.1) = 468 \text{ kips}$$

*Note*: Determination of $F_n$ in steps 4, 5, and 6 could have been delayed until after the controlling, smallest, value of $F_e$ had been determined and then $F_n$ determined only once.

**For LRFD Step 9:** Determine the column design strength.

$$\phi P_n = 0.9(468) = 421 \text{ kips}$$

**For ASD Step 9:** Determine the column allowable strength.

$$\frac{P_n}{\Omega} = \frac{468}{1.67} = 280 \text{ kips}$$

**EXAMPLE 5.16** ***Strength of a WT-Shape Compression Member***

**Goal:** Determine the available strength of a WT-shape compression member with consideration of flexural, torsional, and flexural-torsional buckling.

**Given:** A WT7×34 A992 column is 10.0 ft long and is braced laterally and torsionally at its ends only.

***SOLUTION***

**Step 1:** From *Manual* Table 1-8,

$A_g = 10.0$ in.$^2$, $I_x = 32.6$ in.$^4$, $I_y = 60.7$ in.$^4$, $r_x = 1.81$ in.,
$r_y = 2.46$ in., $t_f = 0.720$, $C_w = 3.21$ in.$^6$, $J = 1.50$ in.$^4$, $d/t_w = 16.9$,
$\bar{y} = 1.29$ in., $b_f/2t_f = 6.97$

**Step 2:** Determine the flange and stem slenderness limits from Table B4.1 cases 1 and 4.

$$\lambda_{rf} = 0.56\sqrt{\frac{E}{F_y}} = 0.56\sqrt{\frac{29{,}000}{50}} = 13.5$$

$$\lambda_{rw} = 0.75\sqrt{\frac{E}{F_y}} = 0.75\sqrt{\frac{29{,}000}{50}} = 18.1$$

**Step 3:** Check slenderness of the flange and stem.

$$\frac{b_f}{2t_f} = 6.97 \le \lambda_{rf} = 13.5$$

$$\frac{d}{t_w} = 16.9 < \lambda_{rw} = 18.1$$

Therefore the WT has nonslender flange and stem.

**Step 4:** Determine the nominal strength for flexural buckling. Since $L_{cx} = L_{cy}$ and the $x$-axis has the smallest radius of gyration, flexural buckling will be controlled by the $x$-axis.

$$\left(\frac{L_c}{r}\right)_x = \frac{10(12)}{1.81} = 66.3 \le 4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29,000}{50}} = 113$$

$$F_{ex} = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} = \frac{\pi^2(29,000)}{(66.3)^2} = 65.1 \text{ ksi} > \frac{F_y}{2.25} = 22.2 \text{ ksi}$$

Therefore, use Equation E3-2:

$$F_n = (0.658)^{\left(\frac{F_y}{F_{e.}}\right)} F_y = (0.658)^{\left(\frac{50}{65.1}\right)}(50) = 36.3 \text{ ksi}$$

and

$$P_n = 36.3(10.0) = 363 \text{ kips}$$

**Step 5:** To determine flexural-torsional buckling, the elastic buckling stress for $y$-axis buckling is required.

$$\left(\frac{L_c}{r}\right)_y = \frac{10(12)}{2.46} = 48.8$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} = \frac{\pi^2(29,000)}{(48.8)^2} = 120 \text{ ksi}$$

**Step 6:** Determine the flexural-torsional elastic buckling stress for $z$-axis buckling using Equation E4-3. The shear center of a WT-shape is at the stem-flange intersection. Thus, the distance from the centroid to the shear center is

$$x_o = 0,\ y_o = \bar{y} - \frac{t_f}{2} = 1.29 - \frac{0.720}{2} = 0.930 \text{ in.}$$

and from Equation E4-9

$$r_o^2 = x_o^2 + y_o^2 + \frac{I_x + I_y}{A_g} = 0.0 + 0.930^2 + \frac{32.6 + 60.7}{10.0} = 10.2$$

and Equation E4-8

$$H = 1 - \frac{x_o^2 + y_o^2}{\bar{r}_o^2} = 1 - \frac{0 + 0.930^2}{10.2} 0.915$$

From the user note in Section E4, take $C_w = 0$ in Equation E4-7. Thus,

$$F_{ez} = \frac{GJ}{A_g \bar{r}_o^2} = \frac{11,200(1.50)}{10.0(10.2)} = 165 \text{ ksi}$$

**Step 7:** Determine the flexural-torsional elastic buckling stress for the singly symmetric member using Equation E4-3.

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left(1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right)$$

$$= \left(\frac{120 + 165}{2(0.915)}\right)\left(1 - \sqrt{1 - \frac{4(120)(165)(0.915)}{(120 + 165)^2}}\right) = 105 \text{ ksi}$$

**Step 8:** Determine the nominal stress using the flexural-torsional elastic buckling stress

$$F_e = 105 \text{ ksi} > \frac{F_y}{2.25} = 22.2 \text{ ksi}$$

Therefore, using Equation E3-2

$$F_n = (0.658)^{\frac{50}{105}}(50) = 41.0 \text{ ksi}$$

**Step 9:** Determine the nominal strength of the compression member for the limit state of flexural-torsional buckling.

$$P_n = 41.0(10.0) = 410 \text{ kips}$$

**For LRFD**

**Step 10:** Determine the compression member design strength. Since the nominal strength for flexural buckling about the $x$-axis is less than the flexural-torsional buckling strength,

$$\phi P_n = 0.9(363) = 327 \text{ kips}$$

**For ASD**

**Step 10:** Determine the compession member allowable strength. Since the nominal strength for flexural buckling about the $x$-axis is less than the flexural-torsional buckling strength,

$$\frac{P_n}{\Omega} = \frac{363}{1.67} = 217 \text{ kips}$$

**EXAMPLE 5.17**
***Strength of a W-Shape Column with Constrained-Axis Torsional Buckling***

**Goal:** Determine the available strength of a W-shape column and consider torsional buckling when the lateral bracing is offset from the shear center.

**Given:** A W14×48 A992 column as shown in Figure 5.30 and considered in Example 5.15 is braced laterally and torsionally at its ends. At mid-height it is braced to resist buckling about the *y*-axis. The y-axis bracing is moved from the shear center, as shown in Figure 5.30c, to the face of the flange. Thus constrained-axis torsional buckling must be assessed.

***SOLUTION***

**Step 1:** From *Manual* Table 1-1,

$$A_g = 14.1 \text{ in.}^2,\ I_x = 484 \text{ in.}^4,\ I_y = 51.4 \text{ in.}^4,\ r_x = 5.85 \text{ in.},\ r_y = 1.91 \text{ in.},$$
$$d = 13.8 \text{ in.},\ h_o = 13.2 \text{ in.},\ t_f = 0.595,\ C_w = 2240 \text{ in.}^6,\ J = 1.45 \text{ in.}^4,$$
$$h/t_w = 33.6$$

**Step 2:** Determine the web slenderness limit.

$$\lambda_{rw} = 1.49\sqrt{\frac{E}{F_y}} = 1.49\sqrt{\frac{29{,}000}{50}} = 35.9$$

**Step 3:** Check slenderness of the web and flange.

$$\frac{h}{t_w} = 33.6 < \lambda_{rw} = 35.9$$

Therefore, the web is not slender. As previously discussed, all W-shapes with $F_y = 50$ ksi have nonslender flanges.

**Step 4:** Determine the nominal stress for *y*-axis buckling.

$$\left(\frac{L_c}{r}\right)_y = \frac{10(12)}{1.91} = 62.8$$

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} = \frac{\pi^2(29{,}000)}{(62.8)^2} = 72.6 \text{ ksi} > \frac{F_y}{2.25} = 22.2 \text{ ksi}$$

Therefore, use Equation E3-2:

$$F_n = (0.658)^{\left(\frac{50}{72.6}\right)}(50) = 37.5 \text{ ksi}$$

**Step 5:** Determine the nominal stress for *x*-axis buckling.

$$\left(\frac{L_c}{r}\right)_x = \frac{20(12)}{5.85} = 41.0$$

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} = \frac{\pi^2 (29{,}000)}{(41.0)^2} = 170 \text{ ksi} > \frac{F_y}{2.25} = 22.2 \text{ ksi}$$

Therefore, use Equation E3-2:

$$F_n = (0.658)^{\left(\frac{50}{170}\right)}(50) = 44.2 \text{ ksi}$$

**Step 6:** Determine the nominal stress for *z*-axis buckling with the bracing offset along the minor axis using Section E4(d) Equation E4-10.

The bracing offset is $y_a = d/2 = 13.8/2 = 6.9$ in. and $x_a = 0$. Thus, from Equation E4-11,

$$r_o^2 = \left(r_x^2 + r_y^2 + y_a^2 + x_a^2\right) = \left(5.85^2 + 1.91^2 + 6.9^2 + 0\right) = 85.5 \text{ in.}^2$$

and

$$F_{ez} = \left[\frac{\pi^2 E I_y}{(L_{cz})^2}\left(\frac{h_o^2}{4} + y_a^2\right) + GJ\right]\frac{1}{A_g r_o^2}$$

$$= \left[\frac{\pi^2 (29{,}000)(51.4)}{\left(20(12)\right)^2}\left(\frac{13.2^2}{4} + 6.9^2\right) + 11{,}200(1.45)\right]\frac{1}{14.1(85.5)}$$

$$= 32.8 \text{ ksi} > \frac{F_y}{2.25} = 22.2 \text{ ksi}$$

Therefore use Equation E3-2:

$$F_n = (0.658)^{\left(\frac{50}{32.8}\right)}(50) = 26.4 \text{ ksi}$$

**Step 7:** Select the lowest nominal stress determined in Steps 4, 5, and 6.

$$F_n = 26.4 \text{ ksi}$$

Since the controlling nominal stress comes from Step 6, the strength of the column is controlled by torsional buckling.

**Step 8:** Determine the nominal strength of the column

$$P_n = 26.4(14.1) = 372 \text{ kips}$$

*Note*: As was the case for Example 5.15, determination of $F_n$ in steps 4, 5, and 6 could have been delayed until after the controlling, smallest, value of $F_e$ had been determined and then $F_n$ determined only once.

Also note that moving the lateral brace from the shear center, as in Example 5-15, to the face of the flange has reduced the strength of the column.

**For LRFD Step 9:** Determine the column design strength.

$$\phi P_n = 0.9(372) = 335 \text{ kips}$$

**For ASD Step 9:** Determine the column allowable strength.

$$\frac{P_n}{\Omega} = \frac{372}{1.67} = 223 \text{ kips}$$

## 5.9 SINGLE-ANGLE COMPRESSION MEMBERS

Single-angle compression members would be designed for flexural-torsional buckling according to the provisions in *Specification* Section E4 except for an exclusion for angles with $b/t \le 0.71\sqrt{E/F_y}$ . All hot rolled, A36 angles satisfy this exclusion limit so they need not be checked for flexural-torsional buckling. However, since the preferred material for angles is A572 Gr. 50, one must check to be sure that the provisions in *Specification* Section E4 are applicable.

Studies show that the compressive strength of single angles can be reasonably predicted using the compression member equations of *Specification* Section E3 if a modified effective length is used and the member satisfies the following limiting criteria as found in *Specification* Section E5.

1. Members are loaded at their ends in compression through the same one leg.
2. Members are attached by either welding or a connection containing a minimum of two bolts.
3. There are no intermediate transverse loads.
4. $L_c/r$ as determined in this section does not exceed 200.
5. For unequal leg angles, the ratio of the long leg width to short leg width is less than 1.7.

Two cases are given for these provisions: (1) angles that are individual members or web members of planar trusses, and (2) angles that are web members in box or space trusses. This distinction is intended to reflect the difference in restraint provided by the elements to which the compression members are attached.

The first set of equations is for angles that

1. are individual members or web members of planar trusses.
2. are equal-leg angles or unequal-leg angles connected through the longer leg.
3. have adjacent web members attached to the same side of a gusset plate or truss chord.

Buckling is assumed to occur about the geometric axis parallel to the attached leg. Since this may be either the *x*- or *y*-axis, the *Specification* uses the subscript *a* and then defines $r_a$ as the radius of gyration about the axis parallel to the attached leg.

If $\dfrac{L}{r_a} \leq 80$,

$$\frac{L_c}{r} = 72 + 0.75\frac{L}{r_a} \qquad \text{(AISC E5-1)}$$

and if $\dfrac{L}{r_a} > 80$

$$\frac{L_c}{r} = 32 + 1.25\frac{L}{r_a} \qquad \text{(AISC E5-2)}$$

These effective lengths must be modified if the unequal-leg angles are attached through the shorter legs. The provisions of *Specification* Section E5 should be reviewed for these angles as well as for similar angles in box or space trusses.

**EXAMPLE 5.18** ***Strength of Single-Angle Compression Member***

**Goal:** Determine the available strength of a 10.0 ft single-angle compression member using A572 Gr. 50 steel and the provisions of *Specification* Section E5.

**Given:** A 4×4×1/2 angle is a web member in a planar truss. It is attached by two bolts at each end through the same leg.

***SOLUTION***

**Step 1:** Check angle leg slenderness,

$$\frac{b}{t} = \frac{4}{0.5} = 8.0 < 0.71\sqrt{\frac{E}{F_y}} = 0.71\sqrt{\frac{29{,}000}{50}} = 17.1$$

Therefore, it is permissible to use the provisions of *Specification* Section E5 and, since

$$\frac{b}{t} = \frac{4}{0.5} = 8.0 < 0.45\sqrt{\frac{E}{F_y}} = 0.45\sqrt{\frac{29{,}000}{50}} = 10.8$$

The provisions of Section E7 do not apply.

**Step 2:** From *Manual* Table 1-7,

$A = 3.75$ in.$^2$ and $r_x = 1.21$.

**Step 3:** Determine the slenderness ratio for the axis parallel to the connected leg, $r_a = r_x$.

$$\frac{L}{r_a} = \frac{L}{r_x} = \frac{10.0(12)}{1.21} = 99.2$$

**Step 4:** Determine which equation will give the effective slenderness ratio.

Because

$$\frac{L}{r_a} = 99.2 > 80$$

use Equation E5-2.

**Step 5:** Determine the effective slenderness ratio from Equation E5.2.

$$\frac{L_c}{r} = 32 + 1.25(99.2) = 156 < 200$$

**Step 6:** Determine which column strength equation to use.

Because

$$\frac{L_c}{r} = 156 > 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

use Equation E3-3.

**Step 7:** Determine the Euler buckling stress.

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} = \frac{\pi^2(29{,}000)}{(156)^2} = 11.8 \text{ ksi}$$

**Step 8:** Determine the nominal stress from Equation E3-3.

$$F_n = 0.877F_e = 0.877(11.8) = 10.3 \text{ ksi}$$

**Step 9:** Determine the nominal strength.

$$P_n = F_{cr}A = 10.3(3.75) = 38.6 \text{ kips}$$

**For LRFD Step 10:** Determine the design strength.

$$\phi P_n = 0.9(38.6) = 34.7 \text{ kips}$$

**For ASD Step 10:** Determine the allowable strength.

$$P_n/\Omega = 38.6/1.67 = 23.1 \text{ kips}$$

For single angle compression members that do not meet the criteria set forth in Section E5 for use of the modified slenderness ratio equations, the provisions of Sections E3 or E7 must be followed. The provisions in Section E4 for torsional or flexural-torsional buckling do not need to be followed for hot-rolled angles that meet the leg slenderness exclusion of $b/t \le 0.71\sqrt{E/F_y}$ . Thus, for these members, the strength for flexural buckling about the principal axes must be assessed.

**EXAMPLE 5.19** ***Strength of Single-Angle Compression Member***

**Goal:** Determine the available strength of a 10.0 ft single-angle compression member using A572 Gr. 50 steel.

**Given:** A 4×4×1/2 angle is a web member in a planar truss. It is attached by single bolts at each end through the same leg.

***SOLUTION***

**Step 1:** From *Manual* Table 1-7,

$$A = 3.75 \text{ in.}^2 \text{ and } r_x = r_y = 1.21,\ r_z = 0.776 \text{ in.}^2.$$

and from Example 5.17, the angle is not a slender element member and the provisions of Section E7 do not apply. Additionally, since

$$\frac{b}{t} = \frac{4}{0.5} = 8.0 < 0.71\sqrt{\frac{E}{F_y}} = 0.71\sqrt{\frac{29{,}000}{50}} = 17.1$$

the provisions of Section E4 need not be checked.

**Step 2:** Determine the slenderness ratio for the minor (weak) principal axis.

$$\frac{L_c}{r_z} = \frac{10.0(12)}{0.776} = 155$$

**Step 3:** Determine which column strength equation to use.

Because

$$\frac{L_c}{r_z} = 155 > 4.71\sqrt{\frac{29{,}000}{50}} = 113$$

Use Equation E3-3.

**Step 4:** Determine the Euler buckling stress.

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} = \frac{\pi^2(29{,}000)}{(155)^2} = 11.9 \text{ ksi}$$

**Step 5:** Determine the nominal stress from Equation E3-3.

$$F_{cr} = 0.877F_e = 0.877(11.9) = 10.4 \text{ ksi}$$

**Step 6:** Determine the nominal strength.

$$P_n = F_{cr}A = 10.4(3.75) = 39.0 \text{ kips}$$

**For LRFD Step 7:** Determine the design strength.

$$\phi P_n = 0.9(39.0) = 35.1 \text{ kips}$$

**For ASD Step 7:** Determine the allowable strength.

$$P_n/\Omega = 39.0/1.67 = 23.4 \text{ kips}$$

## 5.10 BUILT-UP MEMBERS

Members composed of more than one shape are called *built-up members*. Several of these were illustrated in Figure 5.2h through n. Built-up compression members composed of two shapes are covered in *Specification* Section E6. Compressive strength is addressed by establishing the slenderness ratio and referring to *Specification* Section E3, E4, or E7 as appropriate.

If a built-up section buckles so that the fasteners between the shapes are not stressed in shear but simply "go along for the ride," the only requirement is that the slenderness ratio of the shape between fasteners be no greater than 0.75 times the controlling slenderness ratio of the built-up shape. If overall buckling would put the fasteners into shear, then the controlling slenderness ratio will be somewhat greater than the slenderness ratio of the built-up shape. This modified slenderness ratio is used to account for the effect of shearing deformations through the connectors. Thus, the effective slenderness ratio for a built-up member with snug-tight connectors will be greater than the same member with pre-tensioned or welded connectors. In addition, the spacing of the intermediate connectors will influence the modified slenderness ratio.

For intermediate connectors that are bolted snug-tight, the modified slenderness ratio is always greater than the slenderness ratio of the built-up member acting as a unit since there will always be some shearing deformation in the connectors. It is specified as

$$\left(\frac{L_c}{r}\right)_m = \sqrt{\left(\frac{L_c}{r}\right)_o^2 + \left(\frac{a}{r_i}\right)^2} \qquad \text{(AISC E6-1)}$$

If the intermediate connectors are welded or pre-tensioned bolted, the shearing deformation in the connectors is significantly less than for snug-tight connectors and the modified slenderness ratio may be equal to the slenderness ratio of the built-up member acting as a unit. For this case, the modified slenderness ratio is specified as, when $\frac{a}{r_i} \leq 40$,

$$\left(\frac{L_c}{r}\right)_m = \left(\frac{L_c}{r}\right)_o \qquad \text{(AISC E6-2a)}$$

and when $\frac{a}{r_i} > 40$,

$$\left(\frac{L_c}{r}\right)_m = \sqrt{\left(\frac{L_c}{r}\right)_o^2 + \left(\frac{K_i a}{r_i}\right)^2} \qquad \text{(AISC E6-2b)}$$

where

$\left(\frac{L_c}{r}\right)_o$ = column slenderness of built-up member acting as a unit

$K_i$ = 0.5 for angles back-to-back

= 0.75 for channels back-to-back

= 0.86 for all other shapes

$a$ = distance between connectors

$r_i$ = minimum radius of gyration of individual component

The remaining provisions in *Specification* Section E6 address dimensions and detailing requirements. These provisions are based on judgment and experience and are provided to ensure that the built-up member behaves in a way consistent with the strength provisions already discussed. The ends of built-up compression members must be either welded or pre-tensioned bolted in order to ensure that the member can work together as a unit. Even the smallest amount of slip in the end connections could mean that the built-up member is unable to carry any more load than the components individually. Along the length of built-up members, the longitudinal spacing of connectors must be sufficient to provide for transfer of the required shear force in the buckled member. The Commentary of the *Specification* gives guidance on how to determine the magnitude of the forces in the connectors. A built-up compression member with connectors spaced so that the slenderness ratio of the shape between fasteners is no greater than 0.75 times the controlling slenderness ratio of the built-up shape will not automatically satisfy this strength requirement.

The *Manual* provides tables of properties for double angles, double channels, and I-shapes with cap channels in Part 1 and tables of compressive strength for double-angle compression members in Part 4.

**EXAMPLE 5.20** ***Strength of a Built-up Double-Angle Compression Member***

**Goal:** Determine the available strength of a 10.0 ft double-angle compression member using A572 Gr. 50 steel.

**Given:** Two 5×3×5/16 angles, long legs back-to-back with a 3/8 in. gap are used as a chord member in a planar truss. The angles are welded at each end to a gusset plate and along the length at two intermediate points with a spacing of 40 in.

***SOLUTION***

**Step 1:** From *Manual* Table 1-15 for double angles

$A = 4.82$ in.$^2$ , $r_x = 1.61$ in., $r_y = 1.21$ in., $\bar{r}_o = 2.52$ in. and $H = 0.640$ in.

From *Manual* Table 1-7 for single angles

$r_z = 0.649$ in. and $J = 0.0832$ in.$^4$

Check leg slenderness

$$\frac{b}{t} = \frac{5}{0.3125} = 16.0 < 0.71\sqrt{\frac{E}{F_y}} = 0.71\sqrt{\frac{29{,}000}{50}} = 17.1$$

Thus, flexural-torsional buckling of the individual angles need not be considered.

For local buckling,

$$\frac{b}{t} = \frac{5}{0.3125} = 16.0 > 0.45\sqrt{\frac{E}{F_y}} = 0.45\sqrt{\frac{29{,}000}{50}} = 10.8$$

Therefore, local buckling must be considered.

**Step 2:** Determine the slenderness ratio for each axis if the member works as a unit.

$$\frac{L}{r_x} = \frac{10.0(12)}{1.61} = 74.5$$

$$\frac{L}{r_y} = \frac{10.0(12)}{1.21} = 99.2$$

**Step 3:** Determine the effective slenderness ratio for buckling about the *y*-axis, the axis that will put the connectors in shear. Since the intermediate connectors are spaced at 40 in.

$$\frac{a}{r_i} = \frac{a}{r_z} = \frac{40}{0.649} = 61.6 > 40$$

Therefore use Equation E6.2b

$$\left(\frac{L_c}{r}\right)_m = \sqrt{\left(\frac{L_c}{r}\right)_o + \left(\frac{K_i a}{r_i}\right)^2} = \sqrt{99.2^2 + \left(0.5\left(61.6\right)\right)^2} = 104$$

**Step 4:** Check the maximum permitted slenderness ratio between connectors

$$\frac{a}{r_i} = 61.6 < 0.75(104) = 78.0$$

**Step 5:** Determine the elastic buckling stress for flexural buckling using the modified slenderness ratio

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)_m^2} = \frac{\pi^2 (29{,}000)}{(104)^2} = 26.5 \text{ ksi}$$

**Step 6:** Determine the elastic buckling stress for torsional buckling using Equation E4-7 with $C_w = 0$ based on the user note.

$$F_{ez} = \frac{GJ}{A_g \bar{r}_o^2} = \frac{11{,}200(2(0.0832))}{4.82(2.52)^2} = 60.9 \text{ ksi}$$

**Step 7:** Determine the elastic buckling stress for flexural-torsional buckling using Equation E4-3.

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left(1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right)$$

$$= \left(\frac{26.5 + 60.9}{2(0.640)}\right)\left[1 - \sqrt{1 - \frac{4(26.5)(60.9)(0.640)}{(26.5 + 60.9)^2}}\right] = 22.0 \text{ ksi}$$

**Step 8:** Determine the nominal stress.

Since the elastic buckling stress for flexural-torsional buckling is less than that for flexural buckling, use that to determine the critical stress.

$$\frac{F_y}{F_e} = \frac{50}{22.0} = 2.27 > 2.25$$

Therefore, use Equation E3-3

$$F_n = 0.877F_e = 0.877(22.0) = 19.3 \text{ ksi}$$

**Step 9:** Determine if the local buckling must be included.

For the short leg $b/t = 3.0/0.3125 = 9.6$
and for the long leg $b/t = 5.0/0.3125 = 16.0$

From Table B4.1a case 3

$$\lambda_r = 0.45\sqrt{\frac{E}{F_n}} = 0.45\sqrt{\frac{29{,}000}{19.3}} = 17.4$$

Since

$$b/t = 16.0 < \lambda_r = 17.4$$

the legs are not slender.

**Step 10:** Determine the nominal strength.

$$P_n = F_n A_g = 19.3(4.82) = 93.0 \text{ kips}$$

**For LRFD Step 11:** Determine the design strength.

$$\phi P_n = 0.9(93.0) = 83.7 \text{ kips}$$

**For ASD Step 11:** Determine the allowable strength.

$$P_n/\Omega = 93.0/1.67 = 55.7 \text{ kips}$$

## 5.11 COLUMN BASE PLATES

When columns are supported on material other than steel, such as concrete or masonry, it is necessary to distribute their load over an area significantly larger than the gross area of the column. In these situations, a column base plate similar to that shown in Figure 5.31 is used.

Column base plates may be attached to the column in the shop, as shown in Figure 5.31a, or shipped separately to the site and attached in the field. Columns are normally welded to the plate but may be attached with angles when large plates must be shipped separately. In either case, the selection of the dimensions and thickness of the plate follows the same rules.

Column base plates are normally attached to a footing or pier with anchor rods, and the space between the plate and the support is filled with a non-shrink grout. A leveling plate, leveling nuts, or shims (as shown in Figure 5.31b) are used to level the column base plate. In cases where the column supports an axial compression only, anchor rods are not designed to resist a specific force. However, all column base plates must be anchored with a minimum of four anchor rods according to the Occupational Safety and Health Administration (OSHA) regulations in *OSHA 29 CFR 1926 Subpart R Safety Standards for Steel Erection*. Figure 5.32 illustrates a column with base plate in plan (Figure 5.32a) and elevation (Figure 5.32b), including four anchor rods.


(a) (b)

**Figure 5.31** Example of a W-Shape Column and Base Plate
Photos courtesy Douglas Steel Fabricating Corporation

Column base plates are normally attached to a footing or pier with anchor rods, and the space between the plate and the support is filled with a non-shrink grout. A leveling plate, leveling nuts, or shims (as shown in Figure 5.31b) are used to level the column base plate. In cases where the column supports an axial compression only, anchor rods are not designed to resist a specific force. However, all column base plates must be anchored with a minimum of four anchor rods according to the Occupational Safety and Health Administration (OSHA) regulations in *OSHA 29 CFR 1926 Subpart R Safety Standards for Steel Erection*. Figure 5.32 illustrates a column with base plate in plan (Figure 5.32a) and elevation (Figure 5.32b), including four anchor rods.

To determine the area of bearing that is required, the strength of the material upon which the base plate is bearing must be evaluated. For concrete, Section J8 of the *Specification* gives provisions identical to those given in the concrete code, ACI 318. When the bearing plate is covering the full area of the concrete support, the nominal bearing strength is

$$P_n = P_p = 0.85\, f_c' A_1 \qquad \text{(AISC J8-1)}$$

where $f_c'$ is the specified concrete compressive strength and $A_1$ is the area of the plate and concrete. If the plate does not cover all of the concrete, there will be an increase in strength due to the spread of the load as it progresses down through the concrete. In this case the nominal bearing strength is given as

$$P_n = P_p = 0.85\, f_c' A_1 \sqrt{A_2/A_1} \le 1.7\, f_c' A_1 \qquad \text{(AISC J8-2)}$$

Here $A_2$ is the maximum area of concrete with the same shape as the bearing plate. The limit on the right side of the equation imposes a maximum ratio of areas of 4:1. If the supporting element is designed based on the bearing strength of the soil, it will be relatively easy to determine the extent to which the base plate covers the concrete foundation or pier. In all cases, $\phi = 0.65$ and $\Omega = 2.31$.

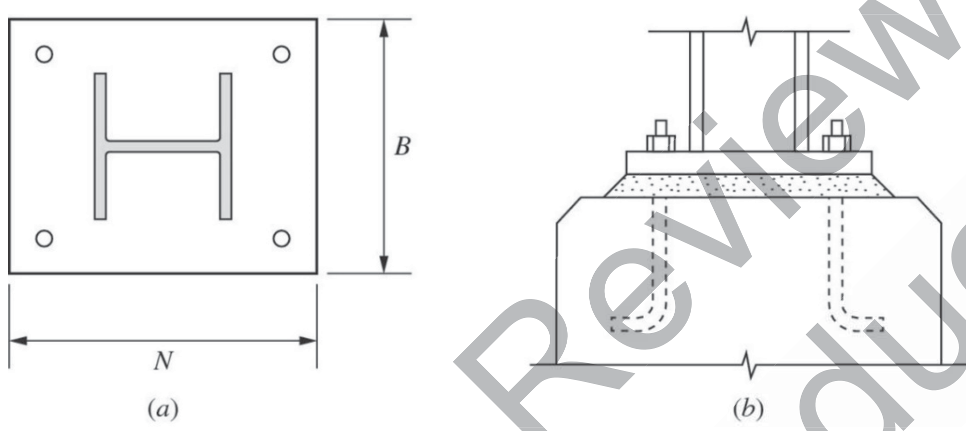



**Figure 5.32** Column and Base Plate Section and Plan Including Anchor Rods

The thickness of a column base plate is a function of the bending strength of the plate. Since bending has not yet been covered in this book, this topic is deferred to Section 11.11. Those wishing to address base plate design further should proceed to Section 11.11 and to the example problems given there as well as AISC Design Guides 1 and 10.

## 5.12 PROBLEMS

**1.** Determine the theoretical buckling strength (Euler buckling load) for a W8×48 A992 column with an effective length of 20 ft. Will the theoretical column buckle or yield at this length?

**2.** Determine the theoretical buckling strength (Euler buckling load) for a W16×77 A36 column with an effective length of 12 ft. Will the theoretical column buckle or yield at this length?

**3.** Determine the theoretical buckling strength (Euler buckling load) for a W24×370 A992 column with an effective length of 20 ft. Will the theoretical column buckle or yield at this length?

**4.** Determine the theoretical buckling strength (Euler buckling load) for an HSS 10×5×3/8 A500 Grade C column with an effective length of 20 ft. Will the theoretical column buckle or yield at this length?

**5.** For a W12×72 A992 column, determine the effective length at which the theoretical buckling strength (Euler buckling load) will equal the yield strength.

**6.** For a W6×25 A992 column, determine the effective length at which the theoretical buckling strength (Euler buckling load) will equal the yield strength.

**7.** For an HP8×36 A572 Grade 50 column, determine the effective length at which the theoretical buckling strength (Euler buckling load) will equal the yield strength.

**8.** A W14×132 column has an effective length for *y*-axis buckling equal to 24 ft. Determine the effective length for the *x*-axis that will provide the same theoretical buckling strength (Euler buckling load).

**9.** A W14×53 column has an effective length for *x*-axis buckling equal to 20 ft. Determine the effective length for the *y*-axis that will provide the same theoretical buckling strength (Euler buckling load).

**10.** An HSS12×6×1/2 column has an effective length for *x*-axis buckling equal to 16 ft. Determine the effective length for the *y*-axis that will provide the same theoretical buckling strength (Euler buckling load).

**11.** A W14×132 A992 column has an effective length of 36 ft about both axes. Determine the available compressive strength for the column. Determine the (a) design strength by LRFD and (b) allowable strength by ASD. Is this an elastic or inelastic buckling condition?

**12.** Determine the available compressive strength for a W12×210 A992 column with an effective length about both axes of 40 ft. Determine the (a) design strength by LRFD and (b) allowable strength by ASD. Is this an elastic or inelastic buckling condition?

**13.** A W6×15 A992 column has an effective length of 8 ft about both axes. Determine the available compressive strength for the column. Determine the (a) design strength by LRFD and (b) allowable strength by ASD. Is this an elastic or inelastic buckling condition?

**14.** Determine the available compressive strength for an M10×7.5 A572 Gr 50 column with an effective length about both axes of 7 ft. Determine the (a) design strength by LRFD and (b) allowable strength by ASD. Is this an elastic or inelastic buckling condition?

**15.** A W14×211 A992 column has an effective length of 40 ft about both axes. Determine the available compressive strength for the column. Determine the (a) design strength by LRFD and (b) allowable strength by ASD. Is this an elastic or inelastic buckling condition?

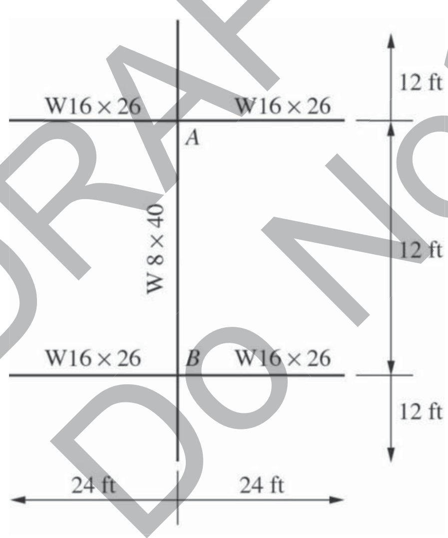
**16.** Determine the available compressive strength for a W12×72 A992 column when the effective length is 20 ft about the *y*-axis and 40 ft about the *x*-axis. Determine the (a) design strength by LRFD and (b) allowable strength by ASD. Is this an elastic or inelastic buckling condition? Describe a common condition where the effective length is different about the different axes.

**17.** A W8×24 A992 column has an effective length of 12.5 ft about the *y*-axis and 28 ft about the *x*-axis. Determine the available compressive strength and indicate whether this is due to elastic or inelastic buckling. Determine the (a) design strength by LRFD and (b) allowable strength by ASD.

**18.** Determine the available compressive strength for an HSS 5×5×3/8 A500 Grade C column where the effective length is 10 ft about the *y*-axis and 15 ft about the *x*-axis. Determine the (a) design strength by LRFD and (b) allowable strength by ASD. Is this an elastic or inelastic buckling condition?

**19.** A round HSS 16.000×0.375 A500 Grade C column has an effective length of 20 ft. Determine the available compressive strength and indicate whether this is due to elastic or inelastic buckling. Determine the (a) design strength by LRFD and (b) allowable strength by ASD.

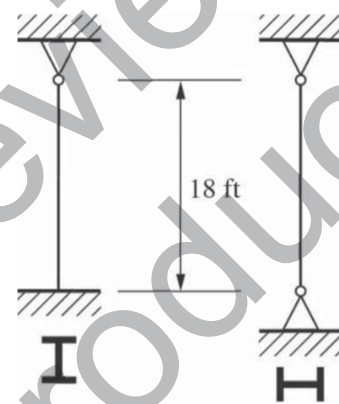
**20.** A W8×40 is used as a 12 ft column in a braced frame with W16×26 beams at the top and bottom as shown in Figure P5.20. The columns above and below are also 12 ft W8×40s. The beams provide moment restraint at each column end. Determine the effective length using the alignment chart and the available compressive strength, and the (a) design strength by LRFD and (b) allowable strength by ASD. Assume that the columns are oriented for (i) buckling about the weak axis and (ii) buckling about the strong axis. All steel is A992.




**P5.20**

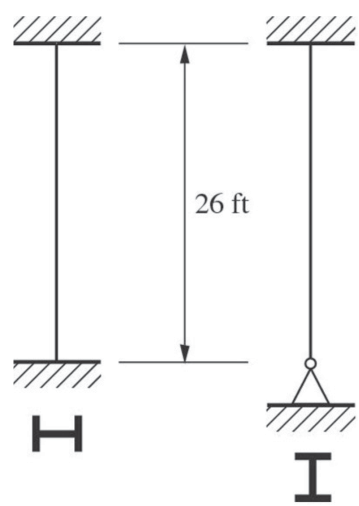
**21.** If the structure described in Problem 20 is an unbraced frame, determine the effective length and compressive strength as requested in Problem 20.

**22.** A W12×136 column is shown in Figure P5.22 with end conditions that approximate ideal conditions. Using the recommended approximate values from Commentary Table C-A-7.1, determine the effective lengths for the *y*-axis and the *x*-axis. Which effective length will control the column strength?




**P5.22**

**23.** A W12×96 column is shown in Figure P5.23 with end conditions that approximate ideal conditions. Using the recommended approximate values from Commentary Table C-A-7.1, determine the effective lengths for the *y*-axis and the *x*-axis. Which effective length will control the column strength?




**P5.23**

**24.** A W10×54 column with an effective length of 30 ft for both axes is called upon to carry a compressive dead load of 80 kips and a compressive live load of 100 kips. Determine whether the column will support the load by (a) LRFD and (b) ASD. Evaluate the strength for (i) $F_y$ = 50 ksi and (ii) $F_y$ = 70 ksi.

**25.** A W14×257 A992 column in a building has effective lengths of 16 ft for both axes. Determine whether the column will carry a compressive dead load of 800 kips and a compressive live load of 1100 kips by (a) LRFD and (b) ASD.

**26.** An A992 W12×53 is used as a column in a building with an effective length for each axis of 15 ft. Determine whether the column will carry a compressive dead load of 85 kips and a compressive live load of 255 kips by (a) LRFD and (b) ASD.

**27.** An A992 W8×58 is used in a structure to support a dead load of 60 kips and a live load of 100 kips. The column has an effective length of 22 ft. Determine whether the column will support the load by (a) LRFD and (b) ASD.

**28.** An A992 W10×54 is used as a column in a building with an effective length of 30 ft. Determine whether the column will carry a compressive dead load of 24 kips and a compressive live load of 72 kips by (a) LRFD and (b) ASD.

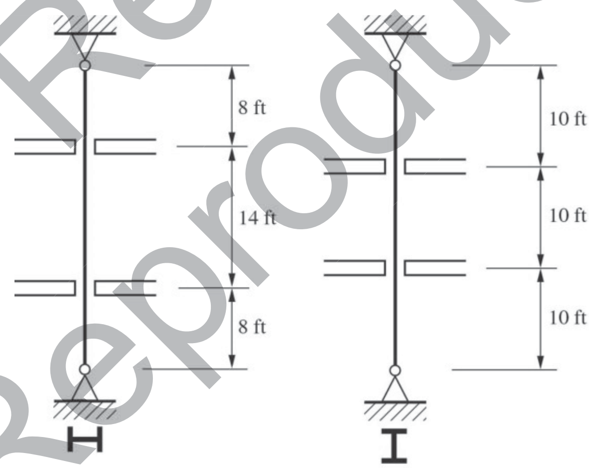
**29.** An A992 W16×77 is used as a column in a building to support a dead load of 130 kips and a live load of 200 kips. The column effective length is 20 ft for the *y*-axis and 30 ft for the *x*-axis. Determine whether the column will support the load by (a) LRFD and (b) ASD.

**30.** An A992 W21×111 is used as a column in a building to support a dead load of 120 kips and a live load of 300 kips. The column effective length is 22 ft for the *y*-axis and 33 ft for the *x*-axis. Determine whether the column will support the load by (a) LRFD and (b) ASD.

**31.** An A992 W24×146 is used as a column in a building to support a dead load of 245 kips and a live load of 500 kips. The column has an effective length about the *y*-axis of 18 ft and an effective length about the *x*-axis of 36 ft. Determine whether the column will support the load by (a) LRFD and (b) ASD.

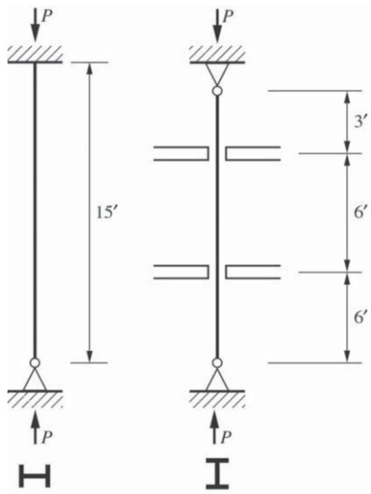
**32.** An A500 Gr. C HSS7×7×1/2 is used as a column to support a dead load of 175 kips and a live load of 100 kips. The column has an effective length of 10 ft. Determine whether the column will support the load by (a) LRFD and (b) ASD.

**33.** For the W10×77 column with bracing and end conditions shown in Figure P5.33, determine the theoretical effective length for each axis and identify the axis that will limit the column strength.



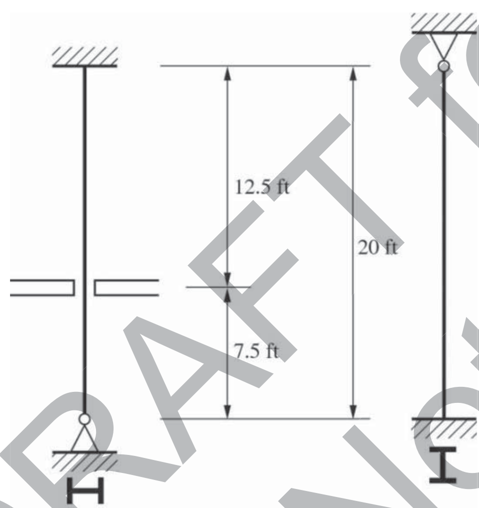

**P5.33**

**34.** For the W8×24 column with bracing and end conditions shown in Figure P5.34, determine the theoretical effective length for each axis and identify the axis that will limit the column strength.



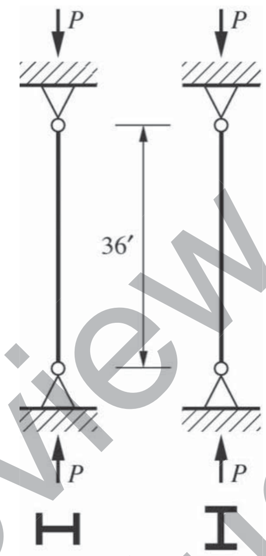

**P5.34**

**35.** A W10×100 column with end conditions and bracing is shown in Figure P5.35. Determine the least theoretical bracing and its location about the *y*-axis, in order that the *y*-axis not control the strength of the column.



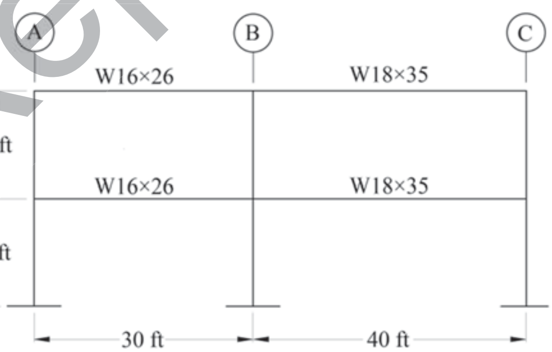

**P5.35**

**36.** A W14×176 column with end conditions and bracing is shown in Figure P5.36. Determine the least theoretical bracing and its location about the *y*-axis, in order that the *y*-axis not control the strength of the column.




**P5.36**

**37.** A W12×72 column is an exterior 2nd story column (gridline A) with strong axis buckling in the plane of the frame in an unbraced multi-story frame. The column below is also a W12×72. Beams and dimensions are as shown in Figure P5.37. Determine the effective length for this condition and the corresponding compressive strength by (a) LRFD and (b) ASD. All steel is A992.




**P5.37**

**38.** A W12×72 column is an exterior 2nd story column (at gridline C) with strong axis buckling in the plane of the frame in an unbraced multi-story frame. The column below is also a W12×72. Beams and dimensions are as shown in Figure P5.37. Determine the effective length for this condition and the corresponding compressive strength by (a) LRFD and (b) ASD. All steel is A992.

**39.** A W12×72 column is an exterior 1st story column (at gridline C) with strong axis buckling in the plane of the frame in an unbraced multi-story frame. The column above is also a W12×72. Beams and dimensions are as shown in Figure P5.37. Using the AISC Commentary recommended stiffness, *G*, for the base of a fixed base column, determine the effective length for this condition and the corresponding compressive strength by (a) LRFD and (b) ASD. All steel is A992.

**40.** A W12×72 column is an interior 2nd story column (at gridline B) with strong axis buckling in the plane of the frame in an unbraced multi-story frame. The column below is also a W12×72. Beams and dimensions are as shown in Figure P5.37. Determine the effective length for this condition and the corresponding compressive strength by (a) LRFD and (b) ASD. All steel is A992.

**41.** A W12×72 column is an interior 1st story column (at gridline B) with strong axis buckling in the plane of the frame in an unbraced multi-story frame. The column above is also a W12×72. Beams and dimensions are as shown in Figure P5.37. Using the AISC Commentary recommended stiffness, *G*, for the base of a fixed base column, determine the effective length for this condition and the corresponding compressive strength by (a) LRFD and (b) ASD. All steel is A992.

**42**. Repeat Problem 39 if the column support were given as a pin.

**43.** Repeat Problem 41 if the column support were given as a pin.

**44.** A W12×50 column is an interior column with strong axis buckling in the plane of the frame in an unbraced multi-story frame. The columns above and below are also W12×50. The beams framing in at the top are W16×31 and those at the bottom are W16×40. The columns are 14 ft and the beam span is 25 ft. The column carries a dead load of 75 kips and a live load of 150 kips. Determine the inelastic effective length for this condition and the corresponding compressive strength by (a) LRFD and (b) ASD. All steel is A992.

**45.** Select the least-weight W12 A992 column to carry a live load of 130 kips and a dead load of 100 kips with an effective length about both axes of 15 ft by (a) LRFD and (b) ASD.

**46.** Select the least-weight W14 A992 column to carry a dead load of 200 kips and a live load of 600 kips if the effective length about both axes is 22 ft by (a) LRFD and (b) ASD.

**47.** Select the least-weight W10 A992 column to carry a dead load of 80 kips and a live load of 280 kips with an effective length about both axes of 15 ft by (a) LRFD and (b) ASD.

**48.** Select the least-weight W8 A992 column to carry a dead load of 20 kips and a live load of 50 kips with an effective length about both axes of 25 ft by (a) LRFD and (b) ASD.

**49.** Select the least-weight W6 A992 column to carry a dead load of 12 kips and a live load of 36 kips with an effective length about both axes of 8 ft by (a) LRFD and (b) ASD.

**50.** Select the least-weight W8 A992 column to carry a dead load of 13 kips and a live load of 39 kips with an effective length about both axes of 14 ft by (a) LRFD and (b) ASD.

**51.** A column with pin ends for both axes must be selected to carry a compressive dead load of 95 kips and a compressive live load of 285 kips. The column is 14 ft long and is in a braced frame. Select the lightest-weight W12 to support this load by (a) LRFD and (b) ASD.

**52.** If the column in Problem 48 had an effective length of 32 ft, select the lightest-weight W12 to support this load by (a) LRFD and (b) ASD.

**53.** A W14 A992 column must support a dead load of 80 kips and a live load of 300 kips. The column is 22 ft long and has end conditions that approximate the ideal conditions of a fixed support at one end and a pin support at the other. Select the lightest-weight W14 to support this load by (a) LRFD and (b) ASD.

**54.** Select the least-weight W8 A992 column to support a dead load of 170 kips with an effective length of 16 ft by (a) LRFD and (b) ASD.

**55.** A column with an effective length of 21 ft must support a dead load of 120 kips, a live load of 175 kips, and a wind load of 84 kips. Select the lightest W14 A992 member to support the load by (a) LRFD and (b) ASD.

**56**. A W14×99 A992 column is 20 ft long, pinned at each end, and braced at mid-height to prevent lateral movement for buckling about the *y*-axis. However, the *y*-axis bracing is not adequate to resist torsion. Considering flexural and torsional buckling, determine the nominal strength of this compression member.

**57.** An A36 single-angle compression web member of a truss is 10 ft long and attached to gusset plates through the same leg at each end with a minimum of two bolts. The member must carry a dead load of 8 kips and a live load of 10 kips. Select the least-weight equal leg angle to carry this load by (a) LRFD and (b) ASD.

**58.** If the compression web member of Problem 57 were loaded concentrically, determine the least-weight single angle to carry the load by (a) LRFD and (b) ASD.

**59**. Determine the web and flange width-to-thickness ratios and determine if a W14×43 A913 Gr 70 compression member requires consideration as a slender element member.

**60**. Determine the web and flange width-to-thickness ratios and determine if a W8×10 A992 compression member requires consideration as a slender element member.

**61.** A W16×36 A992 compression member has a slender web when used in uniform compression. Determine the available strength by (a) LRFD and (b) ASD when the effective length is (i) 6 ft and (ii) 12 ft.

**62.** The W14×43 is the only A992 column shown in the *Manual* column tables that has a slender web. Determine the available strength for this column if the effective length is 8 ft and show whether the slender web impacts that strength by (a) LRFD and (b) ASD.

**63**. Determine the available strength of a WT6×25 A992 steel compression chord of a truss with effective length $L_c$ = 14 ft. Consider the member braced laterally and torsionally at its ends only. Determine by (a) LRFD and (b) ASD.

**64**. Determine the available strength of a WT12×125 A992 steel column with effective length $L_c$ = 18 ft. Consider the member braced laterally and torsionally at its ends only. Determine by (a) LRFD and (b) ASD.

**65**. Determine the available strength of a C9×20 A992 steel compression chord of a truss with effective length, $L_c$ = 16 ft. Consider the member braced laterally and torsionally at its ends only. Determine by (a) LRFD and (b) ASD.

**66**. Determine the available strength of an A572 Gr 50, 20 ft long, 2L6×4×5/8 double-angle compression member in a planar truss. The angles are long legs back-to-back with a 3/8 in. gap. The angles are welded at each end to a gusset plate and along the length at two intermediate points with a spacing of 80 in. Determine the (a) design strength by LRFD and (b) allowable strength by ASD. Compare the results to that found in *Manual* Table 4-9.

**67**. Determine the available strength of an A572 Gr 50, 15 ft long, 2L6×3-1/2×1/2 double-angle compression member in a planar truss. The angles are long legs back-to-back with a 3/8 in. gap. The angles are connected with pretensioned bolts at each end to a gusset plate and along the length at two intermediate points with snug-tight bolts at a spacing of 60 in. Determine the (a) design strength by LRFD and (b) allowable strength by ASD. Compare the results to that found in *Manual* Table 4-9 and explain why they differ.

**68**. Determine the available strength of an A572 Gr 50, 18 ft long, 2L3×3×1/2 double-angle compression member in a planar truss. The angles have a back-to-back gap of 3/8 in. The angles are welded at each end to a gusset plate and along the length at two intermediate points with a spacing of 72 in. Determine the (a) design strength by LRFD and (b) allowable strength by ASD. Compare the results to that found in *Manual* Table 4-9.

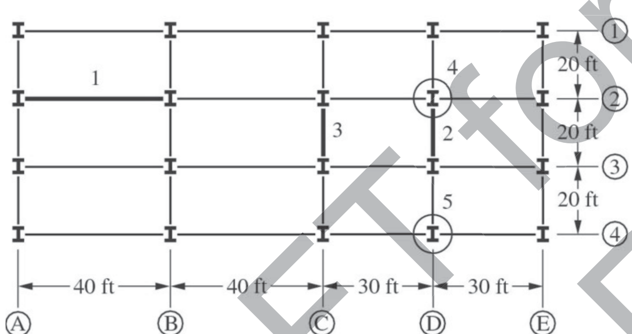
**69**. Redo Problem 68 if the intermediate points are connected with snug-tight bolts and compare the results to Problem 68.

**Multi-Chapter Problems**

**70.** Using the framing plan shown in Figure P5.70 (presented earlier as Figure 2.9), design the columns marked 4 and 5. This is the same structure used in Section 2.5, where load calculations with live load reductions were discussed. Those calculations can be reused here. Load case 2 for dead plus live load is to be considered. The building is an office building with a nominal live load of 50 pounds per square foot (psf) and a calculated dead load of 70 psf.

> 4: Interior column D-2 regardless of deck span direction
> 5: Exterior column D-4 regardless of deck span direction

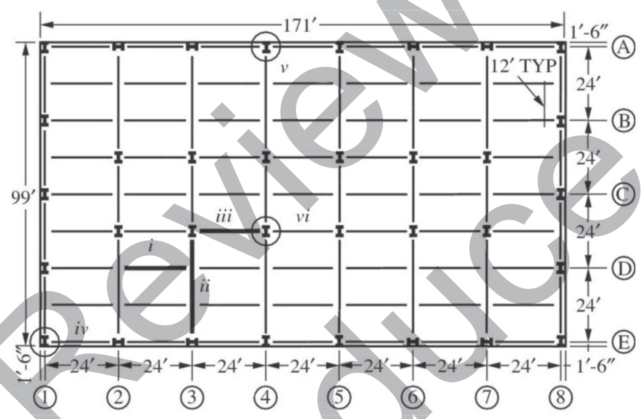
Design for column length $L$ = 14 ft and $K$ = 1.0 using (a) LRFD and (b) ASD.




**P5.70**

**71.** The framing plan shown in Figure P5.71 is the same as the one shown in Figure P2.24 for an 18-story office building. It must support a floor and roof dead load of 80 psf and a floor live load of 50 psf and a roof live load of 30 psf. In all cases, the decking spans in a direction from line A toward line E. Determine the required axial strength for the columns and design the columns as required below for (a) design by LRFD and (b) design by ASD. The required axial load strengths were determined in Problem 24 of Chapter 2. Use a story height of 13.5 ft in a braced frame so that $K$ = 1.0.

> iv: The column at the corner on lines 1 and E that supports eight floor levels plus the roof.
> v: The column on the edge at the intersection of lines 4 and A that supports eight floor levels plus the roof.
> vi: The interior column at the intersection of line 4 and the point between lines C and D that supports three floor levels plus the roof.




**P5.71**

**72. Integrated Design Project.** Using the gravity column loads determined in Chapter 2, design the gravity-only columns. Design columns as single-story members. (It is often more economical to use multi-story columns because of construction costs.) Select the final columns so that they are two-story from below grade to the second floor, two-story from the second floor to the fourth floor, and then use a single-story column to support the roof.

Design the columns in the braced frame for the gravity loads determined in Chapter 2 and the wind load determined in Chapter 4. Remember that the wind load must be considered to act in two directions, so use the largest compression forces from wind to combine with the gravity loads.

Using the wind load analysis from Chapter 4, design all the braces as compression members. Compare the tension design with the compression design and select the appropriate final members.