Discussion of "Nonlinear Truss Analysis by One Matrix Inversion" by A. Fafitis

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The author presents a method of analysis for truss structures with material and geometric nonlinearity. The method consists of a single factorization of the stiffness matrix, followed by its successive application to corrective force vectors in order to find structural equilibrium when nonlinearities arise due to strain-hardening/softening, buckling, breaking, and stiffness degradation. The author points out that the stiffness matrix used for the iteration does not necessarily have to be the exact stiffness matrix of the structure, and the proposed analysis method treats proportional, nonproportional, and cyclic loadings uniformly.

Although the developments are clear and the applications are practical, this method of analysis is not significantly different from the modified Newton–Raphson algorithm, where the stiffness matrix is held constant over the course of a load increment. The modified Newton–Raphson algorithm is discussed extensively in a reference (Crisfield 1991) cited by the author; however, the author neither acknowledges this material nor cites any of the numerous references available in the mathematical literature on the convergence properties of the modified Newton–Raphson algorithm, such as Shamanskii (1967), Stoer and Bulirsch (1993), and Kelley (1995) to name but a few. Due to its slow convergence rate, the excessive number of iterations required to achieve equilibrium with the modified Newton–Raphson algorithm can far outweigh the computational savings afforded by a single matrix factorization for structures with a small to moderate number of degrees of freedom. For such structures, the computational cost of a matrix factorization is cheap relative to the cost of evaluating the corrective force vector, making the full Newton–Raphson algorithm more efficient. Furthermore, the use of equation solvers (Mackay et al. 1991; Demmel et al. 1999; Davis 2003) that exploit the sparse matrix topology generated by a finite-element analysis can mitigate the computational expense of full Newton–Raphson for large structural systems. The discussers acknowledge that the full Newton–Raphson algorithm requires an exact stiffness matrix be computed at every iteration, which may not be possible when using complex numerical models of constitutive behavior. This is not the case, however, for closed-form scalar expressions, such as the bilinear stress-strain law and the Euler buckling formula [Eq. (5)] the author uses in the numerical examples.

The author proposes holding an inexact stiffness matrix, $\mathbf{K}_{inexact}$, constant during the equilibrium iteration. This approach can increase the number of iterations required to reach equilibrium. Consistent with the numerical properties of the modified Newton–Raphson algorithm, if the spectral radius of the matrix $\mathbf{I}-\mathbf{K}_{inexact}^{-1}\mathbf{K}_{exact}$ is greater than one, keeping $\mathbf{K}_{inexact}$ constant will lead to divergence of the iteration (Kelley 1995), where the matrix $\mathbf{K}_{exact}$ is the exact stiffness at the solution point and $\mathbf{I}$ is the identity matrix. In one dimension, this condition implies the inexact stiffness must be greater than one-half of the exact stiffness at the solution point. The condition generalizes to multidimensions in terms of the spectral properties of the exact and inexact stiffness matrices. Therefore, the selection of the inexact stiffness matrix for the equilibrium iteration is not arbitrary, as the author suggests. The only mathematical condition stated by the author is the inexact stiffness matrix be "compatible with the geometry and the constraints" of the structure. The discussers interpret this "compatibility" requirement to mean the inexact stiffness matrix must have the same topology as the exact stiffness matrix. As a result, this requirement alone does not preclude a singular stiffness matrix, e.g., due to perfectly-plastic response in one or more members, in which case the iteration will fail due to numerical instability.

The discussers bring to attention quasi-Newton, accelerated Newton, and line search methods, which are more efficient than the modified Newton–Raphson algorithm and can automatically correct for an inexact stiffness matrix by performing simple, low-cost matrix-vector operations during the equilibrium iteration. The rank-two BFGS quasi-Newton technique (Broyden 1970; Fletcher 1970; Shanno 1970) is appropriate for the symmetric positive–definite systems typically encountered in structural mechanics (Matthies and Strang 1979; Bathe and Cimento 1980; Geradin et al. 1980), while the rank-one procedure of Broyden (1967) is suited to the nonsymmetric systems that arise from nonassociative plasticity. The computational efficiency and convergence properties of the secant-based accelerated Newton algorithm of Crisfield (1979) and the Krylov subspace acceleration method of Carlson and Miller (1998) have been demonstrated in the nonlinear analysis of structures (Crisfield 1984, 1991; Scott and Fenves 2003; Scott 2004). Finally, line search methods have been shown to be robust when combined with a full Newton-Raphson iteration (Crisfield 1991).

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