

Analytical Sensitivity of Plastic Rotations in Beam-Column Elements

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Abstract: Analytical sensitivity equations for the plastic rotation of beam-column finite elements are derived for reliability and optimization algorithms in structural engineering and for the assessment of plastic rotation sensitivity to uncertain design parameters and modeling assumptions. The plastic rotation is defined by elastic unloading of element forces in a basic system, which makes the corresponding sensitivity computations applicable to most material nonlinear beam-column formulations available in the literature. The analytical response sensitivity is verified by finite differences then applied to a first-order reliability analysis of a steel subassembly where the performance function places a limit on plastic rotation.

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Introduction

The plastic rotation of a beam-column member is an important engineering demand parameter in assessing the response and damage of a structure due to earthquakes or other loads causing the formation of plastic hinges. Performance-based seismic design and rehabilitation provisions place limits on plastic rotation depending on the member type and the desired performance level (FEMA 2000). Extensive experimental and analytical research has been conducted to correlate plastic rotation to structural damage due to seismic loading (Tsai and Popov 1988; Engelhardt and Husain 1993; SAC Joint Venture 1996; Roeder and Foutch 1996; Gupta and Krawinkler 2000; Lee and Foutch 2002; Rodgers and Mahin 2006).

To characterize structural performance in a probabilistic setting, applications in structural reliability, optimization, and system identification require the sensitivity of the plastic rotation to be computed when the performance function is defined in terms of the plastic rotation of one or more of the structural members. There are two approaches to compute the sensitivity of a structural response quantity with respect to an uncertain parameter. The first approach is the finite difference method (FDM), which is inefficient because it requires a full reanalysis for each parameter that characterizes the structural system. In addition, for small parameter perturbations, the FDM is prone to numerical round-off

errors that lead to slow convergence of gradient-based algorithms. A more efficient and accurate approach to sensitivity computations is the direct differentiation method (DDM), where the governing equations of structural mechanics are differentiated in closed-form and incorporated in the finite-element analysis (Kleiber et al. 1997).

Previous work in structural response sensitivity has focused on computing the gradient of the nodal response (Zhang and Der Kiureghian 1993; Franchin 2004; Scott et al. 2004; Haukaas and Der Kiureghian 2005). This note points out that it is straightforward to develop expressions for the sensitivity of plastic rotation in beam-column finite elements, which is a response quantity derived from the nodal response. The presentation begins with the definition of plastic rotation of a beam-column element and its direct differentiation to obtain analytical response sensitivity equations. Numerical examples that verify the response sensitivity equations for plastic rotation and demonstrate their application in a first-order reliability analysis conclude the presentation.

Structural Response Sensitivity

For a structural system under static equilibrium, the DDM computes the sensitivity of the nodal response, $\partial \mathbf{U} / \partial \theta$, by the following linear system of equations (Kleiber et al. 1997):

$$\mathbf{K}_T \frac{\partial \mathbf{U}}{\partial \theta} = \frac{\partial \mathbf{P}_f}{\partial \theta} - \frac{\partial \mathbf{P}_r}{\partial \theta} \bigg|_{\mathbf{U}} \quad (1)$$

where θ = parameter that describes an uncertain property of the structural system and applied loading. The matrix $\mathbf{K}_T = \partial \mathbf{P}_r / \partial \mathbf{U}$ is the tangent stiffness of the structure. The right-hand side of Eq. (1) is formed from the derivative of the applied load vector, $\partial \mathbf{P}_f / \partial \theta$, which is nonzero for only the parameters that represent the external loads, and the conditional derivative of the resisting force vector, $\partial \mathbf{P}_r / \partial \theta|_{\mathbf{U}}$, which is assembled from element contributions by standard finite-element procedures. The conditional

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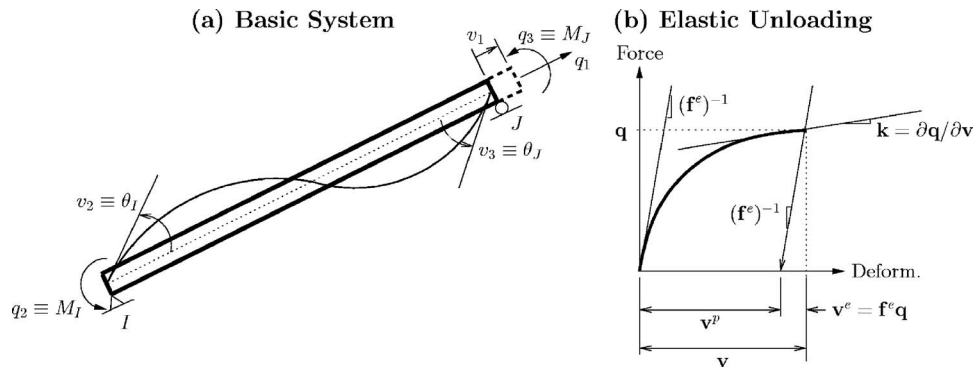


Fig. 1. Basic system of beam-column element: (a) basic forces and deformations; (b) plastic deformations defined by unloading of basic forces using the elastic stiffness

derivative of \mathbf{P} , represents the forces that must be applied to the structure to keep the nodal displacements, \mathbf{U} , fixed due to changes in the parameter θ . As described by Zhang and Der Kiureghian (1993), path-dependent response sensitivity analysis is a two-phase process requiring the assembly of Eq. (1) in phase one followed by the updating of element sensitivity history variables using $\partial\mathbf{U}/\partial\theta$ in phase two. This two-phase process is repeated for each parameter of the structural system. The extension of Eq. (1) to include inertial and damping forces for the case of dynamic equilibrium is straightforward (Franchin 2004).

Plastic Rotation Response Sensitivity

The beam-column finite-element models considered in this note are formulated in a simply supported basic system, as described by Filippou and Fenves (2004) and depicted in Fig. 1(a). The element response is described in terms of the element deformations, $\mathbf{v}=\mathbf{v}(\theta)$, and the corresponding basic forces, $\mathbf{q}=\mathbf{q}(\mathbf{v}(\theta), \theta)$. It is possible to use a displacement-based, force-based, or mixed formulation to compute the basic forces, as summarized by Alemdar and White (2005). Regardless of the numerical formulation, the tangent stiffness matrix of the element in the basic system is the partial derivative of the basic forces with respect to the deformations, $\mathbf{k}=\partial\mathbf{q}/\partial\mathbf{v}$. At every section along the element there are section deformations, $\mathbf{e}=\mathbf{e}(\theta)$, compatible with the element deformations; the corresponding section forces, $\mathbf{s}=\mathbf{s}(\mathbf{e}(\theta), \theta)$, in equilibrium with the basic forces; and the tangent stiffness matrix, $\mathbf{k}_s=\partial\mathbf{s}/\partial\mathbf{e}$.

To define the plastic rotation of a beam-column element, the deformations are decomposed into elastic and plastic components: $\mathbf{v}=\mathbf{v}^e+\mathbf{v}^p$. From this decomposition, the plastic deformation is the difference between the total deformation and the elastic component. A common assumption is the elastic component of deformations represents elastic unloading of the basic forces

$$\mathbf{v}^p = \mathbf{v} - \mathbf{f}^e \mathbf{q} \quad (2)$$

where the matrix \mathbf{f}^e =elastic flexibility of the element, which is assembled easily from the elastic properties, cross-section dimensions, and length of the element. A graphical representation of Eq. (2) is shown in Fig. 1(b), where the element unloads to $\mathbf{q}=\mathbf{0}$ using the elastic flexibility. Alternative definitions of the elastic component of deformation for the purpose of computing the plastic rotation are possible, including those that account for degradation of the unloading stiffness.

To determine the gradient of the plastic rotation, Eq. (2) is differentiated with respect to a parameter, θ , that represents an uncertain property of the structural system

$$\frac{\partial\mathbf{v}^p}{\partial\theta} = \frac{\partial\mathbf{v}}{\partial\theta} - \mathbf{f}^e \frac{\partial\mathbf{q}}{\partial\theta} - \frac{\partial\mathbf{f}^e}{\partial\theta} \mathbf{q} \quad (3)$$

By the chain rule of differentiation, the derivative of the basic forces, $\partial\mathbf{q}/\partial\theta=\mathbf{k}\partial\mathbf{v}/\partial\theta+\partial\mathbf{q}/\partial\theta|_{\mathbf{v}}$, changes Eq. (3) to

$$\frac{\partial\mathbf{v}^p}{\partial\theta} = \frac{\partial\mathbf{v}}{\partial\theta} - \mathbf{f}^e \left(\mathbf{k} \frac{\partial\mathbf{v}}{\partial\theta} + \frac{\partial\mathbf{q}}{\partial\theta} \Big|_{\mathbf{v}} \right) - \frac{\partial\mathbf{f}^e}{\partial\theta} \mathbf{q} \quad (4)$$

The collection of terms in Eq. (4) that multiply the vector $\partial\mathbf{v}/\partial\theta$ gives the following expression for the gradient of the plastic deformation:

$$\frac{\partial\mathbf{v}^p}{\partial\theta} = (\mathbf{I} - \mathbf{f}^e \mathbf{k}) \frac{\partial\mathbf{v}}{\partial\theta} - \mathbf{f}^e \frac{\partial\mathbf{q}}{\partial\theta} \Big|_{\mathbf{v}} - \frac{\partial\mathbf{f}^e}{\partial\theta} \mathbf{q} \quad (5)$$

The derivative of the element deformations, $\partial\mathbf{v}/\partial\theta$, is related to the derivative of the nodal displacement vector, $\partial\mathbf{U}/\partial\theta$, according to the transformation of nodal displacements from the global coordinate system to the basic system of the element. The derivative of the elastic flexibility matrix, $\partial\mathbf{f}^e/\partial\theta$, is equal to zero when θ does not represent the elastic properties, cross-section dimensions, or length of the element. The conditional derivative of the basic force vector, $\partial\mathbf{q}/\partial\theta|_{\mathbf{v}}$, depends on the element formulation for nonlinear material response within the basic system. Thus, Eq. (5) applies to any beam-column formulation where a basic system encapsulates the numerical implementation of the equilibrium, compatibility, and constitutive equations that govern the element response. The computation of $\partial\mathbf{q}/\partial\theta|_{\mathbf{v}}$ is demonstrated in the following section for the force-based formulation.

Force-Based Element Response Sensitivity

In the force-based formulation (Ciampi and Carlesimo 1986; Spacone et al. 1996), element equilibrium is satisfied in strong form

$$\mathbf{s}(x) = \mathbf{b}(x) \mathbf{q} \quad (6)$$

The interpolation matrix relates section forces to forces in the basic system

$$\mathbf{b}(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & x/L - 1 & x/L \end{bmatrix} \quad (7)$$

Without loss of generality, member loads are omitted from Eq. (6). From the principle of virtual forces, compatibility between section and element deformations is satisfied in integral form and evaluated by numerical integration

$$\mathbf{v} = \sum_{i=1}^{N_p} \mathbf{b}^T(x_i) \mathbf{e}(x_i) w_i \quad (8)$$

where x_i and w_i are the location and weight, respectively, of the N_p integration points in the element domain $x=[0, L]$. The force-based state determination procedure to compute basic forces from element deformations is summarized by Neuenhofer and Filippou (1997).

The development of response sensitivity for force-based elements follows the presentation in Scott et al. (2004), where the derivatives of the equilibrium and compatibility relationships are combined to give the conditional derivative of the basic force vector. To this end, Eqs. (6) and (8) are differentiated with respect to θ

$$\frac{\partial \mathbf{s}}{\partial \theta} = \mathbf{b} \frac{\partial \mathbf{q}}{\partial \theta} + \frac{\partial \mathbf{b}}{\partial \theta} \mathbf{q} \quad (9a)$$

$$\frac{\partial \mathbf{v}}{\partial \theta} = \sum_{i=1}^{N_p} \left(\mathbf{b}^T \frac{\partial \mathbf{e}}{\partial \theta} + \frac{\partial \mathbf{b}^T}{\partial \theta} \mathbf{e} \right) w_i + \sum_{i=1}^{N_p} \mathbf{b}^T \mathbf{e} \frac{\partial w_i}{\partial \theta} \quad (9b)$$

Using the derivative of the basic force vector, $\partial \mathbf{q} / \partial \theta = \mathbf{k} \partial \mathbf{v} / \partial \theta + \partial \mathbf{q} / \partial \theta|_v$, and the derivative of the section force vector, $\partial \mathbf{s} / \partial \theta = \mathbf{k}_s \partial \mathbf{e} / \partial \theta + \partial \mathbf{s} / \partial \theta|_e$, the expansion of Eq. (9a) and subsequent solution for the derivative of the section deformations gives

$$\frac{\partial \mathbf{e}}{\partial \theta} = \mathbf{f}_s \mathbf{b} \mathbf{k} \frac{\partial \mathbf{v}}{\partial \theta} + \mathbf{f}_s \left(\mathbf{b} \frac{\partial \mathbf{q}}{\partial \theta} \Big|_v + \frac{\partial \mathbf{b}}{\partial \theta} \mathbf{q} - \frac{\partial \mathbf{s}}{\partial \theta} \Big|_e \right) \quad (10)$$

where $\mathbf{f}_s = \mathbf{k}_s^{-1}$ = section flexibility matrix. Eq. (10) is then substituted into Eq. (9b) and the solution for the conditional derivative of the basic forces gives

$$\frac{\partial \mathbf{q}}{\partial \theta} \Big|_v = \mathbf{k} \sum_{i=1}^{N_p} \mathbf{b}^T \mathbf{f}_s \left(\frac{\partial \mathbf{s}}{\partial \theta} \Big|_e - \frac{\partial \mathbf{b}}{\partial \theta} \mathbf{q} \right) w_i - \mathbf{k} \sum_{i=1}^{N_p} \left(\frac{\partial \mathbf{b}^T}{\partial \theta} \mathbf{e} w_i + \mathbf{b}^T \mathbf{e} \frac{\partial w_i}{\partial \theta} \right) \quad (11)$$

The conditional derivative of the section force vector, $\partial \mathbf{s} / \partial \theta|_e$, is determined from the material properties and cross-section dimensions at each integration point along the element. The terms $\partial \mathbf{b} / \partial \theta$ and $\partial w_i / \partial \theta$ incorporate the derivative of the element length, as well as the derivatives of the locations and weights of the element integration points. When representing distributed plasticity in force-based elements through Gauss-Lobatto quadrature, the derivatives of the integration point locations and weights are zero; however, these derivatives may be nonzero when a prescribed hinge length defines the location and weight of the integration points in plastic hinge regions (Addressi and Ciampi 2002; Scott and Fenves 2006).

Numerical Examples

The response sensitivity equations summarized in this paper have been implemented in the OpenSees software framework (McKenna et al. 2000) with extensions for sensitivity and reliability analysis (Haukaas 2003). An adaptation of specimen PN3 (Popov et al. 1996), a steel subassembly with a web-bolted, flange-welded moment connection, from Phase 1 of the SAC

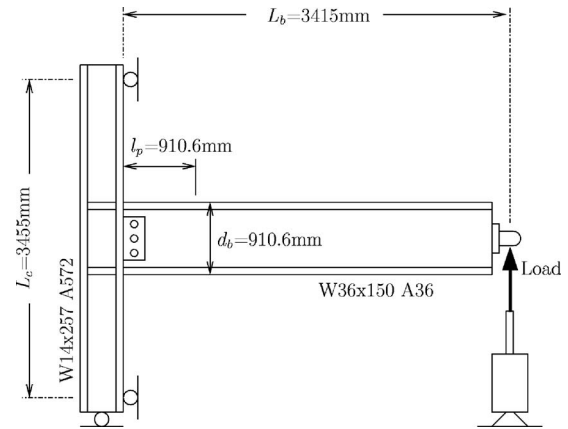


Fig. 2. Analytical model of test specimen PN3 (adapted from Popov et al. 1996)

Steel Project (SAC Joint Venture 1996) is used to verify and demonstrate the application of the plastic rotation sensitivity equations. The subassemblage details for specimen PN3 are shown in Fig. 2. Detailed finite-element analyses of specimen PN3 were conducted by El-Tawil et al. (1999) to assess the nonlinear behavior of the panel zone region.

A single force-based element represents the beam member of specimen PN3. Nonlinear material response in the beam is confined to a plastic hinge region of length equal to the beam depth. Although this subassemblage is determinate and an exact plastic hinge length can be computed from static equilibrium, the assumption that the hinge length is equal to the beam depth is common when simulating the response of indeterminate steel structures. The beam cross section is discretized into fibers whose uniaxial stress-strain behavior is bilinear with 5% kinematic strain hardening. Two Gauss-Radau integration points are located in the plastic hinge region to capture the spread of plasticity (Scott and Fenves 2006). The beam is loaded at its tip through one cycle of peak magnitude 1,000 kN. Material properties and section dimensions are listed in Table 1.

Table 1. Random Variable Distribution properties Assigned to the Uncertain Parameters in Reliability Analysis of Specimen PN3; Parameter Values at Most Probable Failure Point (MPP); and Importance Measures Computed from the Analysis

Parameter	Distribution	COV	Mean	MPP	Importance
Yield stress, f_y (MPa)	Lognormal	0.10	250.0	208.9	-0.7924
Beam depth, d_b (mm)	Normal	0.02	910.6	892.1	-0.4564
Plastic hinge length, l_p (mm)	Normal	0.20	910.6	1044	0.3042
Flange width, b_f (mm)	Normal	0.02	304.2	301.7	-0.1902
Flange thickness, t_f (mm)	Normal	0.02	23.88	23.69	-0.1711
Web thickness, t_w (mm)	Normal	0.02	15.88	15.82	-0.07606

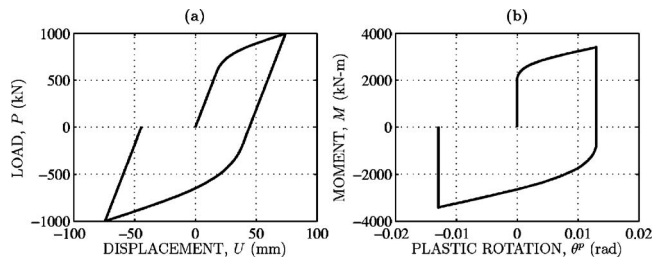


Fig. 3. Computed response for analytical model of specimen PN3: (a) tip load–displacement; (b) beam moment–plastic rotation

Verification of Plastic Rotation Sensitivity

The equations for the sensitivity of plastic rotation are validated by comparison with the finite difference computation

$$\lim_{\varepsilon \rightarrow 0} \frac{v^p(\theta + \varepsilon\theta) - v^p(\theta)}{\varepsilon\theta} = \frac{\partial v^p}{\partial \theta} \quad (12)$$

As the parameter perturbation ε approaches zero, the finite difference computations should converge to the analytical sensitivity, thereby validating the DDM implementation. The nonlinear static analysis of specimen PN3 shows the plastic rotation response sensitivity equations satisfy Eq. (12) for path-dependent behavior under cyclic loading.

The moment–rotation and moment–plastic rotation response for the beam member are shown in Fig. 3. The sensitivity of the plastic rotation response is computed with respect to the beam depth, d_b , and yield stress, f_y , in Fig. 4. For each parameter, the finite difference computation for the plastic rotation sensitivity converges to that obtained by the DDM as the parameter perturbation decreases. In addition to verifying the DDM computations, the results shown in Fig. 4 indicate both the yield stress and the beam depth are resistance variables because an increase in either of these parameters will reduce the magnitude of the plastic rotation.

Reliability Analysis of Steel Subassembly

A first-order reliability (FORM) analysis is conducted to assess the effect of uncertain parameters on the computed response of specimen PN3 (see Fig. 2). The steel yield stress and cross-section dimensions of the beam are treated as significant sources of aleatory uncertainty, whereas the assumed plastic hinge length contributes to epistemic uncertainty, giving a total of six random variables. The distribution, mean, and coefficient of variation as-

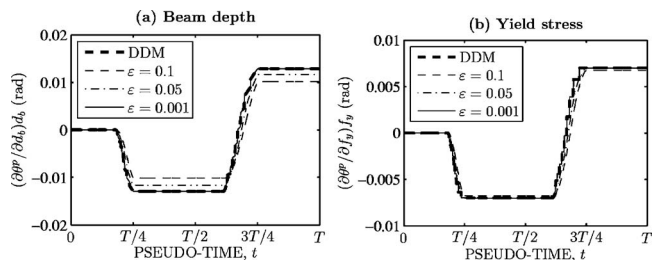


Fig. 4. Steel beam plastic rotation response sensitivity with respect to: (a) beam depth; (b) the yield stress computed by the direct differentiation and finite difference methods

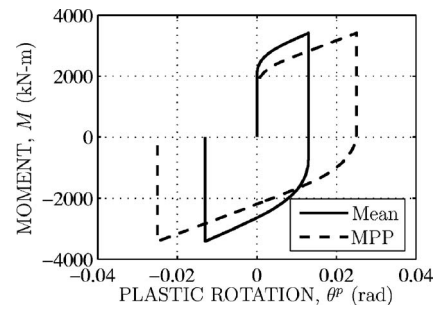


Fig. 5. Computed moment–plastic rotation response for analytical model of specimen PN3 using the mean and MPP values of the random variables

signed to each random variable are shown in Table 1. All random variables are uncorrelated and the applied load is assumed deterministic. Based on reliability analyses of steel frame structures (Haukaas and Scott 2006), the hardening ratio and elastic modulus of the steel material rank low in importance and are thus assumed to be deterministic.

The performance function for this analysis places a 0.025 rad limit on the plastic rotation of the beam after one load cycle

$$g = 0.025 - \theta^p \quad (13)$$

More precise performance functions for the plastic rotation can be derived from statistical data compiled by Roeder (2002) for various connection types. For this performance function, however, the most probable failure point in the FORM analysis is found after eight evaluations of the function and its gradient with respect to each random variable. The resulting reliability index is $\beta = 2.229$, which corresponds to a 1.29% probability that the beam plastic rotation will exceed 0.025 rad given the uncertain properties of the random variables. The MPP values and importance factors (α values) of each random variable are listed in Table 1 and the plastic rotation response at the mean and MPP values is shown in Fig. 5. The steel yield stress and beam depth rank highest in importance and thus have the greatest influence on the system performance. The positive value of importance associated with the plastic hinge length indicates the model should be updated with a longer hinge length in order to better capture the spread of plasticity.

Conclusions

Analytical equations have been derived for response sensitivity analysis and gradient-based reliability and optimization algorithms in structural engineering where the performance function is defined in terms of plastic rotation. Direct differentiation of plastic rotation defined by elastic unloading provides the necessary response sensitivity equations, which are verified for the force-based element formulation of nonlinear material response. It is straightforward to extend the results of this work to definitions of plastic rotation that account for stiffness degradation and to alternative element formulations for nonlinear material response. Further applications of this work include system reliability analyses of frame structures where plastic rotation performance functions are defined for each member and applications where structural performance is optimized based on the plastic rotation response of one or more members.

Notation

The following symbols are used in this technical note:

- \mathbf{b} = section force interpolation matrix;
- \mathbf{e} = section deformation vector;
- \mathbf{f} = element flexibility matrix;
- \mathbf{f}^e = elastic element flexibility matrix;
- \mathbf{f}_s = section flexibility matrix;
- \mathbf{k} = element stiffness matrix in the basic system;
- \mathbf{k}_s = section stiffness matrix;
- l_p = plastic hinge length;
- N_p = number of element integration points;
- \mathbf{q} = element basic force vector;
- \mathbf{s} = section force vector;
- \mathbf{v} = element deformation vector;
- \mathbf{v}^e = element elastic deformation vector;
- \mathbf{v}^p = element plastic deformation vector;
- x = integration point location;
- w = integration point weight;
- θ = uncertain parameter; and
- θ^p = plastic rotation.

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