Geometric Modeling for Computer Graphics

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1

What do we mean by "Modeling"?

How we model geometry depends on what we would like to use the geometry for:

- Looking at its appearance
- Will we need to interact with its shape?
- How does it interact with its environment?
- How does it interact with other objects?
- What is its surface area and volume?
- Will it need to be 3D-printed?
- Etc.

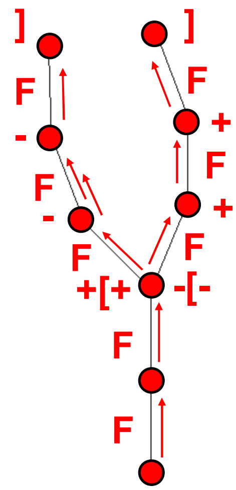
2

L-Systems as a Special Way to Model 3D Geometry

Introduced and developed in 1968 by Aristid Lindenmayer, L-systems are a way to apply grammar rules for generating fractal (self-similar) geometric shapes. For example, take the string:

"FF+[+F-F-F]-[-F+F+F]"

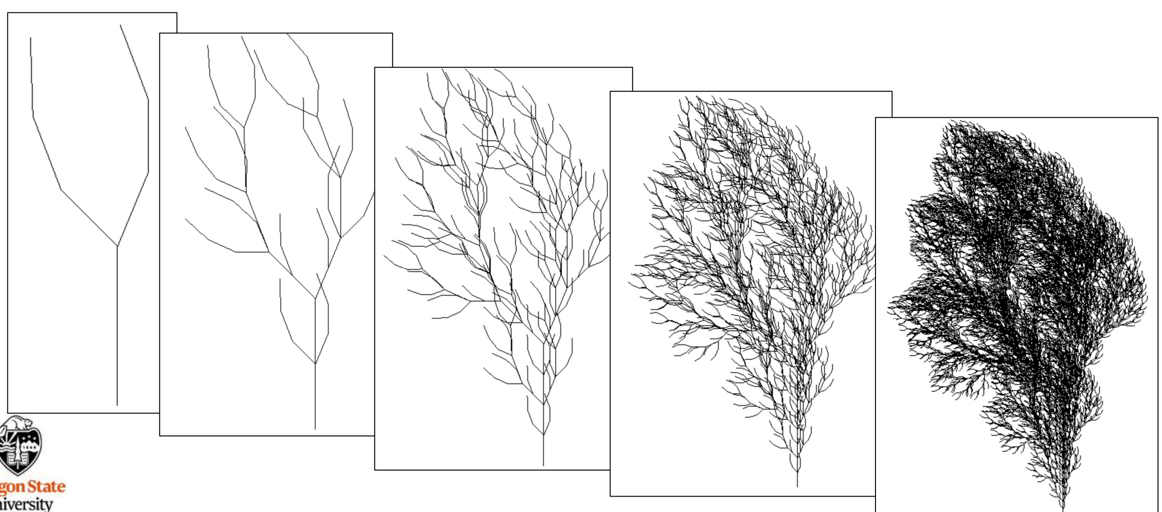
F	move forward one step
+	turn right
-	turn left
[	push state
]	pop state



L-Systems as a Special Way to Model 3D Geometry

But the *real* fun comes when you call that string recursively. For every **F**, replicate that string but with smaller geometry:

"F → FF+[+F-F-F]-[-F+F+F]"

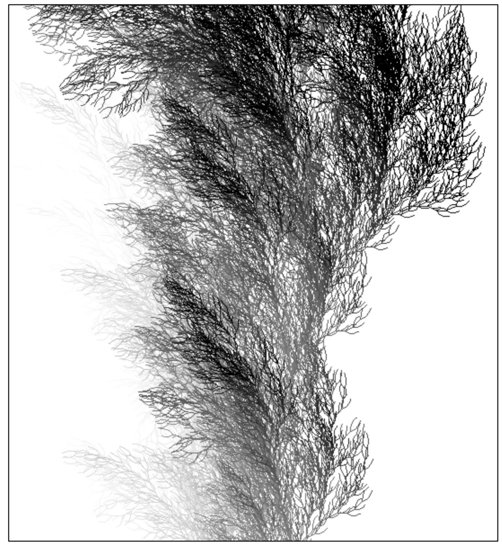


L-Systems as a Special Way to Model 3D Geometry

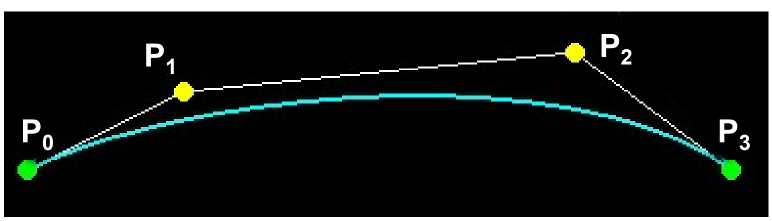
And, of course we can introduce more grammar to swing it into 3D

"F → FF+[+F-<F->F]-[-F+^F+vF]"

+	rotate + about Z
-	rotate - about Z
<	rotate + about Y
>	rotate – about Y
v	rotate + about X
^	rotate – about X



Another way to Model: Curve Sculpting – Bézier Curve Sculpting



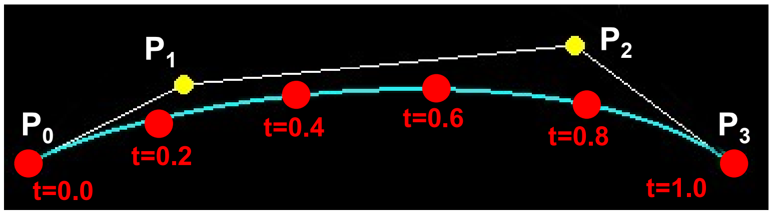
$$P(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t) P_2 + t^3 P_3$$

$$0. \le t \le 1.$$

where ***P*** represents $\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$

t goes from 0.0 to 1.0 in whatever increment you'd like

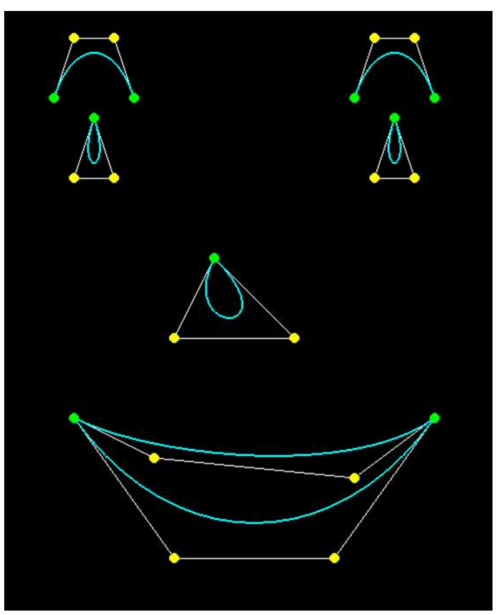
$$0. \le t \le 1.$$

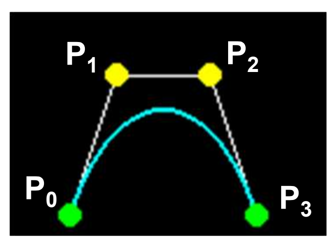


You draw the curve as a series of lines

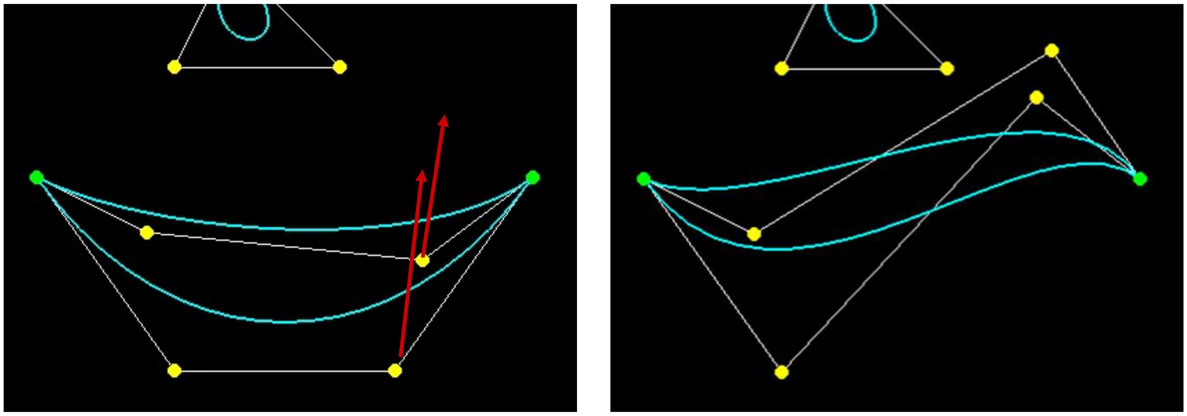
GL_LINE_STRIP is a good topology for this

Curve Sculpting – Bézier Curve Sculpting Example





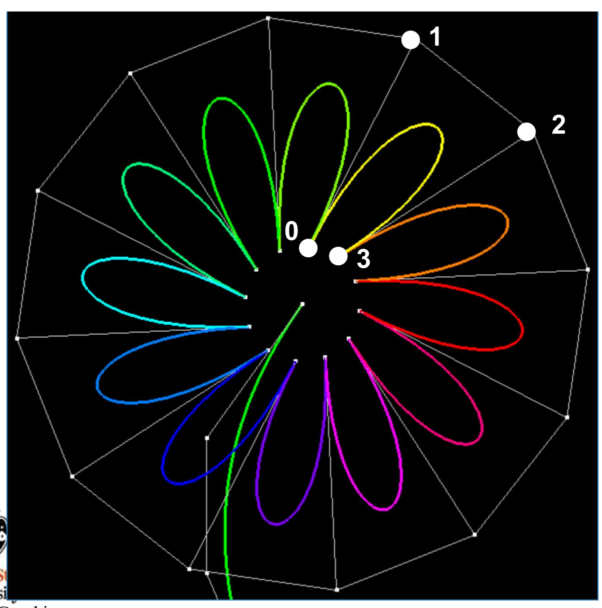
Curve Sculpting – Bézier Curve Sculpting Example

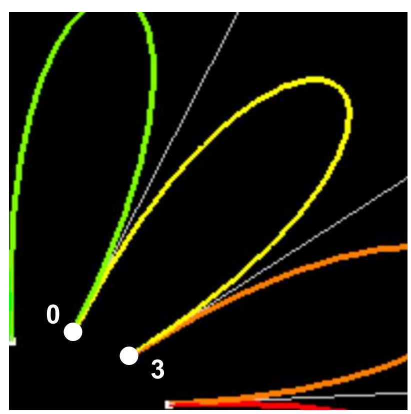


Moving a *single* control point moves its *entire* curve

A *Small* Amount of Input Change Results in a *Large* Amount of Output Change

The Yellow 4-Point Bézier Curve





Another way to Model:
Curve Sculpting – Catmull-Rom Curve Sculpting

The Catmull-Rom curve consists of any number of points.
The first point influences how the curve starts.
The last point influences how the curve ends.
The overall curve goes smoothly through all other points.

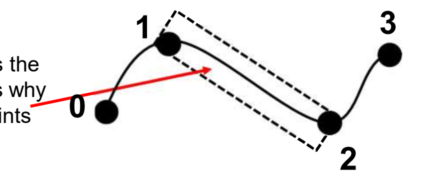
To draw the curve, grab points 0, 1, 2, and 3, call them P_0, P_1, P_2, and P_3, and loop through the following equation, varying t from 0. to 1. in an increment of your own choosing:

$$P(t) = 0.5 * [2 * P_1 + t * (-P_0 + P_2) + t^2(2 * P_0 - 5. * P_1 + 4P_2 - P_3) + t^3(-P_0 + 3P_1 - 3P_2 + P_3)]$$

$$0. \leq t \leq 1.$$

where ***P*** represents $\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$

For each set of 4 points, this equation just draws the line between the second and third points. That's why you keep having to use subsequent sets of 4 points



Another way to Model:
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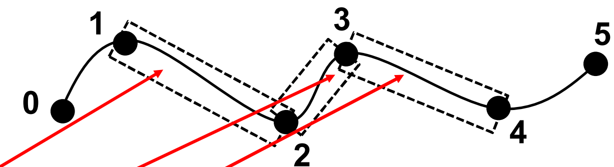


To draw the curve, grab points 0, 1, 2, and 3, call them P_0, P_1, P_2, and P_3, and loop through the equation, varying t from 0. to 1. in an increment of your own choosing.

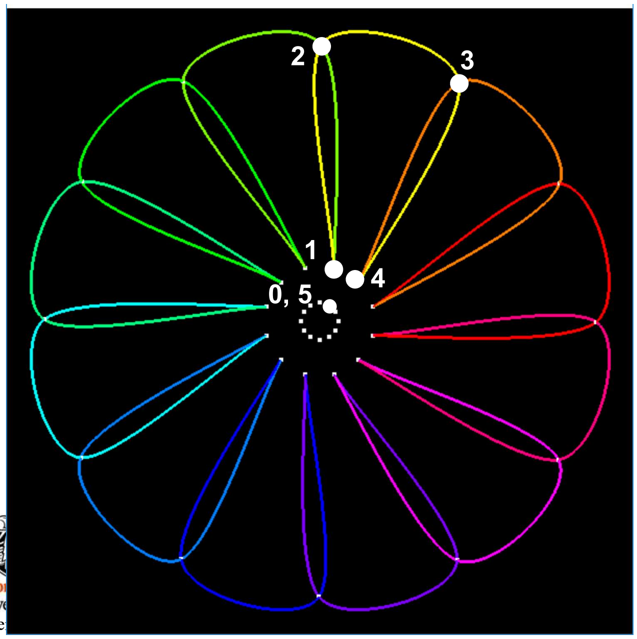
Then, grab points 1, 2, 3, and 4, call them P_0, P_1, P_2, and P_3, and loop through the same equation.

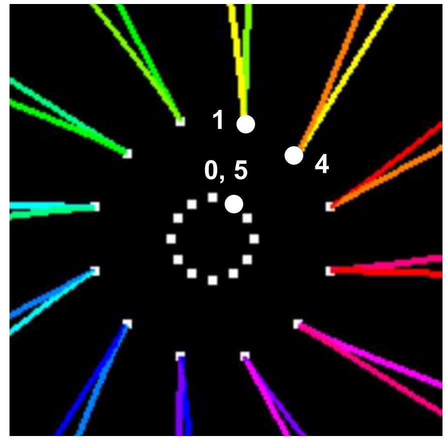
Then, grab points 2, 3, 4, and 5, call them P_0, P_1, P_2, and P_3, and loop through the same equation

And so on…

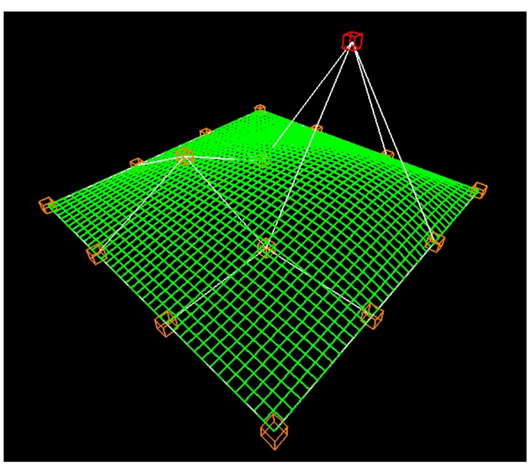
A *Small* Amount of Input Change Results in a *Large* Amount of Output Change

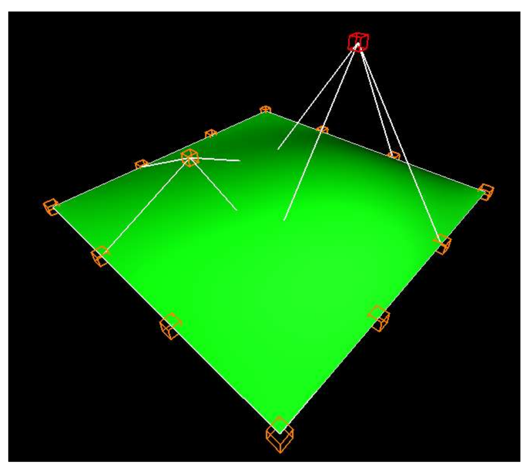
The Yellow 6-Point Catmull-Rom Curve





Another way to Model:
Bézier Surface Sculpting





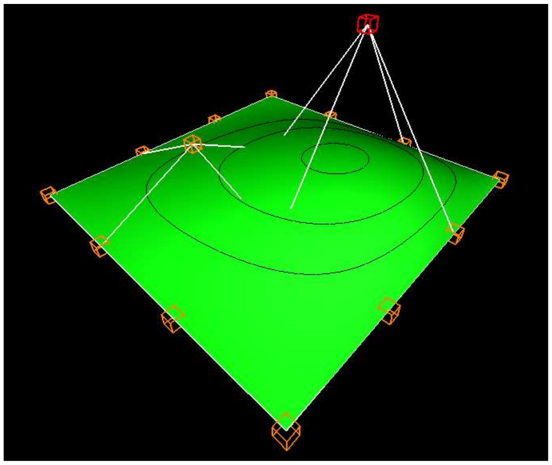
Wireframe **Surface**

Moving a single point moves its entire surface

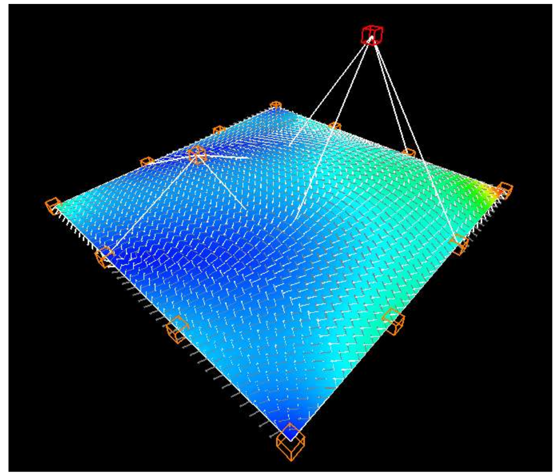
A *Small* Amount of Input Change Results in a *Large* Amount of Output Change

Surface Equations can also be used for Analysis

15



Showing Contour Lines



Showing Curvature

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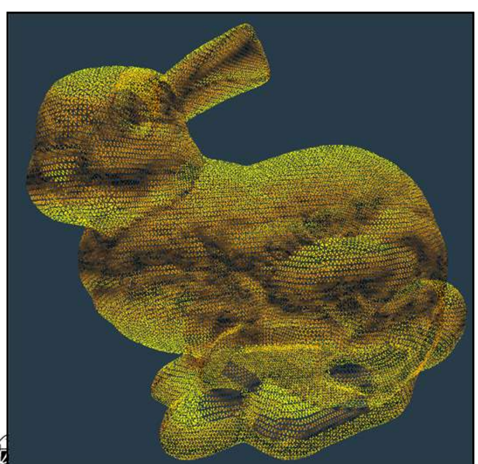
15

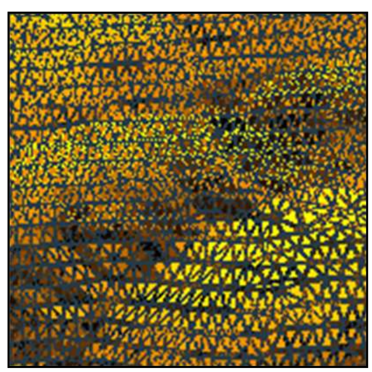
Explicitly Listing Geometry and Topology

16

Models can consist of thousands of vertices and faces – we need some way to list them efficiently



http://graphics.stanford.edu/data/3Dscanrep



This is called a **Mesh.**

If it's in nice neat rows like this, it is called a **Regular Mesh**.

If it's not, it is called an **Irregular Mesh**, or oftentimes called a **Triangular Irregular Network**, or **TIN**.

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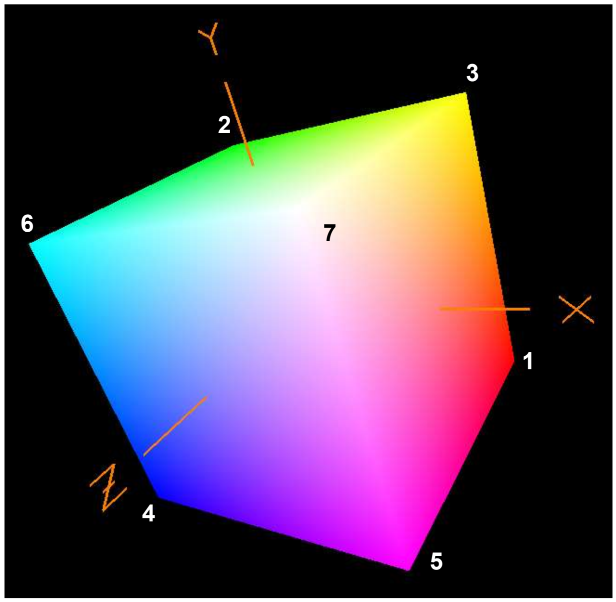
16

Cube Example



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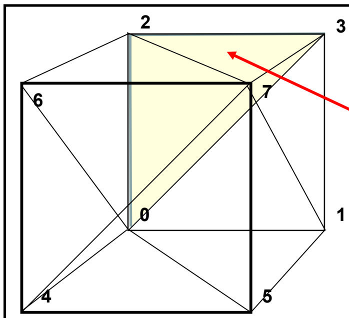
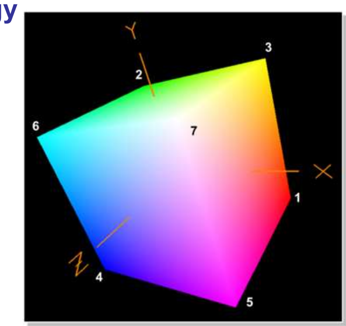
17

Explicitly Listing Geometry and Topology






```
GLuint CubeTriangleIndices[ ][3] =
{
        { 0, 2, 3 },
        { 0, 3, 1 },
        { 4, 5, 7 },
        { 4, 7, 6 },
        { 1, 3, 7 },
        { 1, 7, 5 },
        { 0, 4, 6 },
        { 0, 6, 2 },
        { 2, 6, 7 },
        { 2, 7, 3 },
        { 0, 1, 5 }
        { 0, 5, 4 }
};
```

```
static GLfloat CubeVertices[ ][3] =
{
        { -1., -1., -1. },
        {  1., -1., -1. },
        { -1.,  1., -1. },
        {  1.,  1., -1. },
        { -1., -1.,  1. },
        {  1., -1.,  1. },
        { -1.,  1.,  1. },
        {  1.,  1.,  1. }
};
```

```
static GLfloat CubeColors[ ][3] =
{
        { 0., 0., 0. },
        { 1., 0., 0. },
        { 0., 1., 0. },
        { 1., 1., 0. },
        { 0., 0., 1. },
        { 1., 0., 1. },
        { 0., 1., 1. },
        { 1., 1., 1. },
};
```

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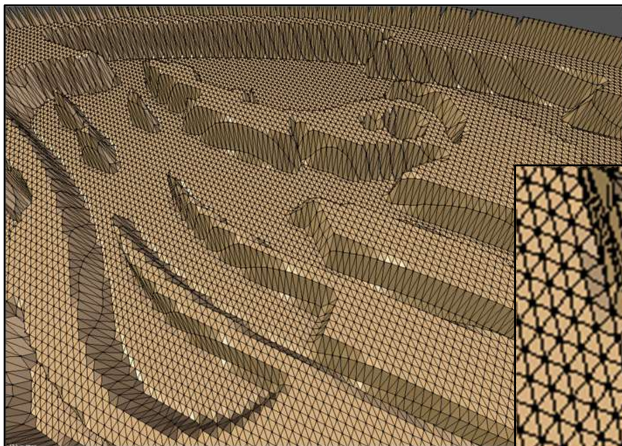
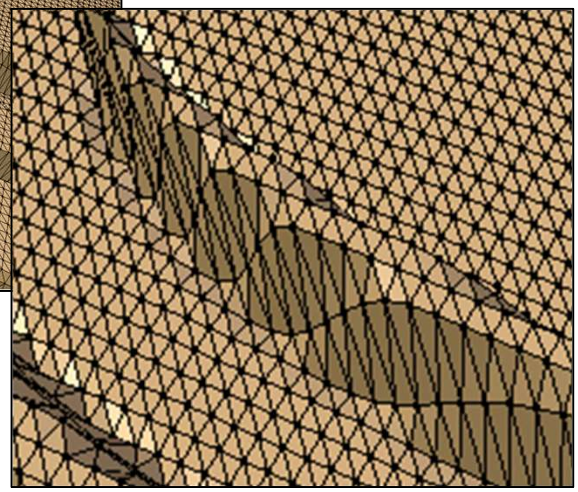
18

3D Printing uses an Irregular Triangular Mesh Data Format

19





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19

3D Printing uses an Irregular Triangular Mesh Data Format

20

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20

Go Beavs – mmmmmm! ☺

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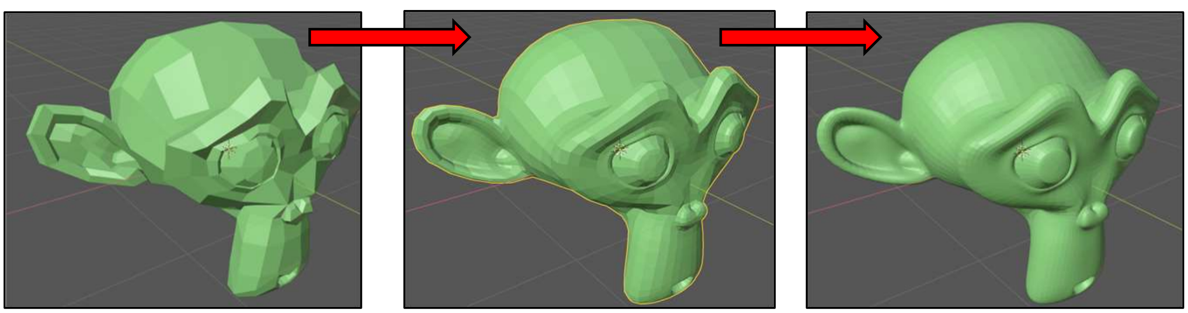
Meshes Can Be Smoothed



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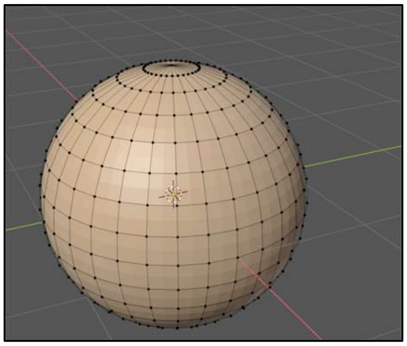
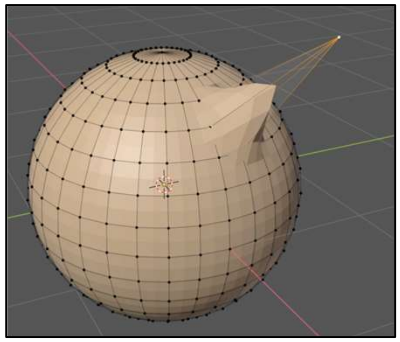
Meshes Can Be Edited

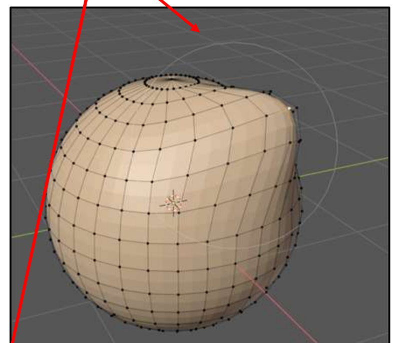
"Circle of Influence"



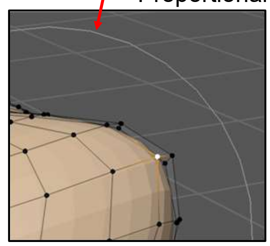
Original



Pulling on a single Vertex



Pulling on a Vertex with Proportional Editing Turned On



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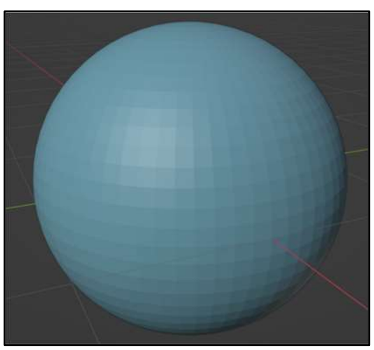
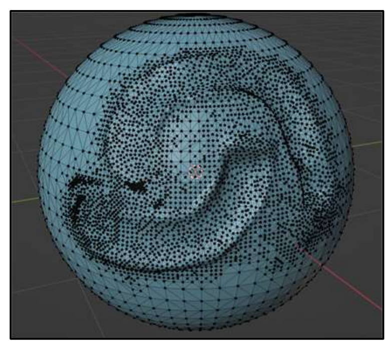
Meshes Can Be Sculpted

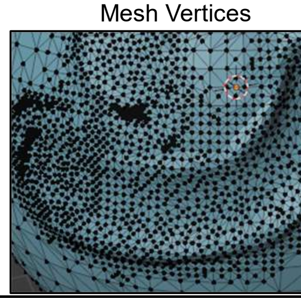


Original

"Clay Thumb" Sculpting



Sculpting Can Produce Additional Mesh Vertices



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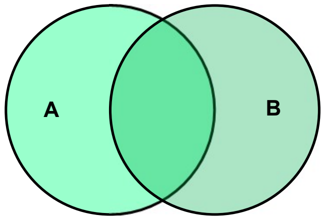
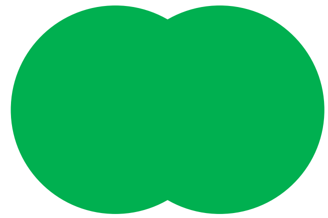
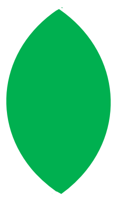
Remember Venn Diagrams (2D Boolean Operators) from High School?



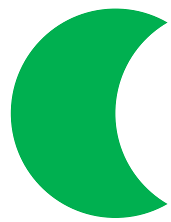
Two Overlapping Shapes



Union: A∪B



Intersection: A∩B



Difference: A-B

I thought I left Venn Diagrams behind in High School !

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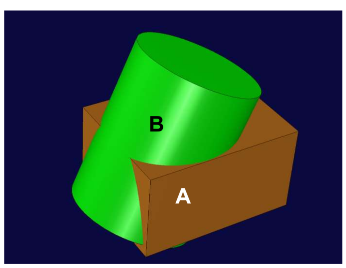
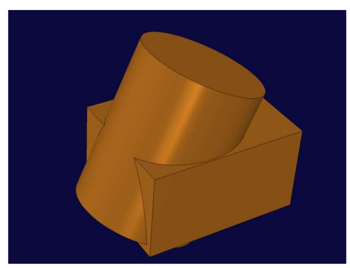
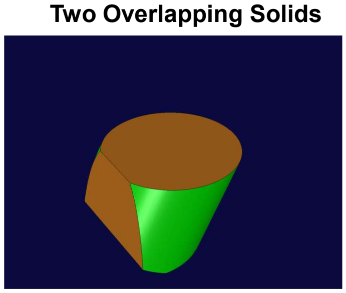
Well, Welcome to Venn Diagrams in 3D



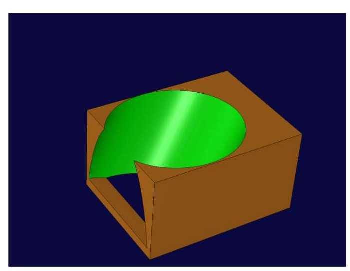
Two Overlapping Solids



Union: A∪B



Intersection: A∩B



Difference: A-B

This is often called **Constructive Solid Geometry**, or **CSG**

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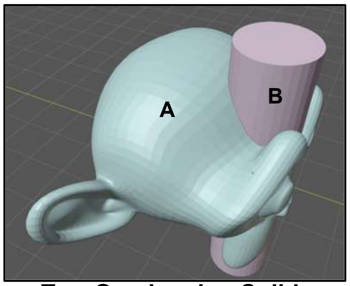
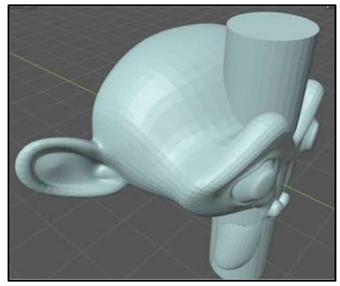
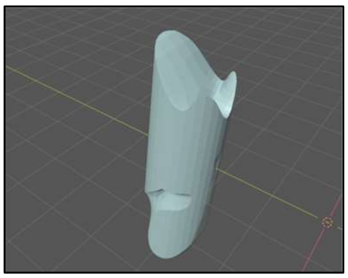
Geometric Modeling Using 3D Boolean Operators on Meshes



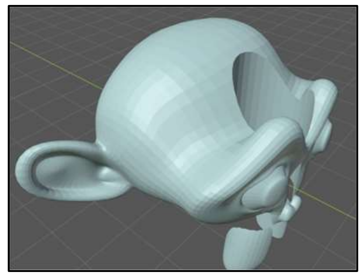
Two Overlapping Solids



Union: A∪B



Intersection: A∩B



Difference: A-B

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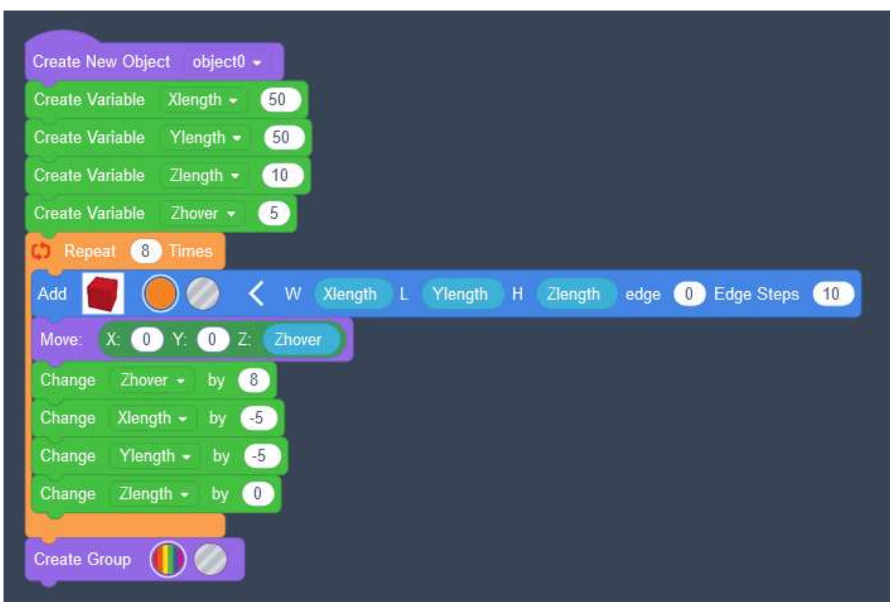
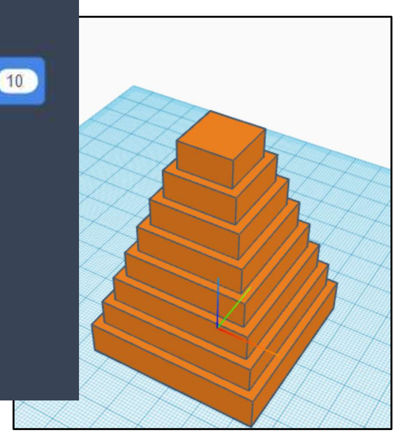
Procedural Geometric Modeling Using TinkerCad/Codeblocks





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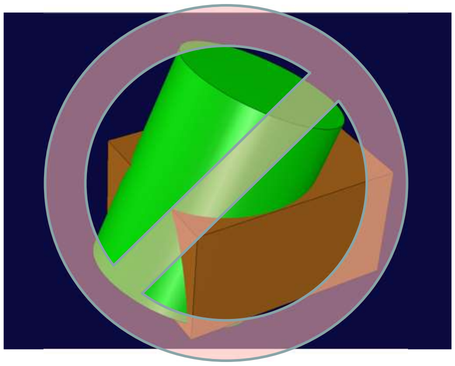
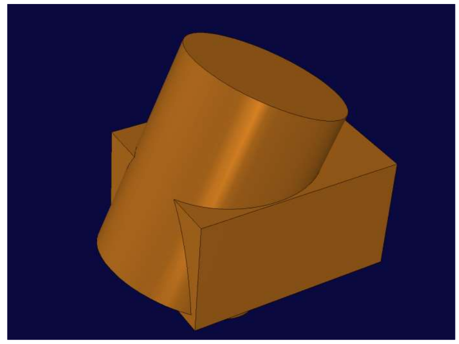
3D Boolean Operators are Important in 3D Printing as well as General Modeling



Two Overlapping Solids – Cannot Be 3D Printed



Two Overlapping Solids Unioned – Now Can Be 3D Printed

Intersected and Differenced Solids Can be 3D Printed as Well

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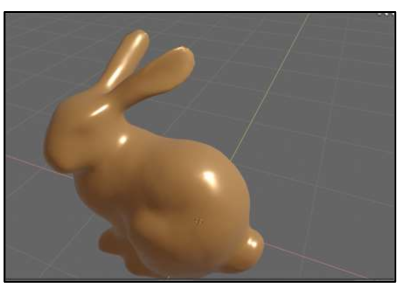
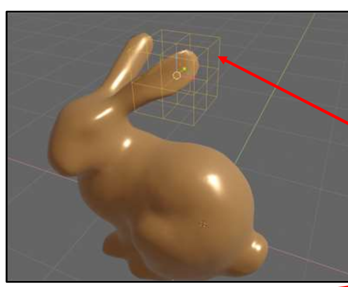
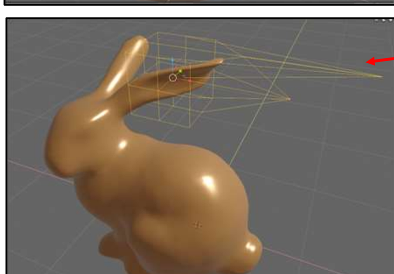
Another Way to Edit Meshes: Lattice Sculpting

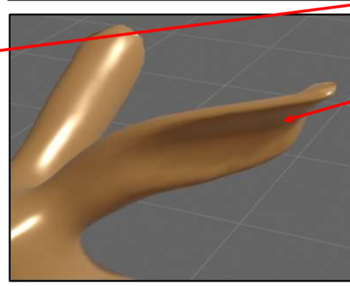




This is often called a **"Lattice"** or a **"Cage".**

Slip a simpler object (e.g., a subdivided cube) around some of the object's vertices. As you sculpt the simpler object, all those object vertices get sculpted too.





lattice.mp4

A *Small* Amount of Input Change Results in a *Large* Amount of Output Change

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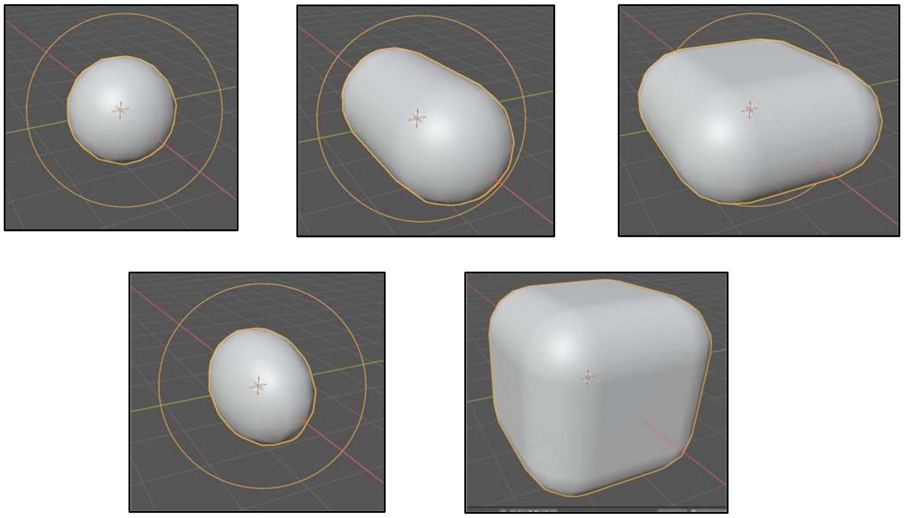
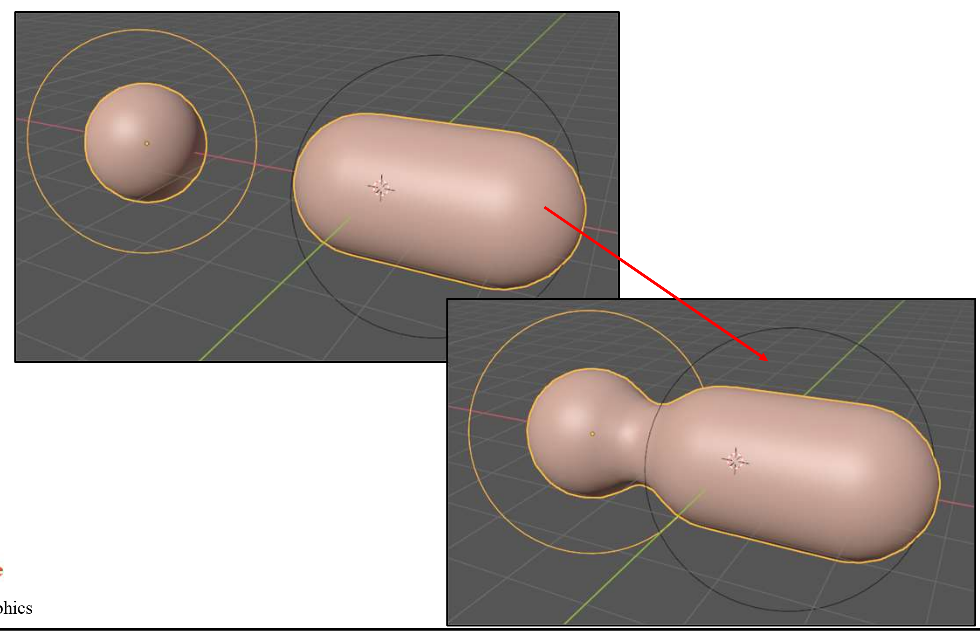
Another Way to Model: Metaball Objects



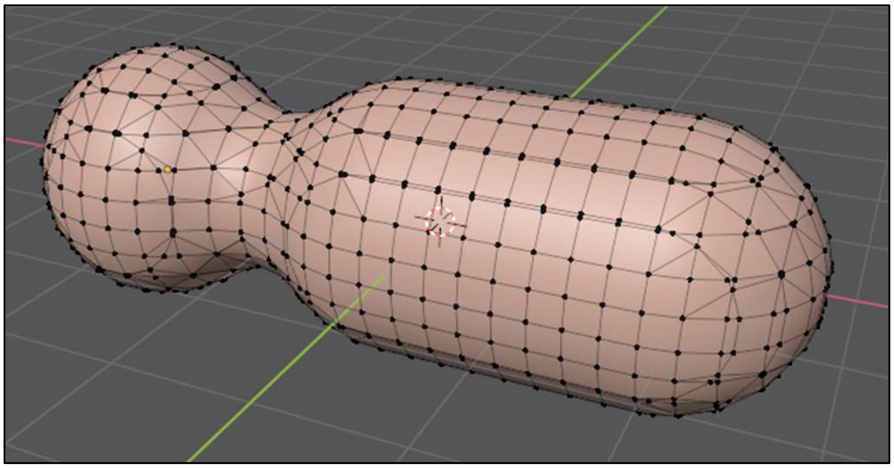
Metaball Objects

The cool thing is that, if you move them close enough together, they will “glom” into a single object



Metaball Objects Can Be Turned into Meshes for Later Editing

33



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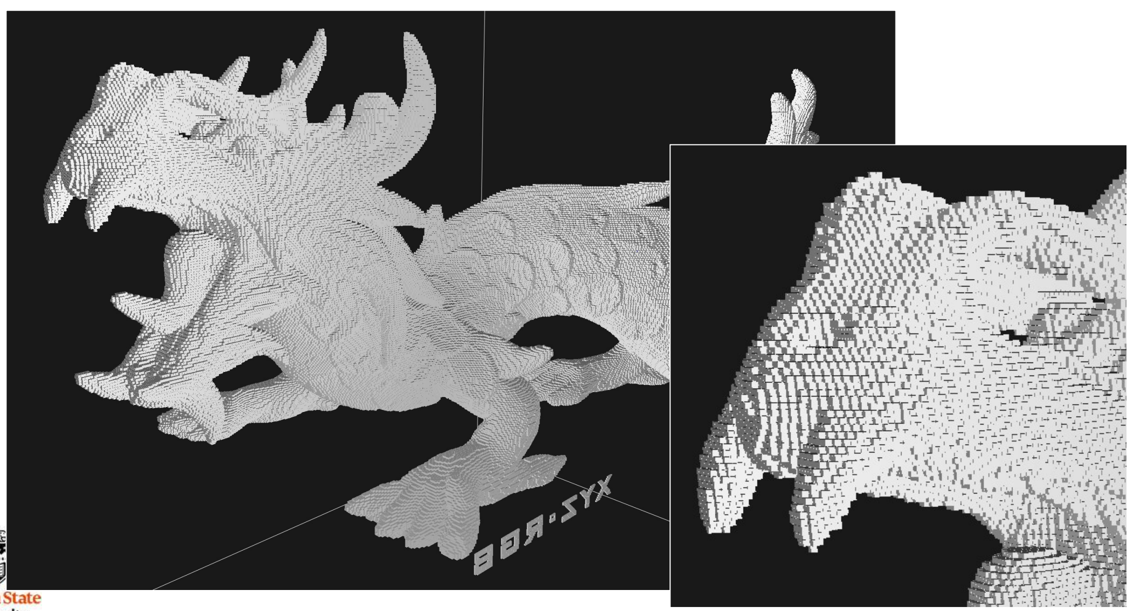
33

Voxelization as a Special Way to Model 3D Geometry

34



Randy Rauwendaal

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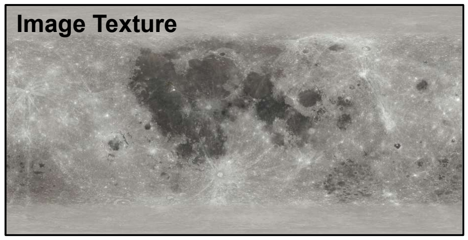
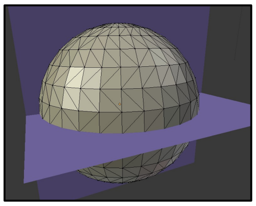
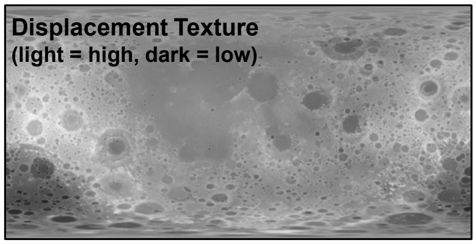
34

Displacement Textures as a Special Way to Model 3D Geometry





Vertex-described Object



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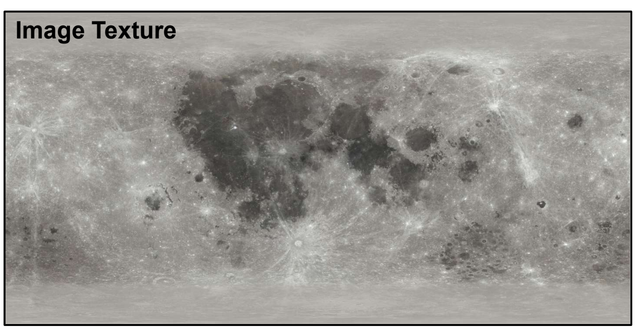
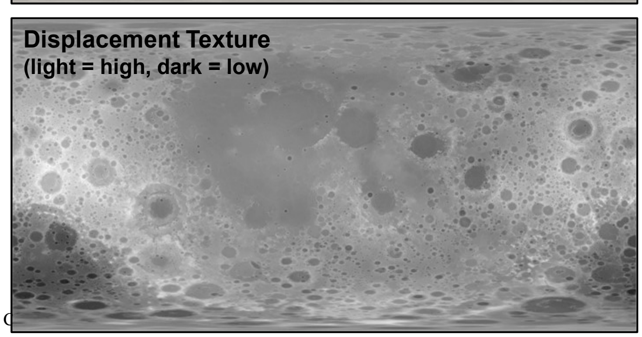
Displacement Textures as a Special Way to Model 3D Geometry





Displacement Textures as a Special Way to Model 3D Geometry

moondisp.vert

```
#version 330 compatibility
uniform float  uLightX, uLightY, uLightZ;
uniform float  uHeightScale;
uniform float  uSeaLevel;
uniform sampler2D        uDispUnit;
uniform bool  uDoElevations;

out vec2        vST;
out vec3        vN;          // normal vector
out vec3        vL;          // vector from point to light

void  main( )
{
        vec2 st = gl_MultiTexCoord0.st;
        vST = st;

        vec3 norm = normalize( gl_NormalMatrix * gl_Normal );          // normal vector
        vN = norm;
        vec3 LightPos = normalize( vec3( uLightX, uLightY, uLightZ ) );
        vec4 ECposition = gl_ModelViewMatrix * gl_Vertex;              // eye coordinate position
        vL = LightPos - ECposition.xyz;                                // vector from the point to the light position

        vec3 vert = gl_Vertex.xyz;
        if( uDoElevations )
        {
                float disp = texture( uDispUnit, st ).r;
                disp -= uSeaLevel;
                disp *= uHeightScale;
                vert += normalize(gl_Normal) * disp;
        }

        gl_Position = gl_ModelViewProjectionMatrix * vec4( vert, 1. );
}
```

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Displacement Textures as a Special Way to Model 3D Geometry

moondisp.frag, I

```
#version 330 compatibility
uniform bool              uDoBumpMapping;
uniform float             uKa, uKd;
uniform float             uHeightScale;
uniform float             uNormalScale;
uniform sampler2D         uColorUnit;
uniform sampler2D         uDispUnit;

in vec2                           vST;
in vec3                           vN;
in vec3                           vL;
#define DELTA                     0.01

void main()
{
        vec3 newColor = texture( uColorUnit, vST ).rgb;
        gl_FragColor = vec4( newColor, 1. );
        if( uDoBumpMapping )
        {
                . . .             // see next slide
        }
}
```

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Displacement Textures as a Special Way to Model 3D Geometry

39

moondisp.frag, II

```
        if( uDoBumpMapping )
        {
                vec2 stp0 = vec2( DELTA, 0. );
                vec2 st0p = vec2( 0. , DELTA );
                float west  = texture2D( uDispUnit, vST-stp0 ).r;
                float east  = texture2D( uDispUnit, vST+stp0 ).r;
                float south = texture2D( uDispUnit, vST-st0p ).r;
                float north = texture2D( uDispUnit, vST+st0p ).r;
                vec3 stangent = vec3( 2.*DELTA, 0., uNormalScale * ( east - west ) );
                vec3 ttangent = vec3( 0., 2.*DELTA, uNormalScale * ( north - south ) );
                vec3 Normal = normalize( cross( stangent, ttangent ) );
                vec3 Light  = normalize(vL);

                vec3 ambient = uKa * newColor;
                float d = 0.;
                if( dot(Normal,Light) > 0. ) // only do diffuse if the light can see the point
                {
                        d = dot(Normal,Light);
                }
                vec3 diffuse = uKd * d * newColor;
                gl_FragColor = vec4( ambient+diffuse, 1. );
        }
}
```

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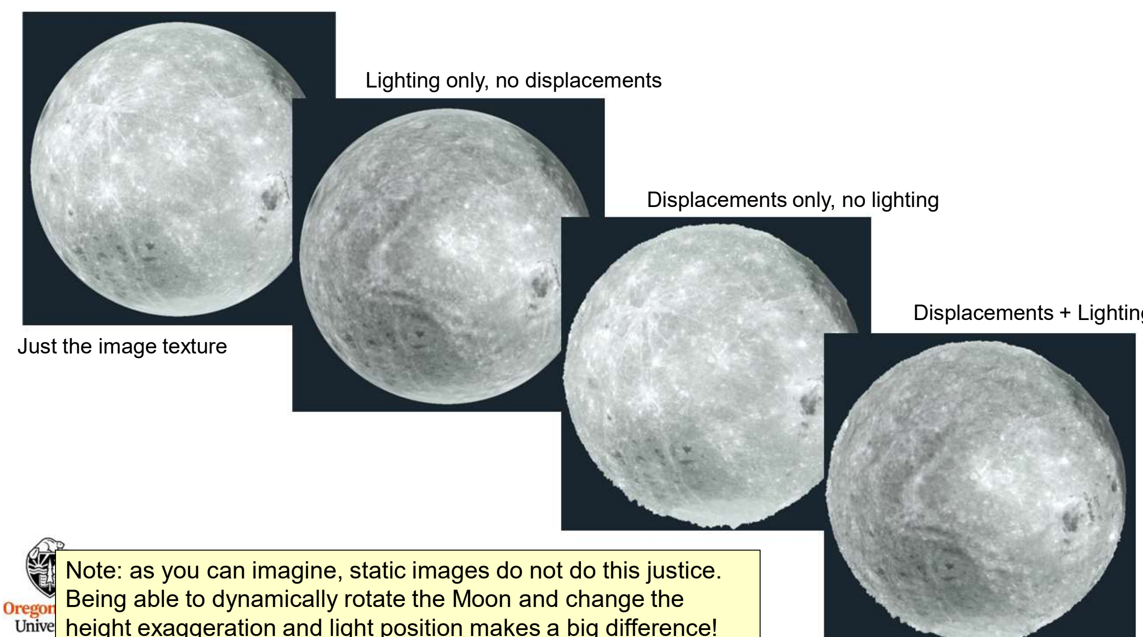
39

Displacement Textures as a Special Way to Model 3D Geometry

40



Lighting only, no displacements

Displacements only, no lighting

Displacements + Lighting

Just the image texture

Note: as you can imagine, static images do not do this justice. Being able to dynamically rotate the Moon and change the height exaggeration and light position makes a big difference!

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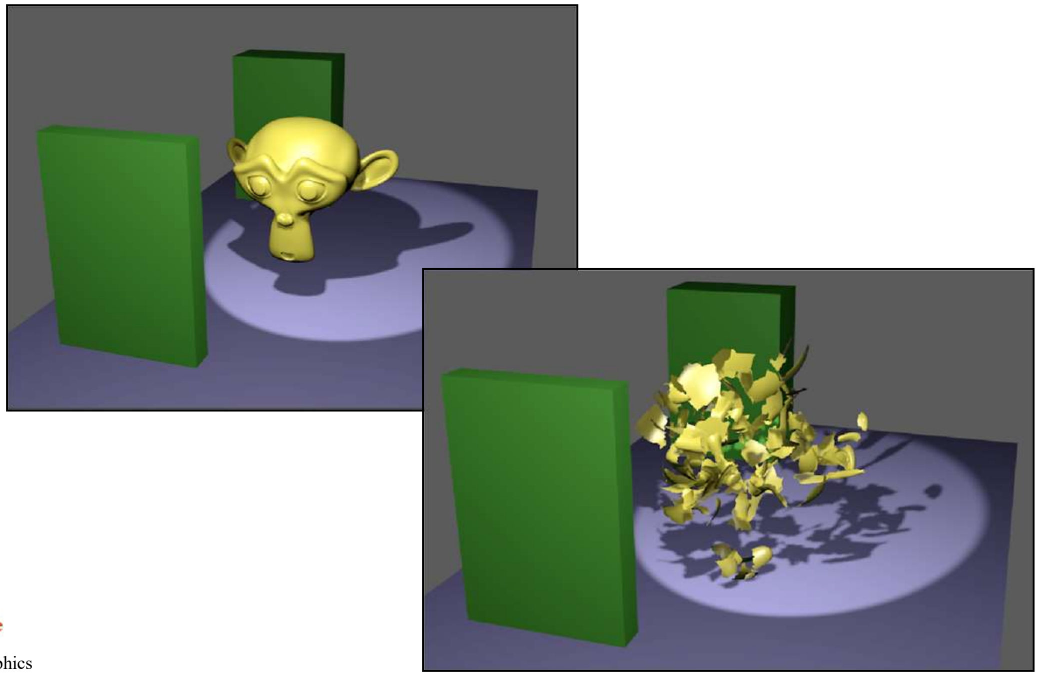
40

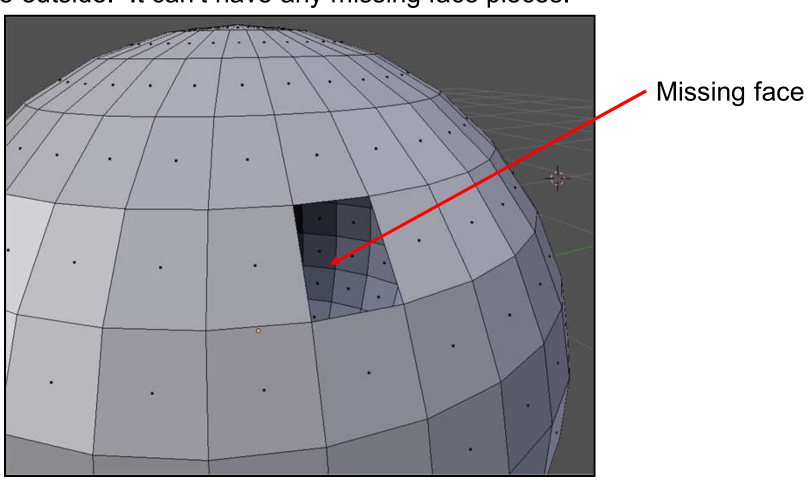
Modeling as an Initial Step in Simulation (Explosion)



41

Object Modeling Rules for 3D Printing

The object must be a legal solid. It must have a definite inside and a definite outside. It can't have any missing face pieces.

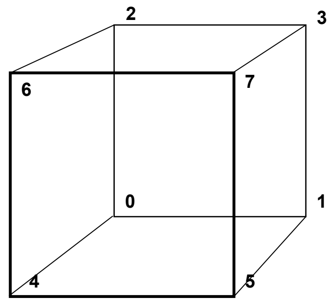


"Definite inside and outside" is sometimes called **"Two-manifold"** or **"Watertight"**

42

The Simplified Euler's Formula* for Legal Solids

*sometimes called the Euler-Poincaré formula



F – E + V = 2

F	**Faces**
E	**Edges**
V	**Vertices**

For a cube, 6 – 12 + 8 = 2

We will talk more about this in the 3D Printing notes!

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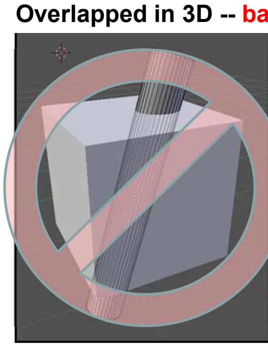
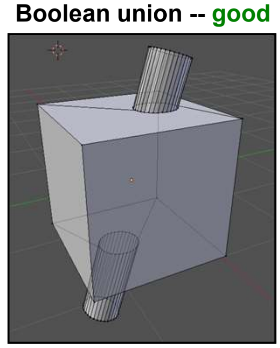
Object Modeling Rules for 3D Printing

Objects cannot pass through other objects. If you want two shapes together, do a Boolean union on them so that they become one complete object.

Overlapped in 3D -- bad



Boolean union -- good



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