Sines and Cosines for Animating Computer Graphics

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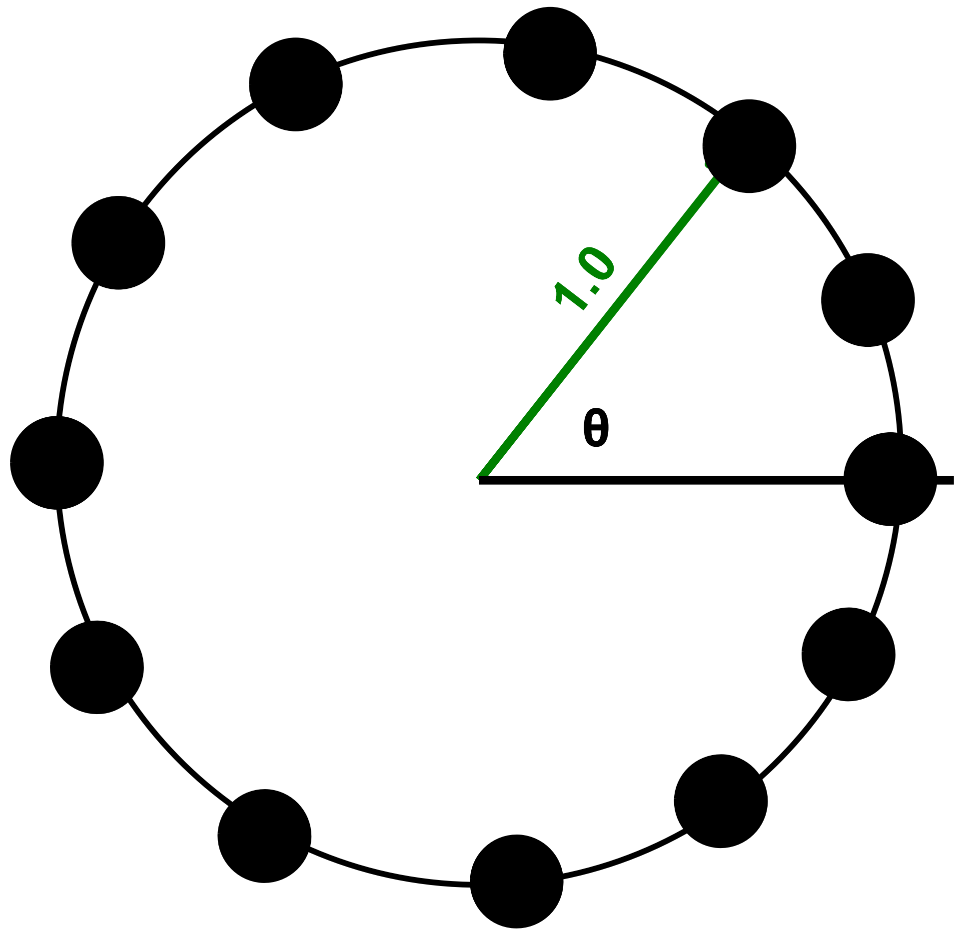
You Know about Sines and Cosines from Math, but They are Very Useful for Animating Computer Graphics

First, We Need to Understand Something about Angles:



If a circle has a radius of 1.0, then we can march around it by simply changing the angle that we call **θ.**

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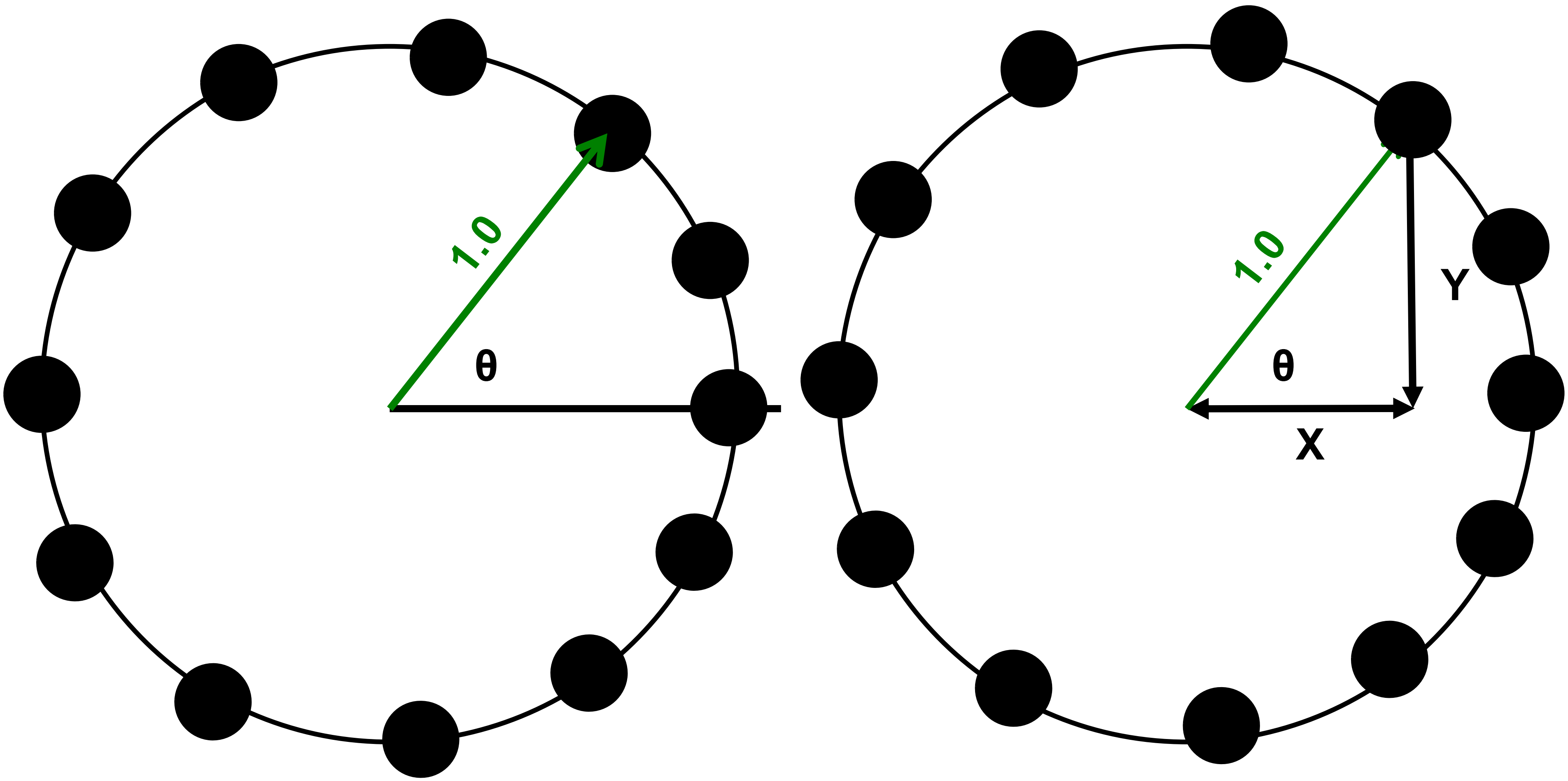
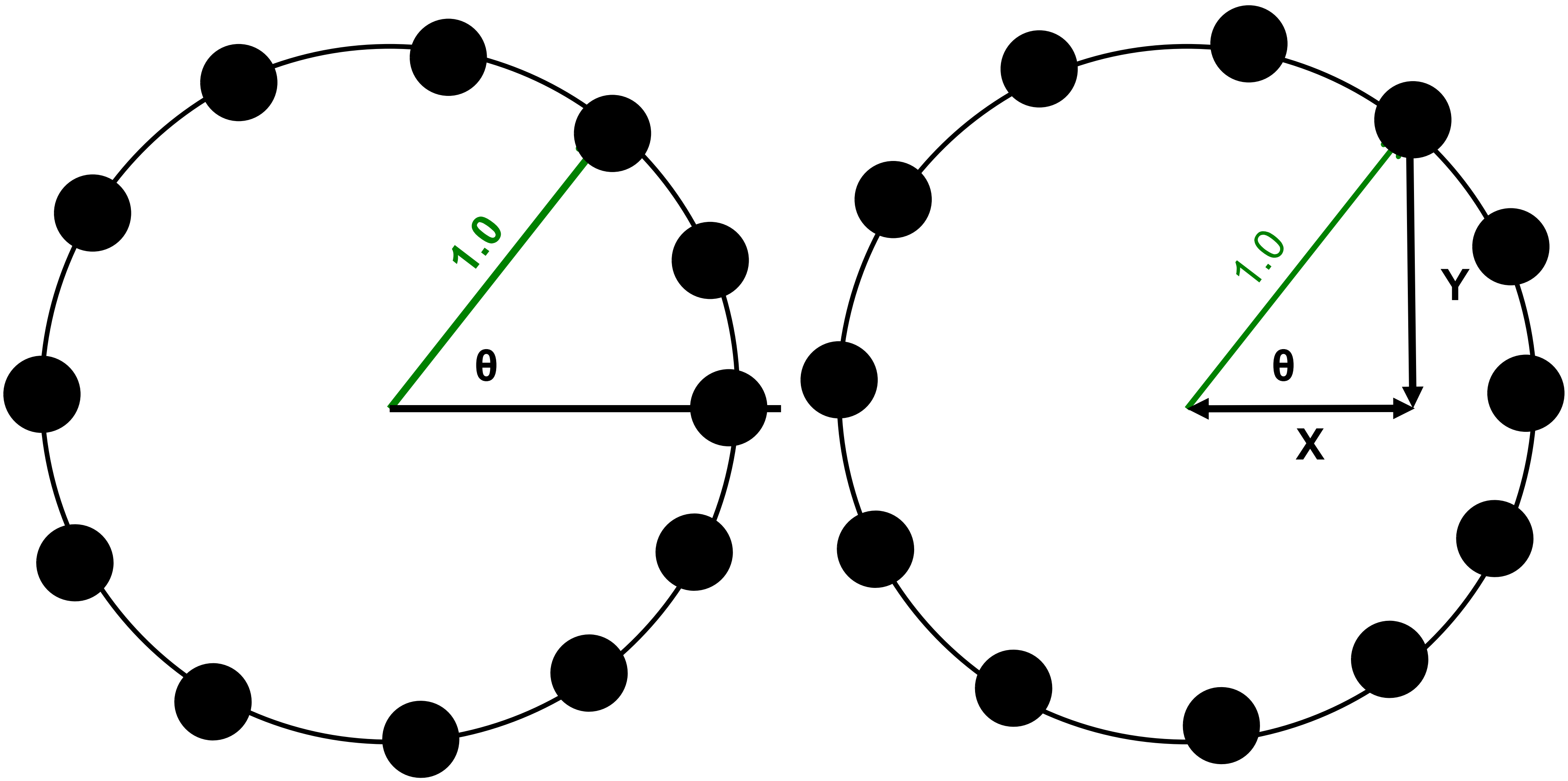


One of the things we notice is that each angle **θ** has a unique **X** and **Y** that goes with it.

These are different for each **θ**.

First, We Need to Understand Something about Angles



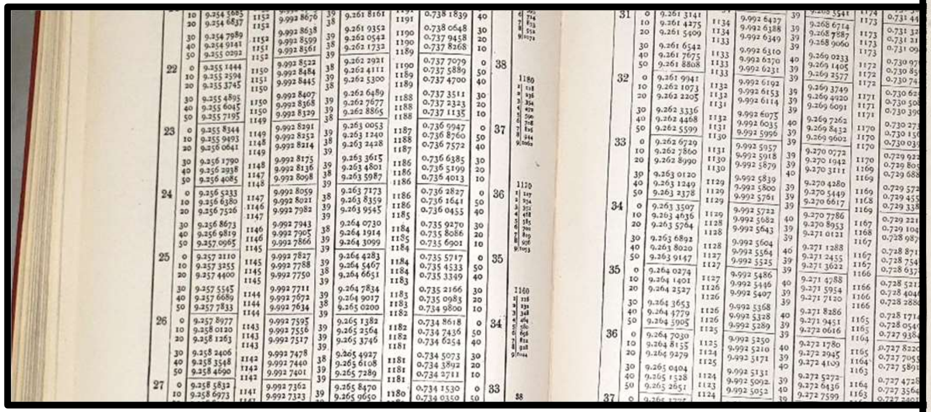
Fortunately, centuries ago, people developed tables of those X and Y values as functions of θ.

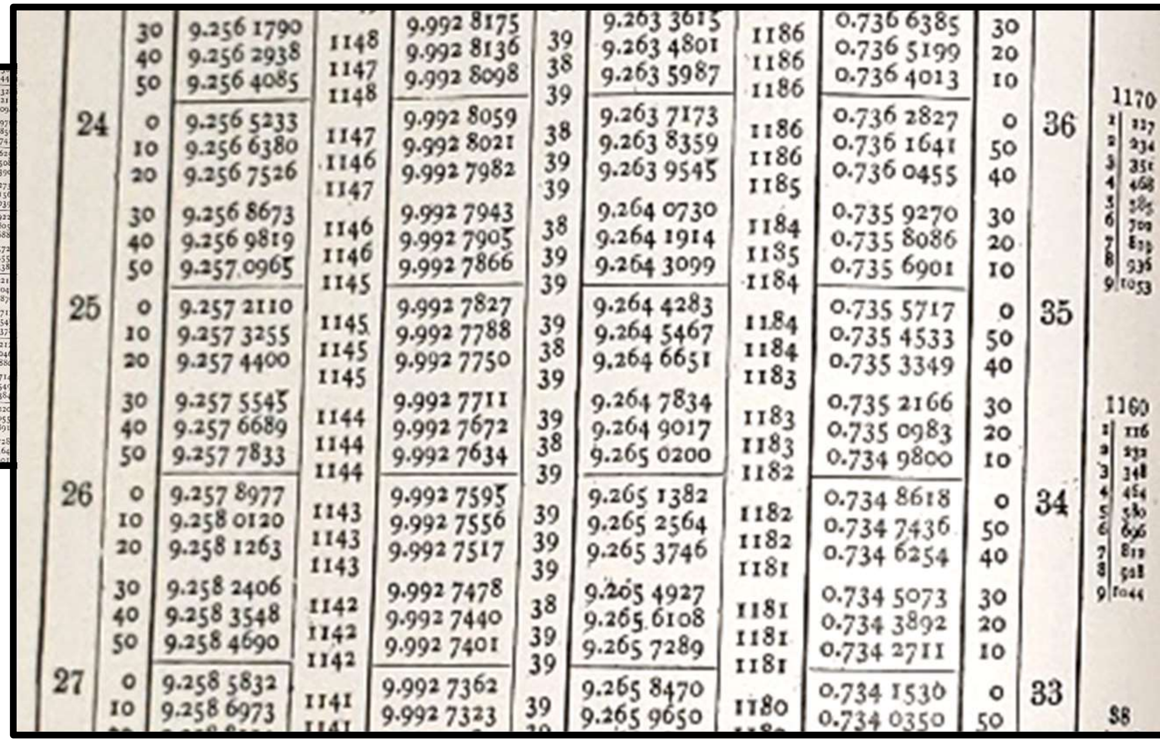
They called the X values **cosines** and the Y values **sines**. These are abbreviated cos and sin.

$$\cos\theta = X$$
$$\sin\theta = Y$$

How People used to Lookup Sines and Cosines – Yuch! Fortunately We Now Have Calculators and Computers

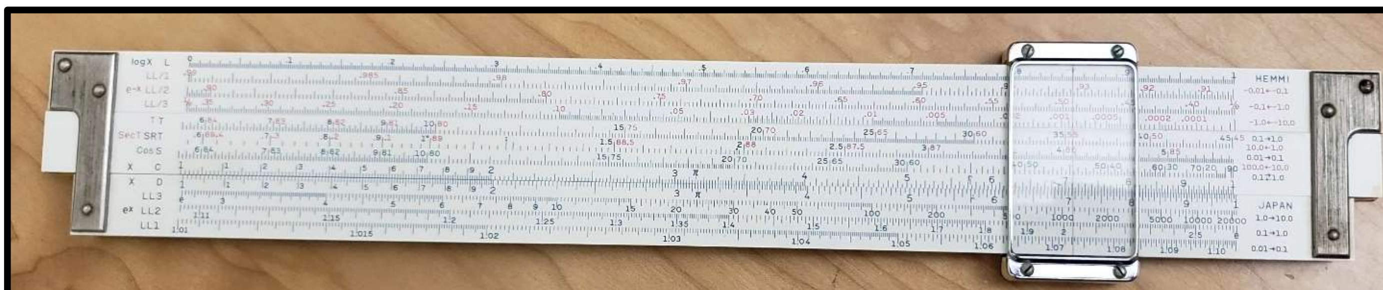
Book of sines and cosines

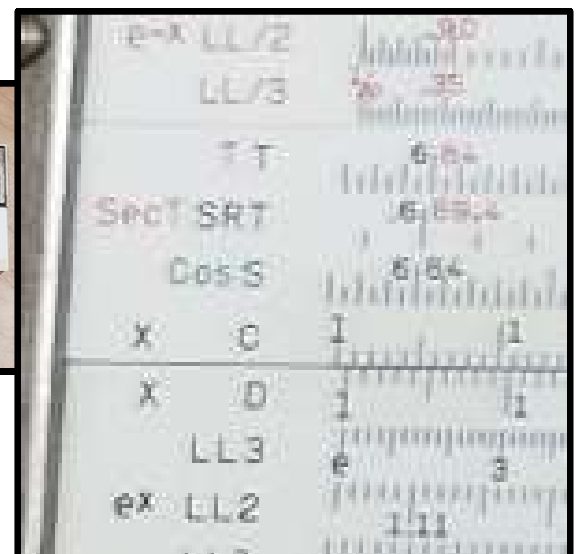




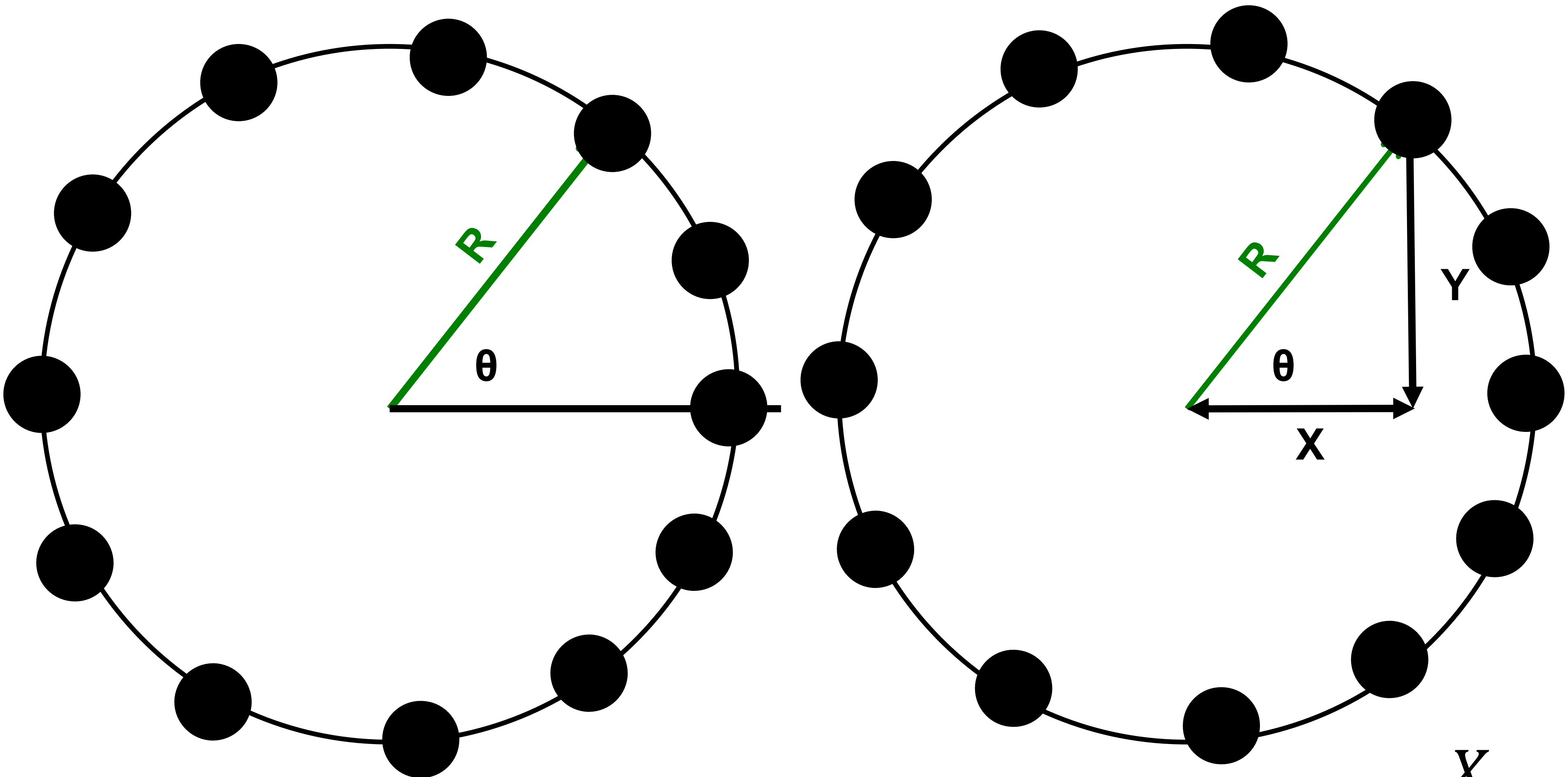
	30	9.256 1790	1148	9.992 8175	39	9.263 3615	1186	0.736 6385	30	
	40	9.256 2938	1147	9.992 8136	38	9.263 4801	1186	0.736 5199	20	
	50	9.256 4085	1148	9.992 8098	39	9.263 5987	1186	0.736 4013	10	
24	0	9.256 5233	1147	9.992 8059	38	9.263 7173	1186	0.736 2827	0	36
	10	9.256 6380	1146	9.992 8021	39	9.263 8359	1186	0.736 1641	50	
	20	9.256 7526	1147	9.992 7982	39	9.263 9545	1185	0.736 0455	40	
	30	9.256 8673	1146	9.992 7943	38	9.264 0730	1184	0.735 9270	30	
	40	9.256 9819	1146	9.992 7905	39	9.264 1914	1185	0.735 8086	20	
	50	9.257 0965	1145	9.992 7866	39	9.264 3099	1184	0.735 6901	10	
25	0	9.257 2110	1145	9.992 7827	39	9.264 4283	1184	0.735 5717	0	35
	10	9.257 3255	1145	9.992 7788	38	9.264 5467	1184	0.735 4533	50	
	20	9.257 4400	1145	9.992 7750	39	9.264 6651	1183	0.735 3349	40	
	30	9.257 5545	1144	9.992 7711	39	9.264 7834	1183	0.735 2166	30	
	40	9.257 6689	1144	9.992 7672	38	9.264 9017	1183	0.735 0983	20	
	50	9.257 7833	1144	9.992 7634	39	9.265 0200	1182	0.734 9800	10	
26	0	9.257 8977	1143	9.992 7595	39	9.265 1382	1182	0.734 8618	0	34
	10	9.258 0120	1143	9.992 7556	39	9.265 2564	1182	0.734 7436	50	
	20	9.258 1263	1143	9.992 7517	39	9.265 3746	1181	0.734 6254	40	
	30	9.258 2406	1142	9.992 7478	38	9.265 4927	1181	0.734 5073	30	
	40	9.258 3548	1142	9.992 7440	39	9.265 6108	1181	0.734 3892	20	
	50	9.258 4690	1142	9.992 7401	39	9.265 7289	1181	0.734 2711	10	
27	0	9.258 5832	1141	9.992 7362	39	9.265 8470	1180	0.734 1530	0	33
	10	9.258 6973	1141	9.992 7323	39	9.265 9650	1180	0.734 0350	50	

Slide rule





First, We Need to Understand Something about Angles



$$\cos\theta = \frac{X}{R}$$

If we were to double the radius of the circle, all of the X's and Y's would also double.

So, really the cos and sin are *ratios* of X and Y to the circle Radius

$$\sin\theta = \frac{Y}{R}$$

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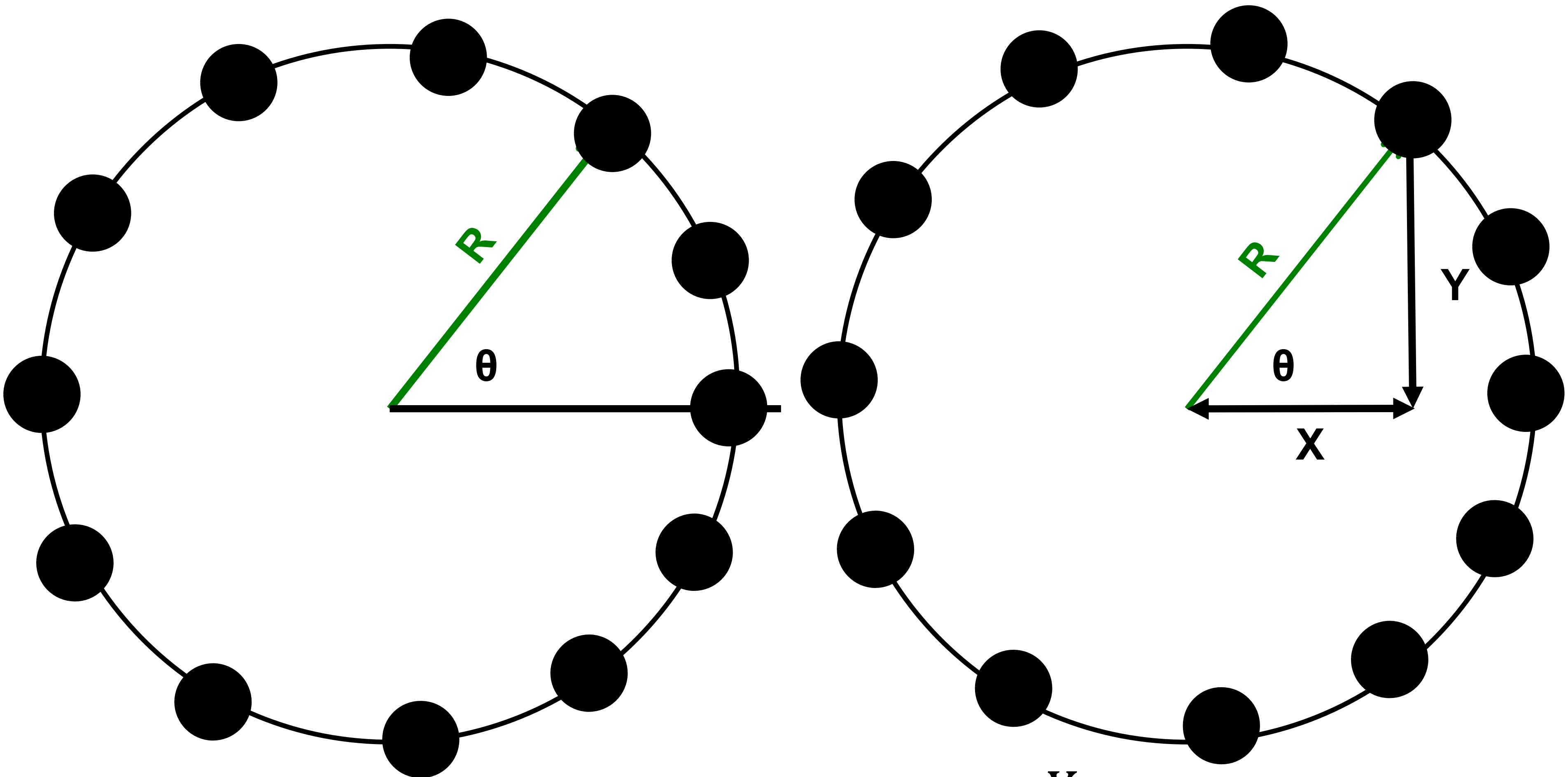
First, We Need to Understand Something about Angles



So, if we know the circle Radius, and we march through a bunch of θ angles, we can determine all of the X's and Y's that we need to draw a circle.

$$\cos\theta = \frac{X}{R}$$

$$\sin\theta = \frac{Y}{R}$$

$$X = R * \cos\theta$$
$$Y = R * \sin\theta$$

Draw to this point

Thus, We Could Create Our Very Own Circle-Drawing Function

Circle center

Circle radius

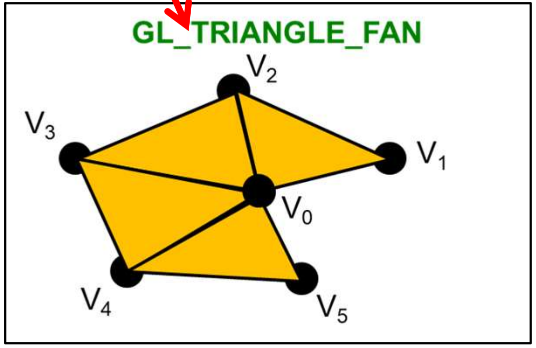
numsegs is the number of line segments making up the circumference of the circle.

numsegs=20 gives a nice circle.

5 gives a pentagon.
8 gives an octagon.
4 gives you a square. Etc.

```
void
Circle( float xc, float yc, float r, int numsegs )
{
  float dang = 2.f * F_PI / (float)numsegs;
  float ang = 0.;
  glBegin( GL_TRIANGLE_FAN );
  glVertex3f( xc, yc, 0. );

  for( int i = 0; i <= numsegs; i++)
  {
    float x = xc + r * cosf(ang);
    float y = yc + r * sinf(ang);
    glVertex3f(  x, y, 0. );
    ang += dang;
  }

 glEnd( );
}
```

2π is how many radians are in a full circle



The C/C++ sin( ) and cos( ) functions use double-precision floating point.

The C/C++ sinf( ) and cosf( ) functions use single-precision floating point, and are faster.

Why 2.*PI ?

```
float dang = 2.f*F_PI / (float)numsegs;
```

*We humans commonly measure angles in **degrees**, but science and computers like to measure them in something else called **radians**.*

There are 360° in a complete circle.
There are 2π radians in a complete circle.

*The built-in cosf( ) and sinf( ) functions expect angles to be given in **radians**.*

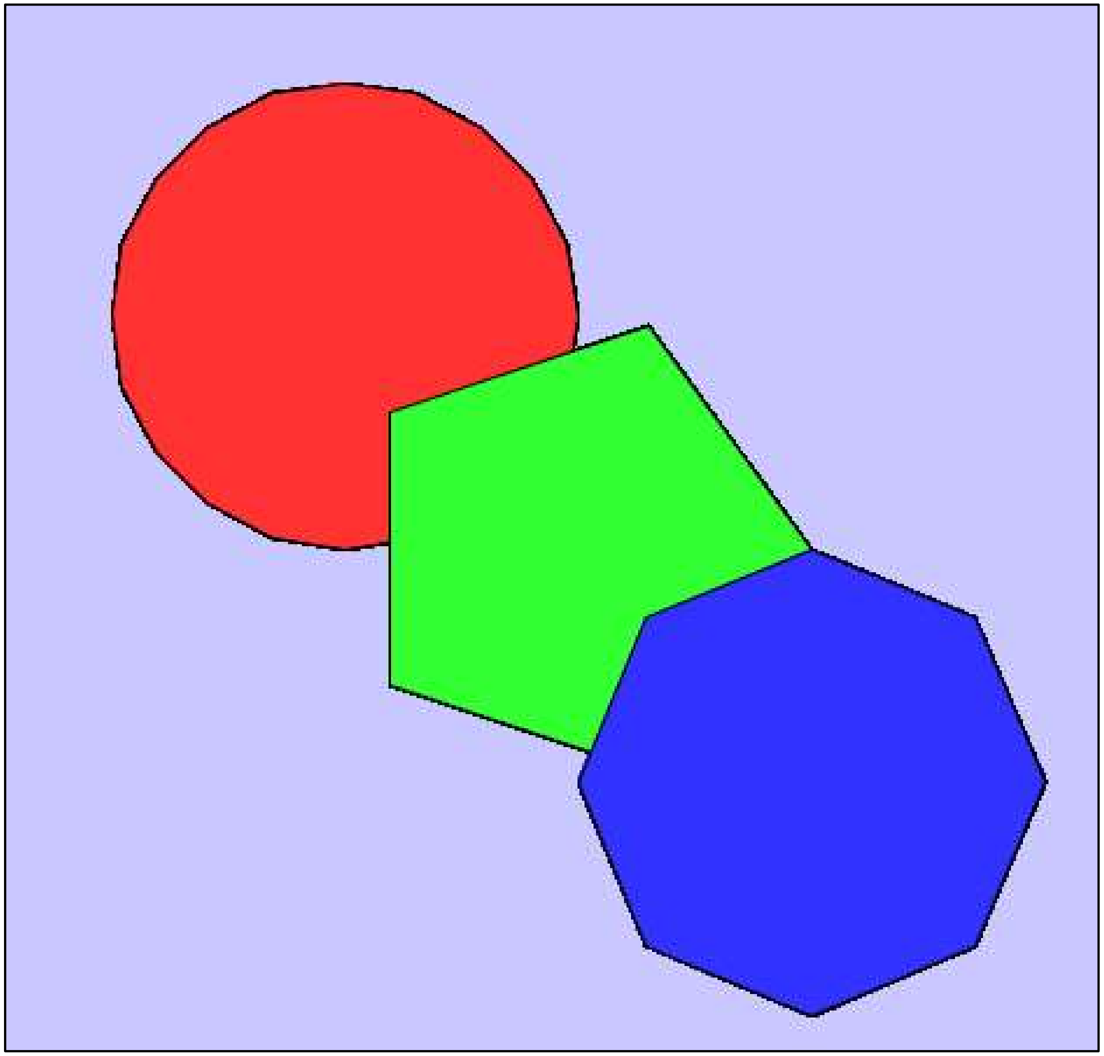
To convert between the two:

```
float rad  = deg * ( F_PI/180.f );
float deg = rad *  ( 180.f/F_PI );
```

glRotatef( ) and gluPerspective( ) are the only two programming functions I can think of that use degrees. All others use radians!

Circles and Pentagons and Octagons, Oh My!

```
glColor3f( 1., 0., 0. );
Circle( 1.f, 3.f, 1.f, 20 )

glColor3f( 0., 1., 0. );
Circle( 2.f, 2.f, 1.f, 5 )

glColor3f( 0., 0., 1. );
Circle( 3.f, 1.f, 1.f, 8 )
```



Easy as π:
M_PI vs. F_PI

The math.h include file has a definition of π that looks like this:

```
#define M_PI        3.14159265358979323846
```

Which will work just fine for whatever you need it for.

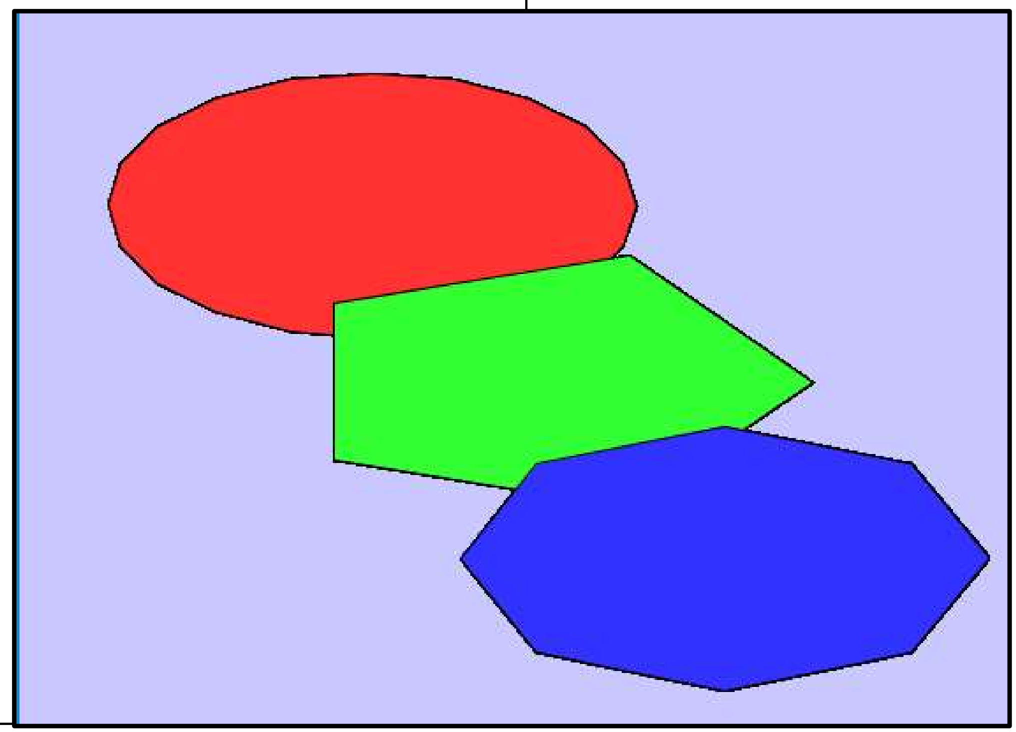
But, Visual Studio goes a little crazy complaining about mixing doubles (which is what M_PI is in) and floats (which is probably what you use most often). So, your sample code has these lines in it:

```
#define F_PI        ((float)(M_PI))       π
#define F_2_PI      ((float)(2.f*F_PI))   2π
#define F_PI_2      ((float)(F_PI/2.f))   π/2
```

I use the *F_* version a lot because it keeps VS quiet. You can use either.

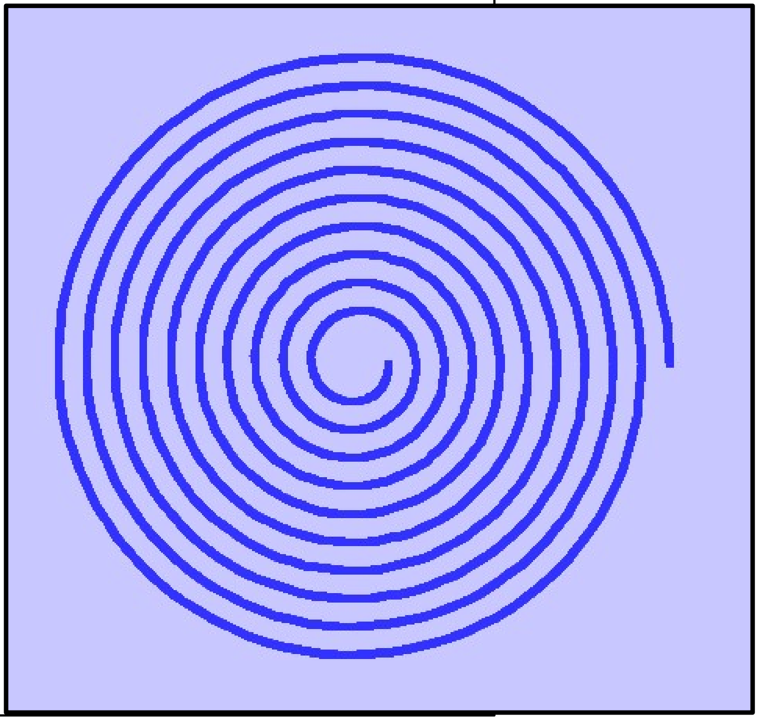
And, there is no reason the X and Y radii need to be the same…

```
void
Ellipse( float xc, float yc, float rx, float ry, int numsegs )
{
  float dang =  2.f * F_PI / (float)numsegs;
  float ang = 0.;
  glBegin( GL_TRIANGLE_FAN );
  glVertex3f( xc, yc, 0. );

  for( int i = 0; i <= numsegs; i++ )
  {
    float x = xc + rx * cosf(ang);
    float y = yc + ry * sinf(ang);
    glVertex3f( x, y, 0. );
    ang += dang;
  }

  glEnd( );
}
```



There is also no reason we can't gradually change the radius …

```
void
Spiral( float xc, float yc, float r0, float r1, int numsegs, int numturns )
{
  float dang = (float)numturns * 2.f * F_PI / (float)numsegs;
  float ang = 0.;
  glBegin( GL_LINE_STRIP );

  for( int i = 0; i <= numsegs; i++ )
  {
    float t = (float)i / (float)numsegs;      // 0.-1.
    float newrad = (1.-t)*r0 + t*r1;
                            // linearly interpolate from r0 to r1
    float x = xc + newrad * cosf(ang);
    float y = yc + newrad * sinf(ang);
    glVertex3f( x, y, 0. );
    ang += dang;
  }

  glEnd( );
}
```



Parametric Linear Interpolation (Blending)

What's this code all about?

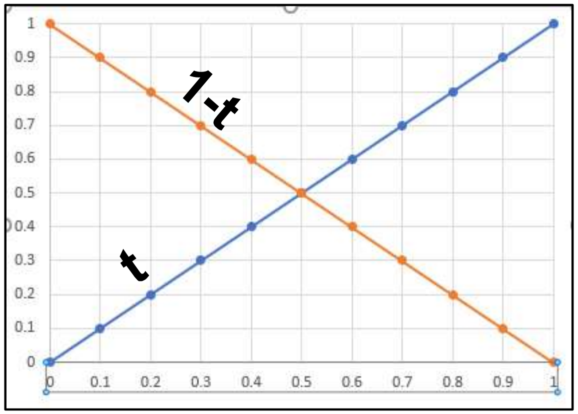
```
float t = (float)i / (float)numsegs;          // 0.-1.
float newrad = (1.-t)*r0 + t*r1;
```

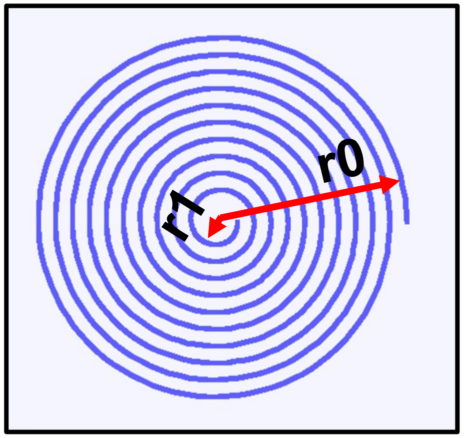
In computer graphics, we do a lot of linear interpolation between two input values. Here is a good way to do that:

1. Setup a float variable, ***t***, such that it ranges from 0. to 1.
   The line **float t = (float)i / (float)numsegs;** does this.

2. Step through as many ***t*** values as you want interpolation steps.
   The line **for(int i = 0; i <= numsegs; i++)** does this.

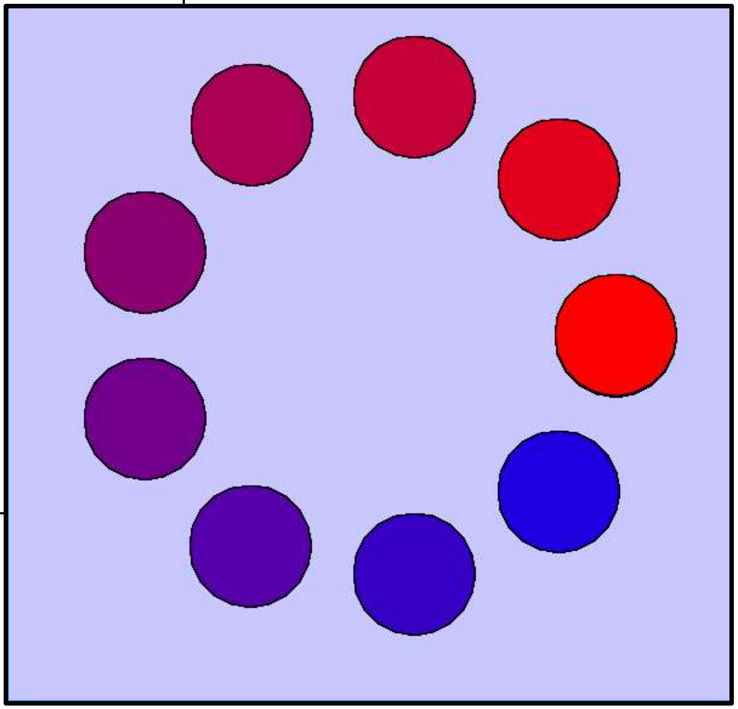
3. For each ***t***, multiply one input value by ***(1.-t)*** and multiply the other input value by **t** and add them together.
   The line **float newrad = (1.-t)*r0 + t*r1;** does this.





We Can Also Use This Same Idea to Arrange Things in a Circle and Linearly Blend Their Colors

```
int numObjects = 9;
float radius = 2.f;
float xc = 3.f;
float yc = 3.f;
int numSegs = 20;
float r = 50.f;
float dang = 2.f*F_PI / (float) ( numObjects - 1 );
float ang = 0.;
for( int i = 0; i < numObjects; i++ )
{
  float x = xc + radius * cosf(ang);
  float y = yc + radius * sinf(ang);
  float t = (float)i / (float)(numObjects-1);    // 0.-1.
  float red   = t;                               // ramp up
  float blue = 1.f - t;                          // ramp down
  glColor3f red, 0., blue );
  Circle( x, y, r, numSegs );
  ang += dang;
}
```



By Understanding what the Sine Function Looks Like, We Can Also Use it to Control Animations Based on Time

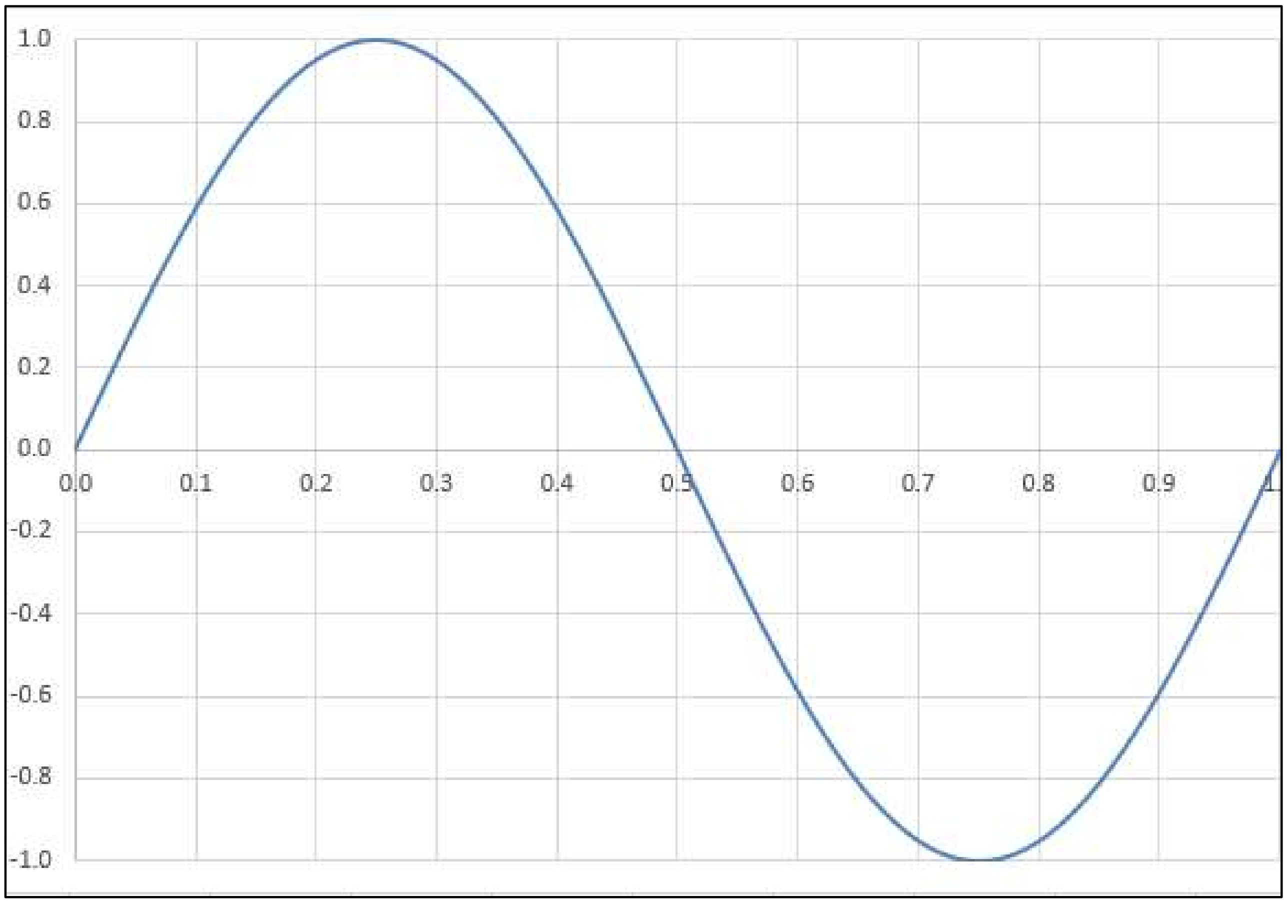
In your sample.cpp file, we have some code that looks like this:

```
float Time;                 // global variable intended to lie between [0.,1.)

. . .

const int MS_PER_CYCLE = 10000;    // 10000 milliseconds = 10 seconds

. . .

// in Animate( ):
        int ms = glutGet(GLUT_ELAPSED_TIME);
        ms %= MS_PER_CYCLE;
                        // makes the value of ms between 0 and MS_PER_CYCLE-1
        Time = (float)ms / (float)MS_PER_CYCLE;
                        // makes the value of Time between 0. and slightly less than 1.
```

By Understanding what the Sine Function Looks Like, We Can Also Use it to Control Animations Based on Time

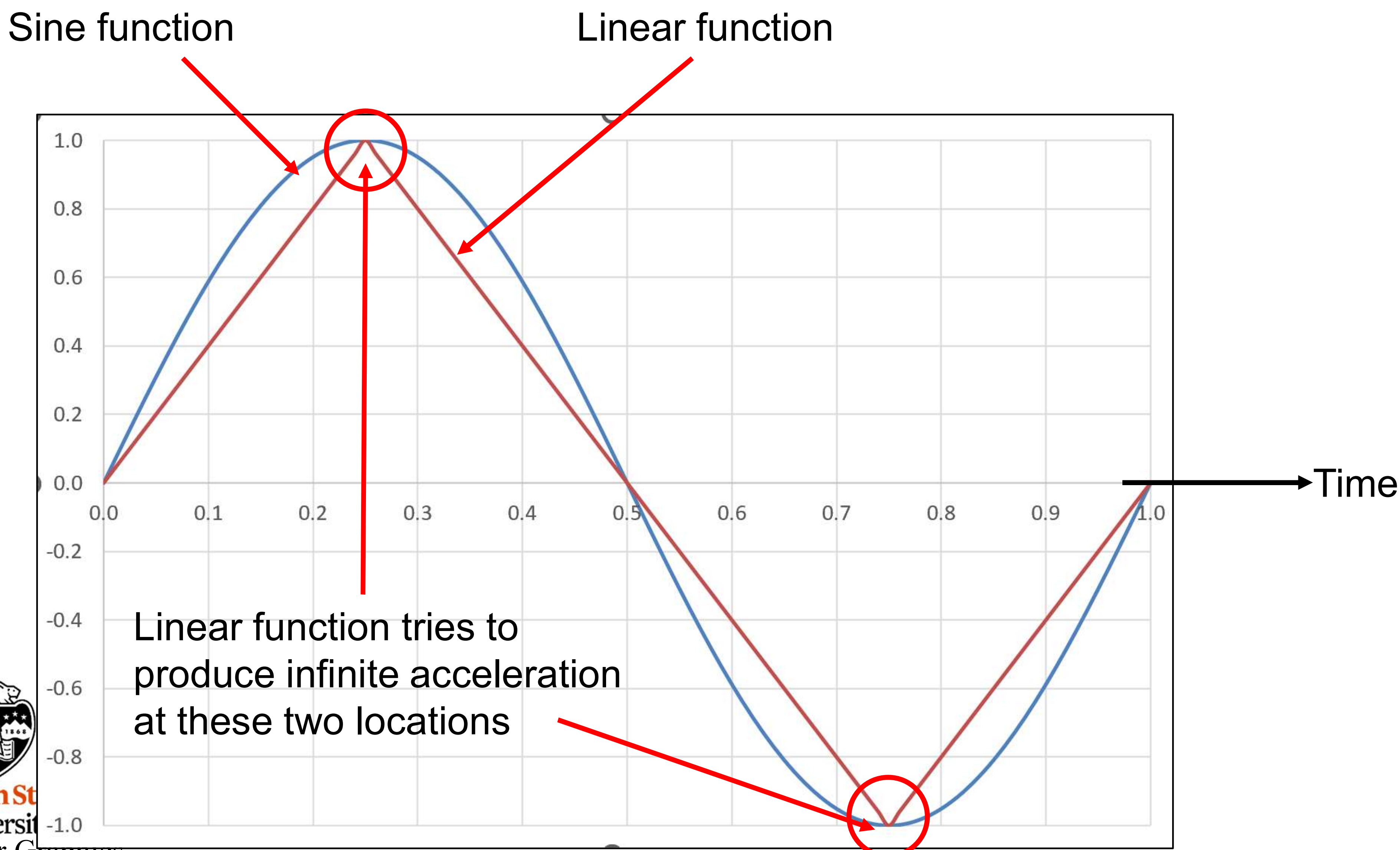
The sine function goes from -1. to +1., and does it very smoothly

$$y=\sin(2.*\pi*Time)$$



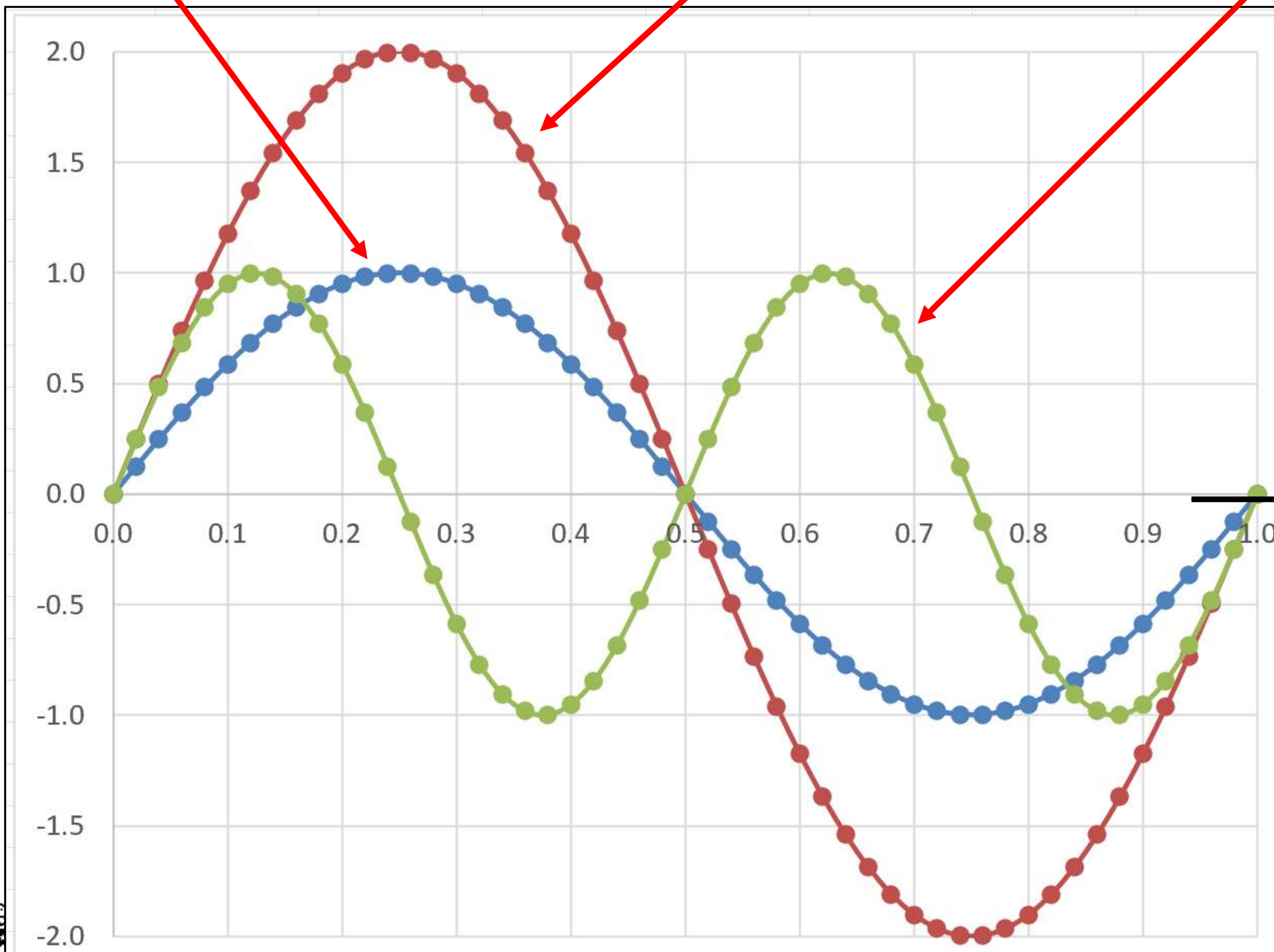
By Understanding what the Sine Function Looks Like, We Can Also Use it to Control Animations Based on Time

Sine functions produce a smoother set of motions than linear functions do (that's why we use them):



Increasing the Amplitude, Increasing the Frequency

$sin(2.* \pi * Time)$ $2.*sin(2.* \pi * Time)$ $sin(2.*(2.* \pi * Time))$



Time

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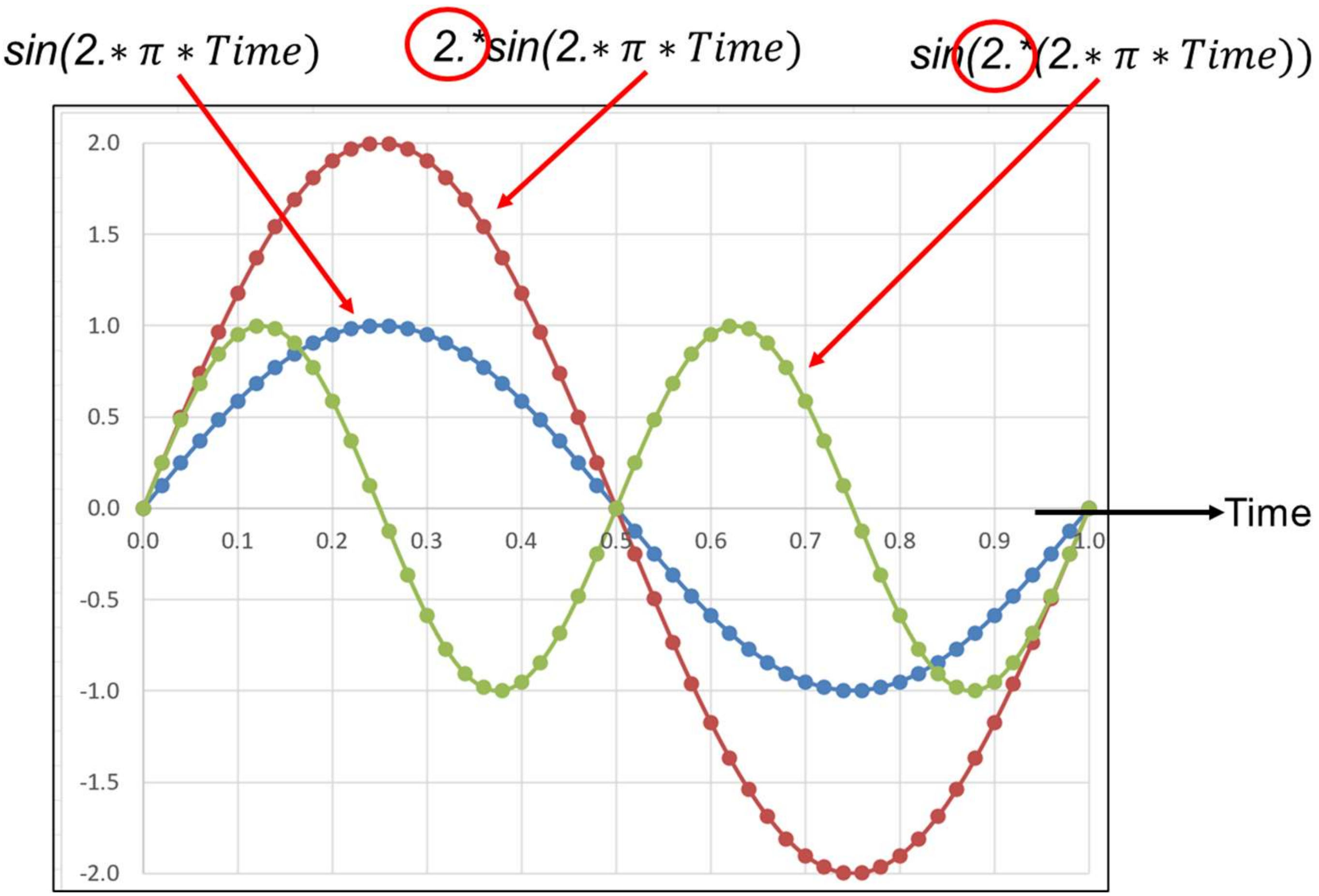
Increasing the Amplitude, Increasing the Frequency



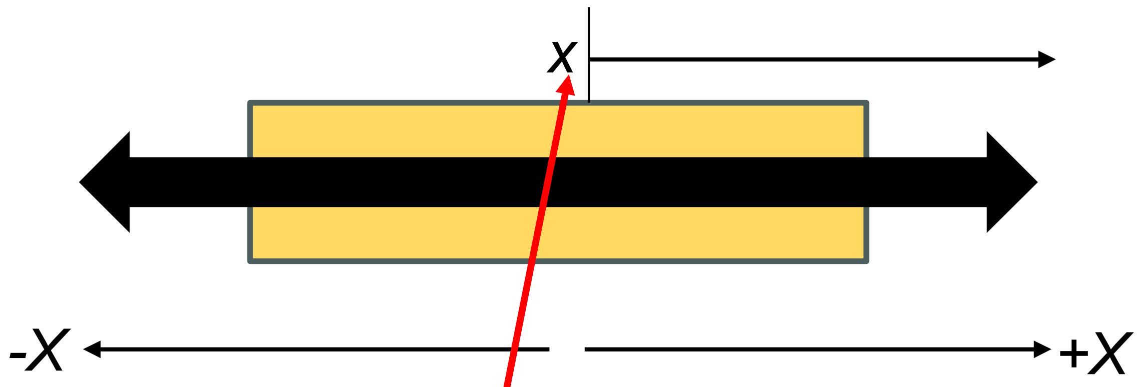
$$A*sin(F*(2.*\pi*Time))$$

Changing this number changes the Amplitude

Changing this number changes the Frequency

Oscillating Motion

Let’s say you want a block to oscillate back and forth in x:

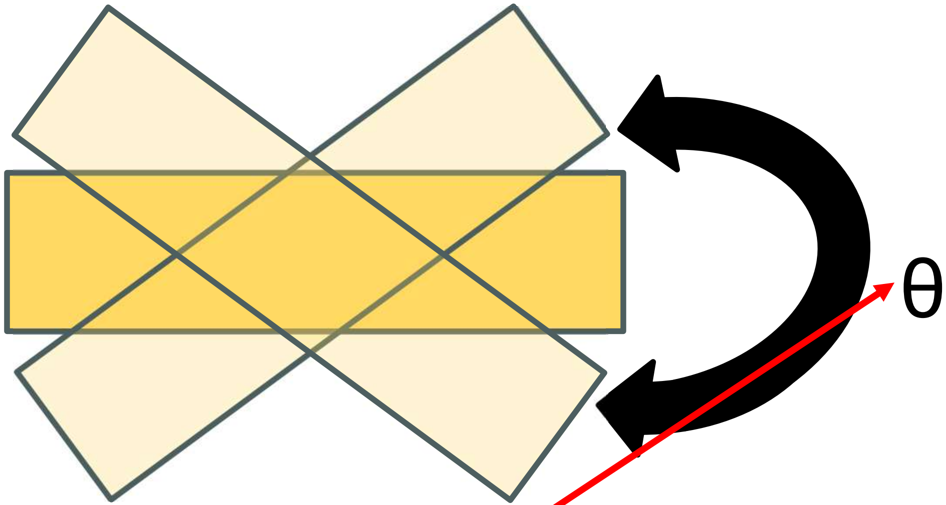


This code would cause it to do that:

```
// in Display( ):
        float x = X*sin(F*(2.∗ π ∗ Time) )
        . . .
        glTranslatef( x, 0., 0. );
        glCallList( BlockList );
```

Rocking Motion

Let's say you want a block to rock back and forth:



This code would cause it to do that:

```
// in Display( ):
        float theta = 45.f * sin(F *(2.* π * Time) )
        . . .
        glRotatef( theta,    0., 0., 1. );
        glCallList( BlockList );
```