Sines and Cosines for Animating Computer Graphics

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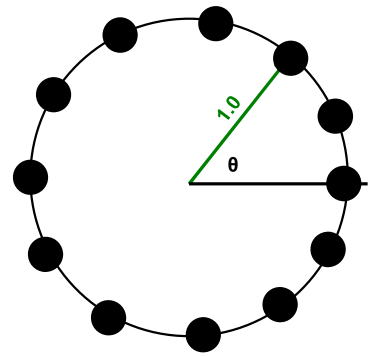
You Know about Sines and Cosines from Math, but They are Very Useful for Animating Computer Graphics

First, We Need to Understand Something about Angles:



If a circle has a radius of 1.0, then we can march around it by simply changing the angle that we call **θ.**

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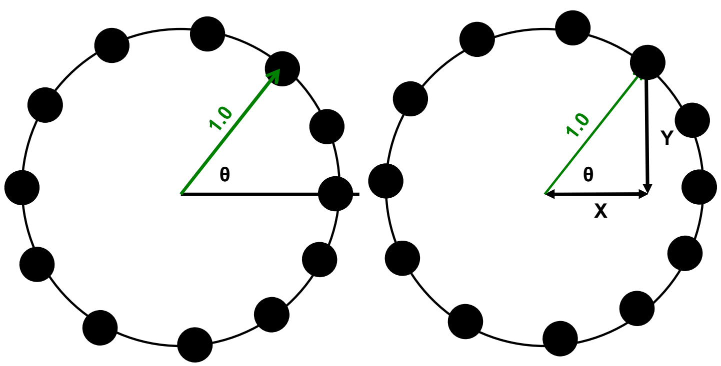
First, We Need to Understand Something about Angles



One of the things we notice is that each angle **θ** has a unique **X** and **Y** that goes with it.

These are different for each **θ**.

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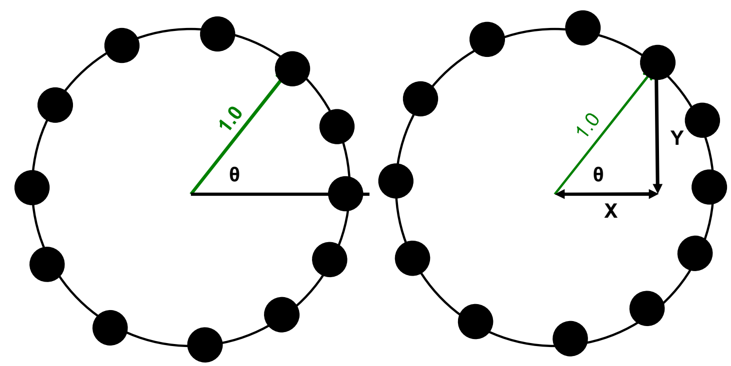
First, We Need to Understand Something about Angles



Fortunately, centuries ago, people developed tables of those X and Y values as functions of θ.

They called the X values **cosines** and the Y values **sines**. These are abbreviated cos and sin.

$\cos\theta = X$

$\sin\theta = Y$

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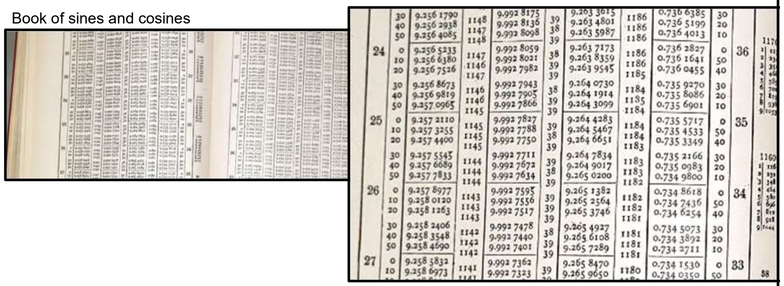
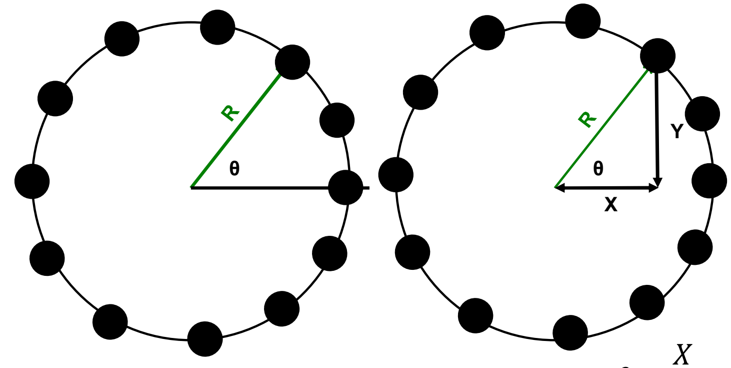
How People used to Lookup Sines and Cosines – Yuch! Fortunately We Now Have Calculators and Computers

Book of sines and cosines



Slide rule

First, We Need to Understand Something about Angles



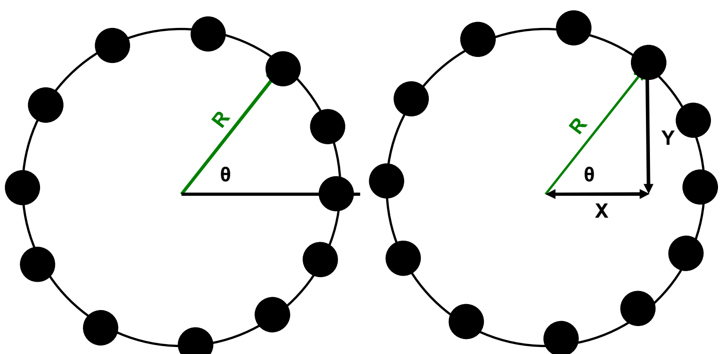
$$\cos\theta = \frac{X}{R}$$

If we were to double the radius of the circle, all of the X's and Y's would also double.

So, really the cos and sin are *ratios* of X and Y to the circle Radius

$$\sin\theta = \frac{Y}{R}$$

First, We Need to Understand Something about Angles



So, if we know the circle Radius, and we march through a bunch of θ angles, we can determine all of the X's and Y's that we need to draw a circle.

$$\cos\theta = \frac{X}{R} \qquad \sin\theta = \frac{Y}{R}$$

$$X = R * \cos\theta$$
$$Y = R * \sin\theta$$

Draw to this point

Thus, We Could Create Our Very Own Circle-Drawing Function

Circle center

Circle radius

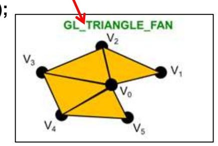
numsegs is the number of line segments making up the circumference of the circle.

numsegs=20 gives a nice circle.

5 gives a pentagon.
8 gives an octagon.
4 gives you a square. Etc.

```
void
Circle( float xc, float yc, float r, int numsegs )
{
  float dang = 2.f * F_PI / (float)numsegs;
  float ang = 0.;
  glBegin( GL_TRIANGLE_FAN );
  glVertex3f( xc, yc, 0. );

  for( int i = 0; i <= numsegs; i++)
  {
    float x = xc + r * cosf(ang);
    float y = yc + r * sinf(ang);
    glVertex3f( x, y, 0. );
    ang += dang;
  }

 glEnd( );
}
```

2π is how many radians are in a full circle



The C/C++ sin( ) and cos( ) functions use double-precision floating point.

The C/C++ sinf( ) and cosf( ) functions use single-precision floating point, and are faster.

Why 2.*PI ?

```
float dang = 2.f*F_PI / (float)numsegs;
```

*We humans commonly measure angles in **degrees**, but science and computers like to measure them in something else called **radians**.*

There are 360° in a complete circle.
There are 2π radians in a complete circle.

*The built-in cosf( ) and sinf( ) functions expect angles to be given in **radians**.*

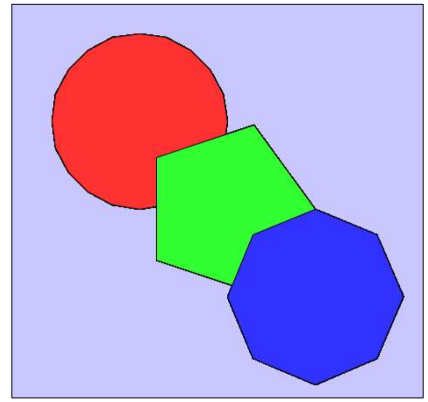
To convert between the two:

float rad = deg * (F_PI/180.f);
float deg = rad * (180.f/F_PI);

glRotatef( ) and gluPerspective( ) are the only two programming functions I can think of that use degrees. All others use radians!

Circles and Pentagons and Octagons, Oh My!

```
glColor3f( 1., 0., 0. );
Circle( 1.f, 3.f, 1.f, 20 )

glColor3f( 0., 1., 0. );
Circle( 2.f, 2.f, 1.f, 5 )

glColor3f( 0., 0., 1. );
Circle( 3.f, 1.f, 1.f, 8 )
```



Easy as π: M_PI vs. F_PI

The math.h include file has a definition of π that looks like this:

```
#define M_PI        3.14159265358979323846
```

Which will work just fine for whatever you need it for.

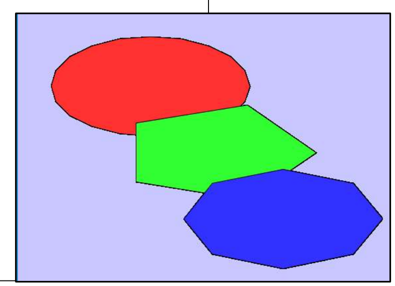
But, Visual Studio goes a little crazy complaining about mixing doubles (which is what M_PI is in) and floats (which is probably what you use most often). So, your sample code has these lines in it:

```
#define F_PI        ((float)(M_PI))        ← π
#define F_2_PI      ((float)(2.f*F_PI))    ← 2π
#define F_PI_2      ((float)(F_PI/2.f))    ← π/2
```

I use the *F_* version a lot because it keeps VS quiet. You can use either.

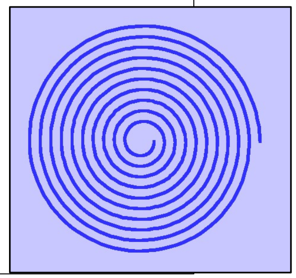
And, there is no reason the X and Y radii need to be the same…

```
void
Ellipse( float xc, float yc, float rx, float ry, int numsegs )
{
  float dang = 2.f * F_PI / (float)numsegs;
  float ang = 0.;
  glBegin( GL_TRIANGLE_FAN );
  glVertex3f( xc, yc, 0. );

  for( int i = 0; i <= numsegs; i++ )
  {
    float x = xc + rx * cosf(ang);
    float y = yc + ry * sinf(ang);
    glVertex3f( x, y, 0. );
    ang += dang;
  }

  glEnd( );
}
```



There is also no reason we can't gradually change the radius …


```
void
Spiral( float xc, float yc, float r0, float r1, int numsegs, int numturns )
{
  float dang = (float)numturns * 2.f * F_PI / (float)numsegs;
  float ang = 0.;
  glBegin( GL_LINE_STRIP );

  for( int i = 0; i <= numsegs; i++ )
  {
    float t = (float)i / (float)numsegs;     // 0.-1.
    float newrad = (1.-t)*r0 + t*r1;
                   // linearly interpolate from r0 to r1
    float x = xc + newrad * cosf(ang);
    float y = yc + newrad * sinf(ang);
    glVertex3f( x, y, 0. );
    ang += dang;
  }

  glEnd( );
}
```



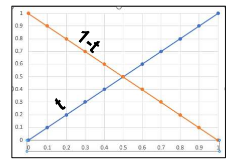
Parametric Linear Interpolation (Blending)

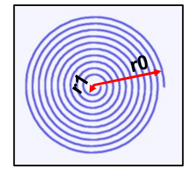
What's this code all about?

```
float t = (float)i / (float)numsegs;          // 0.-1.
float newrad = (1.-t)*r0 + t*r1;
```

In computer graphics, we do a lot of linear interpolation between two input values. Here is a good way to do that:

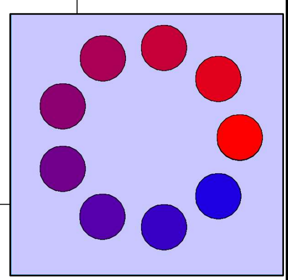
1. Setup a float variable, ***t***, such that it ranges from 0. to 1.
   The line **float t = (float)i / (float)numsegs;** does this.
2. Step through as many ***t*** values as you want interpolation steps.
   The line **for(int i = 0; i <= numsegs; i++)** does this.
3. For each ***t***, multiply one input value by ***(1.-t)*** and multiply the other input value by **t** and add them together.
   The line **float newrad = (1.-t)*r0 + t*r1;** does this.





We Can Also Use This Same Idea to Arrange Things in a Circle and Linearly Blend Their Colors


```
int numObjects = 9;
float radius = 2.f;
float xc = 3.f;
float yc = 3.f;
int numSegs = 20;
float r = 50.f;
float dang = 2.f*F_PI / (float) ( numObjects - 1 );
float ang = 0.;
for( int i = 0; i < numObjects; i++ )
{
  float x = xc + radius * cosf(ang);
  float y = yc + radius * sinf(ang);
  float t = (float)i / (float)(numObjects-1);   // 0.-1.
  float red  = t;                               // ramp up
  float blue = 1.f - t;                         // ramp down
  glColor3f red, 0., blue );
  Circle( x, y, r, numSegs );
  ang += dang;
}
```



By Understanding what the Sine Function Looks Like, We Can Also Use it to Control Animations Based on Time

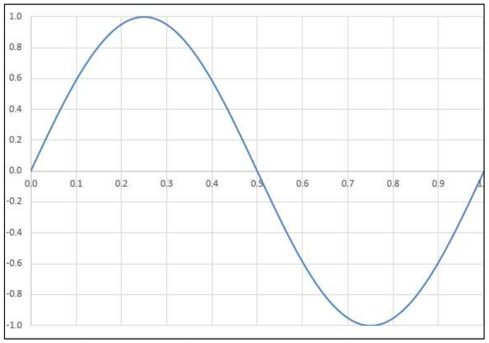
In your sample.cpp file, we have some code that looks like this:

```
float Time;                      // global variable intended to lie between [0.,1.)

. . .

const int MS_PER_CYCLE = 10000;     // 10000 milliseconds = 10 seconds

. . .

// in Animate( ):
      int ms = glutGet(GLUT_ELAPSED_TIME);
      ms %= MS_PER_CYCLE;
                     // makes the value of ms between 0 and MS_PER_CYCLE-1
      Time = (float)ms / (float)MS_PER_CYCLE;
                     // makes the value of Time between 0. and slightly less than 1.
```

By Understanding what the Sine Function Looks Like, We Can Also Use it to Control Animations Based on Time

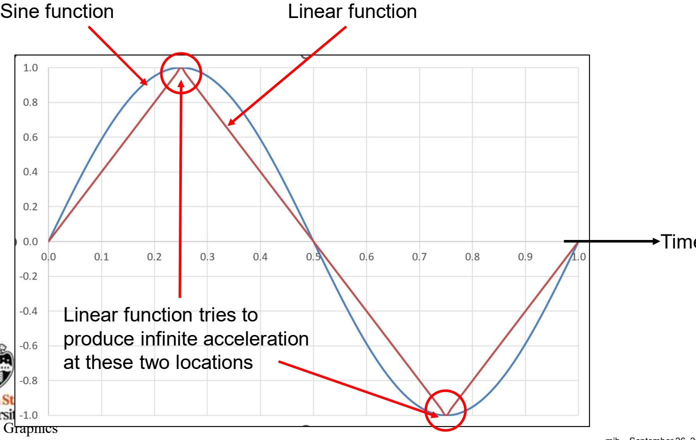
The sine function goes from -1. to +1., and does it very smoothly

$$y = sin(2. * \pi * Time)$$

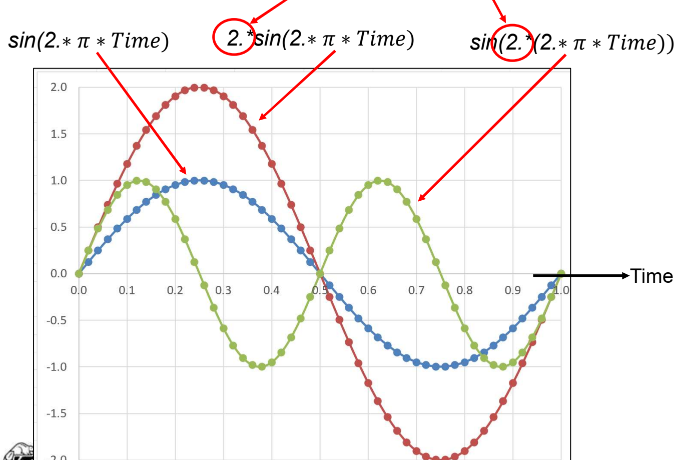


By Understanding what the Sine Function Looks Like, We Can Also Use it to Control Animations Based on Time

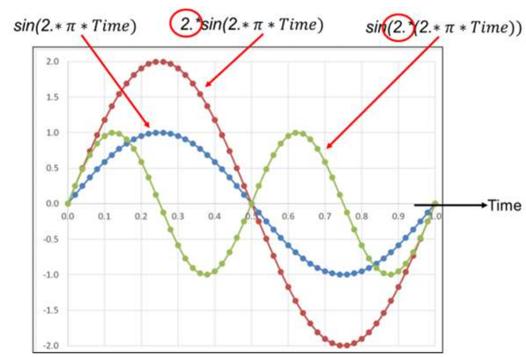
Sine functions produce a smoother set of motions than linear functions do (that's why we use them):



Increasing the Amplitude, Increasing the Frequency



Increasing the Amplitude, Increasing the Frequency



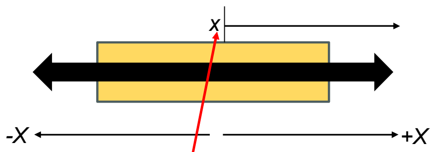
$$A*sin(F*(2. * \pi * Time))$$

Changing this number changes the Amplitude

Changing this number changes the Frequency

Oscillating Motion

Let's say you want a block to oscillate back and forth in x:



This code would cause it to do that:

```
// in Display( ):
        float x = X*sin(F*(2.∗ π ∗ Time) )
        . . .
        glTranslatef( x, 0., 0. );
        glCallList( BlockList );
```

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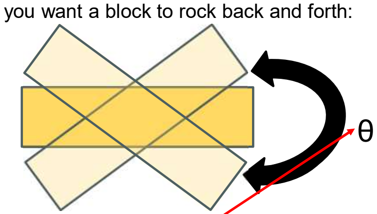
Rocking Motion

Let's say you want a block to rock back and forth:



This code would cause it to do that:

```
// in Display( ):
        float theta = 45.f ∗ sin(F ∗(2.∗ π ∗ Time) )
        . . .
        glRotatef( theta,    0., 0., 1. );
        glCallList( BlockList );
```

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