Forward Kinematics

(aka, Hierarchical Transformations)

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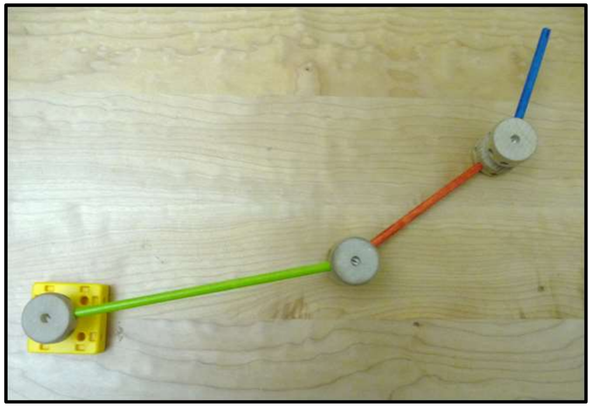
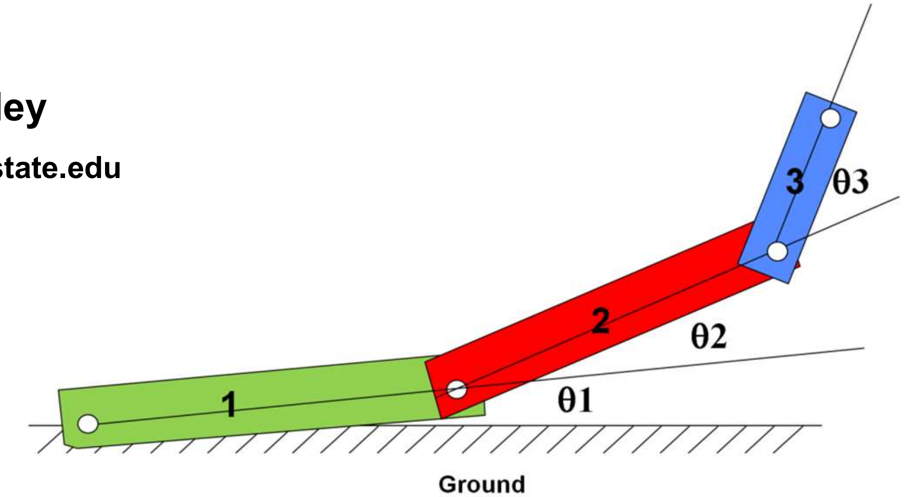
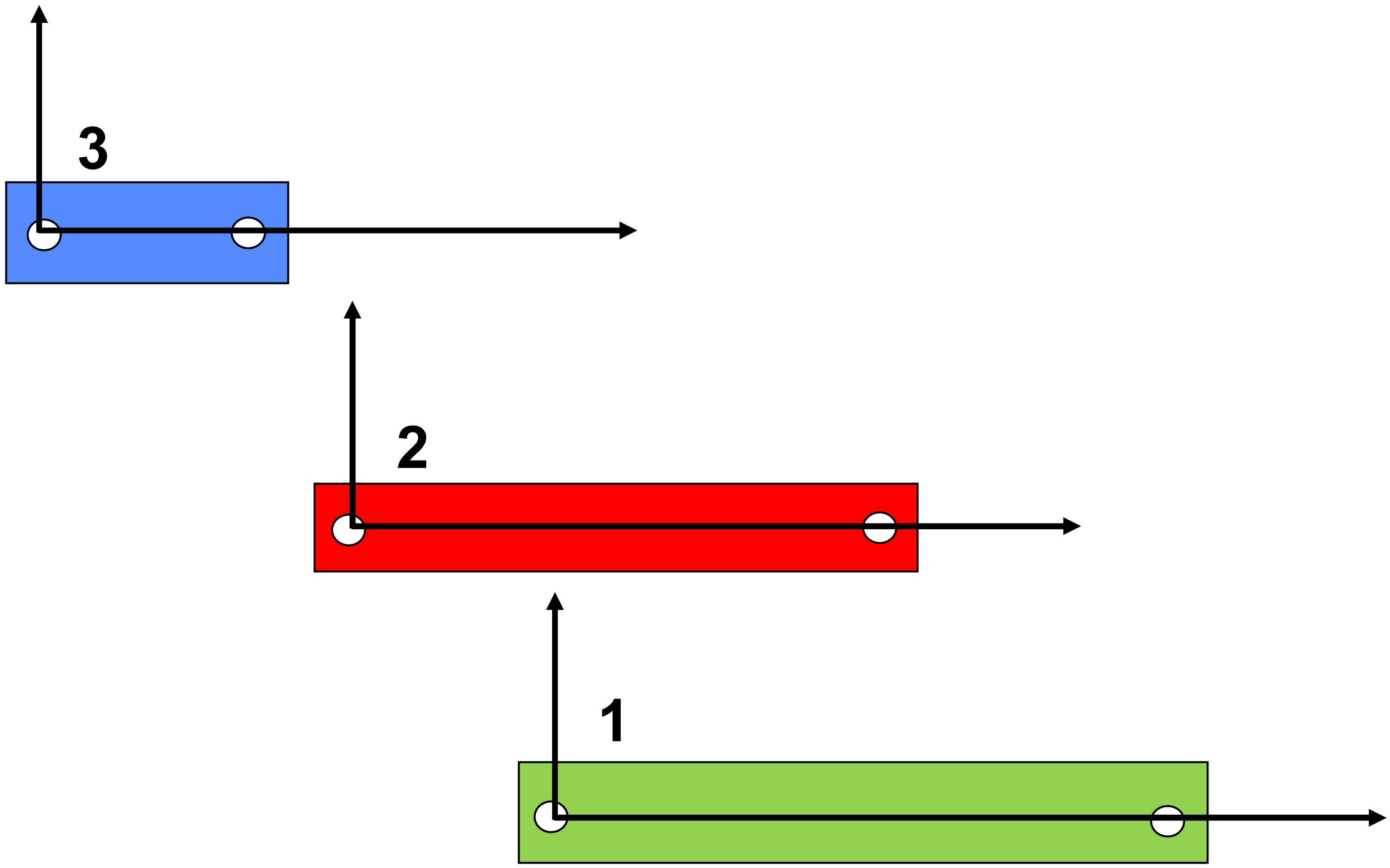
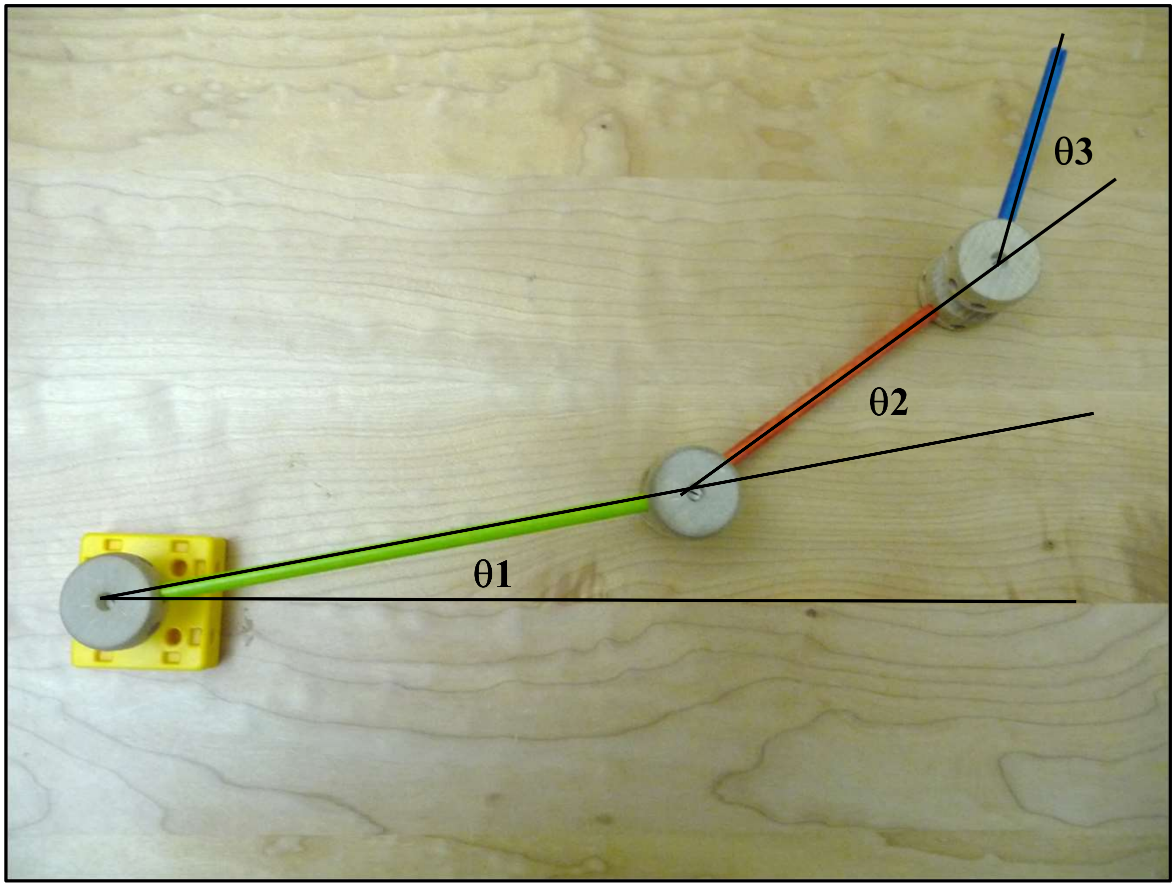




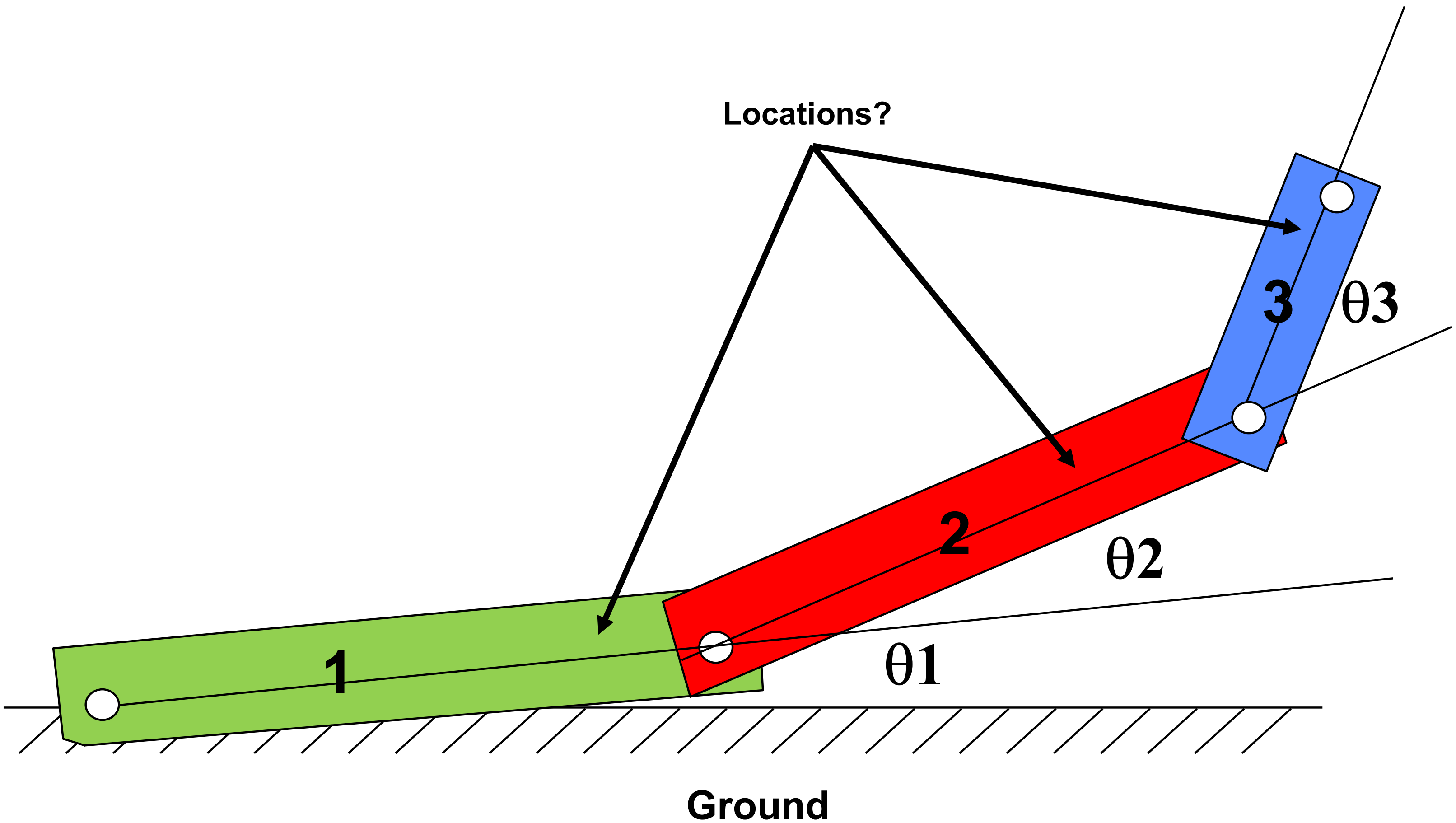
Forward Kinematics:
You Start with Separate Pieces, all Defined in their Own Local Coordinate System



Forward Kinematics:
Hook the Pieces Together, Change Parameters, Things Move
(All Children Understand This)



Forward Kinematics: Where do the Pieces Move To?



Positioning Part #1 With Respect to Ground

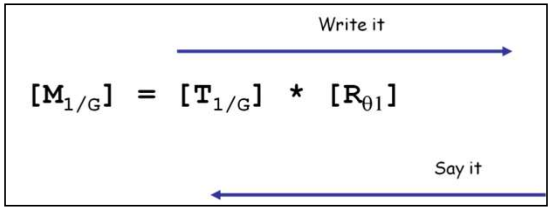
1. Rotate by Θ1
2. Translate by $T_{1/G}$

Write it →

$$\left[M_{1/G}\right] = \left[T_{1/G}\right] * \left[R_{\theta 1}\right]$$

← Say it

Why Do We Say it Right-to-Left?



It's because computer graphics has adopted the convention that the coordinates are multiplied on the right side of the matrix:

$$\begin{Bmatrix} x' \\ y' \\ z' \\ 1 \end{Bmatrix} = \begin{bmatrix} A & B & C & D \\ E & F & G & H \\ I & J & K & L \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{Bmatrix} x \\ y \\ z \\ 1 \end{Bmatrix}$$

$$\begin{Bmatrix} x' \\ y' \\ z' \\ 1 \end{Bmatrix} = \left[M_{1/G} \right] \begin{Bmatrix} x \\ y \\ z \\ 1 \end{Bmatrix} = \left[T_{1/G} \right] * \left[R_{\theta 1} \right] * \begin{Bmatrix} x \\ y \\ z \\ 1 \end{Bmatrix}$$

So the right-most transformation in the sequence multiplies the (x,y,z,1) *first* and the left-most transformation multiples it *last*

Positioning Part #2 With Respect to Ground

1. Rotate by Θ2
2. Translate the length of part 1
3. Rotate by Θ1
4. Translate by $T_{1/G}$

Write it

$$[M_{2/G}] = [T_{1/G}]*[R_{\theta 1}]*[T_{2/1}]*[R_{\theta 2}]$$

$$[M_{2/G}] = [M_{1/G}]*[M_{2/1}]$$

Say it

Positioning Part #3 With Respect to Ground

1. Rotate by Θ3
2. Translate the length of part 2
3. Rotate by Θ2
4. Translate the length of part 1
5. Rotate by Θ1
6. Translate by $T_{1/G}$

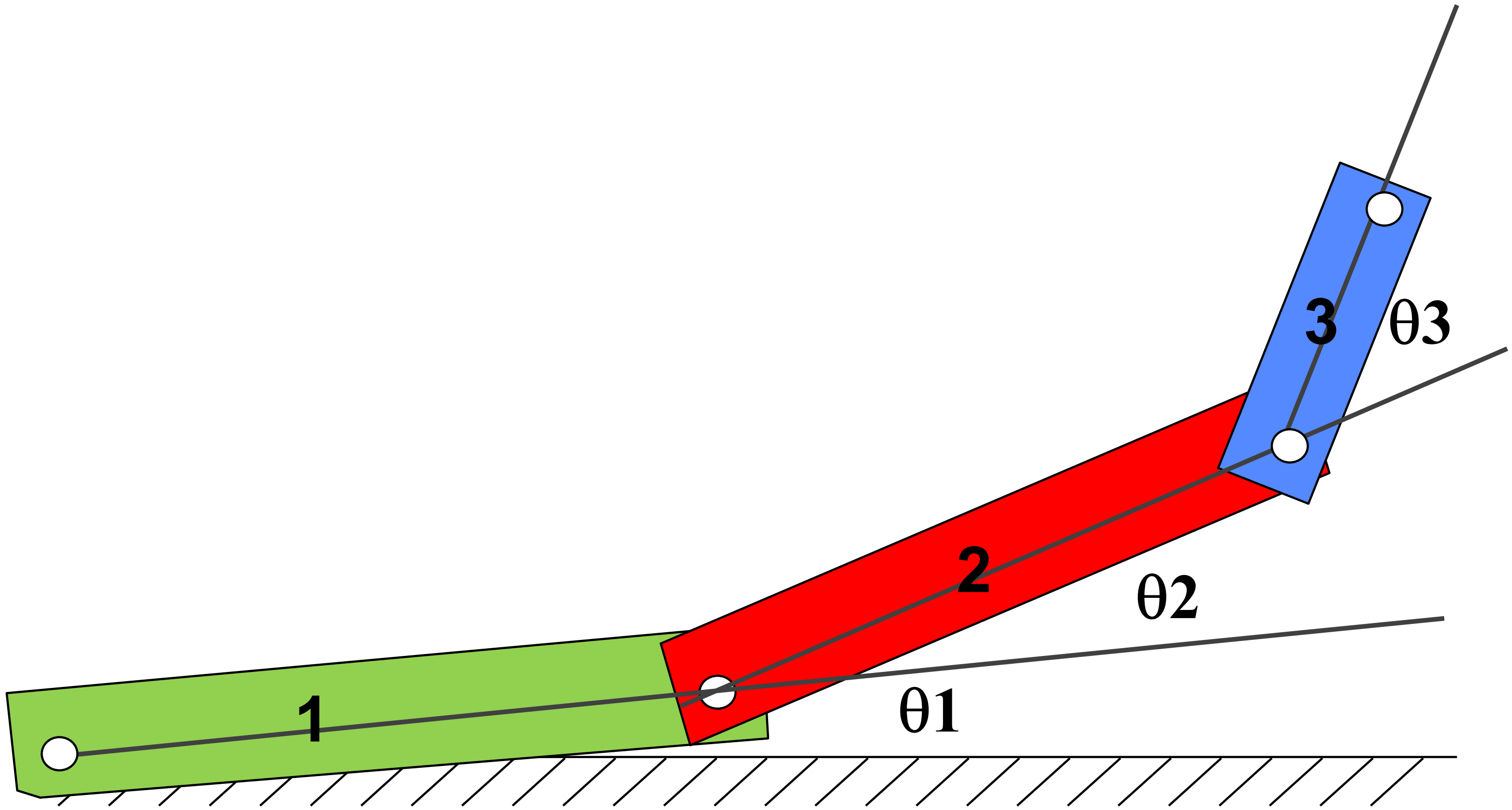
Write it

$$[M_{3/G}] = [T_{1/G}]*[R_{\theta 1}]*[T_{2/1}]*[R_{\theta 2}]*[T_{3/2}]*[R_{\theta 3}]$$

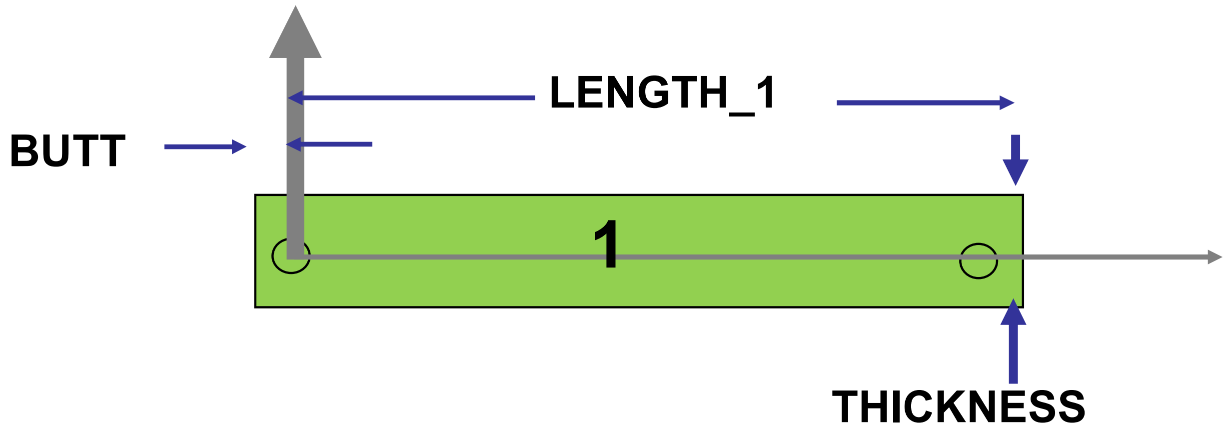
$$[M_{3/G}] = [M_{1/G}]*[M_{2/1}]*[M_{3/2}]$$

Say it

Sample Program



Sample Program, using OpenGL's Built-in Transformation Concatenation




```
DrawLinkOne( )
{
    glColor3f( 1., 0., 0. );  // red, green blue
    glBegin( GL_QUADS );
        glVertex2f(    -BUTT, -THICKNESS/2 );
        glVertex2f( LENGTH_1, -THICKNESS/2 );
        glVertex2f( LENGTH_1,  THICKNESS/2 );
        glVertex2f(    -BUTT,  THICKNESS/2 );
    glEnd( );
}
```

Sample Program

Write it

```
DrawMechanism( float θ1, float θ2, float θ3 )
{
    glPushMatrix( );
        glRotatef( θ1,  0., 0., 1. );
        glColor3f( 1., 0., 0. );
        DrawLinkOne( );

        glTranslatef( LENGTH_1, 0., 0. );
        glRotatef( θ2,  0., 0., 1. );
        glColor3f( 0., 1., 0. );
        DrawLinkTwo( );

        glTranslatef( LENGTH_2, 0., 0. );
        glRotatef( θ3,  0., 0., 1. );
        glColor3f( 0., 0., 1. );
        DrawLinkThree( );
    glPopMatrix( );
}
```

Say it

$$[M_{1/G}] = [T_{1/G}] * [R_{\theta 1}]$$

$$[M_{2/G}] = [T_{1/G}] * [R_{\theta 1}] * [T_{2/1}] * [R_{\theta 2}]$$

$$[M_{3/G}] = [T_{1/G}] * [R_{\theta 1}] * [T_{2/1}] * [R_{\theta 2}] * [T_{3/2}] * [R_{\theta 3}]$$

Sample Program

```
DrawMechanism( float θ1, float θ2, float θ3 )
{
    glPushMatrix( );
        glRotatef( θ1,  0., 0., 1. );
        glColor3f( 1., 0., 0. );
        DrawLinkOne( );

        glTranslatef( LENGTH_1, 0., 0. );
        glRotatef( θ2,  0., 0., 1. );
        glColor3f( 0., 1., 0. );
        DrawLinkTwo( );

        glTranslatef( LENGTH_2, 0., 0. );
        glRotatef( θ3,  0., 0., 1. );
        glColor3f( 0., 0., 1. );
        DrawLinkThree( );
    glPopMatrix( );
}
```

$[M_{1/G}] = [T_{1/G}] * [R_{\theta 1}]$

$[M_{2/G}] = [T_{1/G}] * [R_{\theta 1}] * [T_{2/1}] * [R_{\theta 2}]$

$[M_{3/G}] = [T_{1/G}] * [R_{\theta 1}] * [T_{2/1}] * [R_{\theta 2}] * [T_{3/2}] * [R_{\theta 3}]$

Sample Program

Where in the window to display (pixels) → glViewport

Viewing Info: field of view angle, x:y aspect ratio, near, far → gluPerspective

Set the eye position → gluLookAt

```
glViewport( 100, 100,   500, 500 );

glMatrixMode( GL_PROJECTION );
glLoadIdentity( );
gluPerspective( 90., 1.0, 1.,  10. );

glMatrixMode( GL_MODELVIEW );
glLoadIdentity( );

done = FALSE;
while( ! done )
{
    <<  Determine θ1, θ2, θ3  >>
    glPushMatrix();
    gluLookAt( eyex, eyey, eyez,
               centerx, centery, centerz,
               upx, upy, upz );
    DrawMechanism( θ1, θ2, θ3 );
    glPopMatrix();
}
```

Another Great Example of Hierarchical Transformations