Simple Keyframe Animation for Scientific and Engineering Conceptualization

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The Problem

• Sometimes “visualization” means explaining an idea or a concept or a process to someone

• Often, this idea is not data-driven or equation-driven, but rather is general concept-driven

• How can we create such an animation without doing a lot of work?

Approaches to Animation

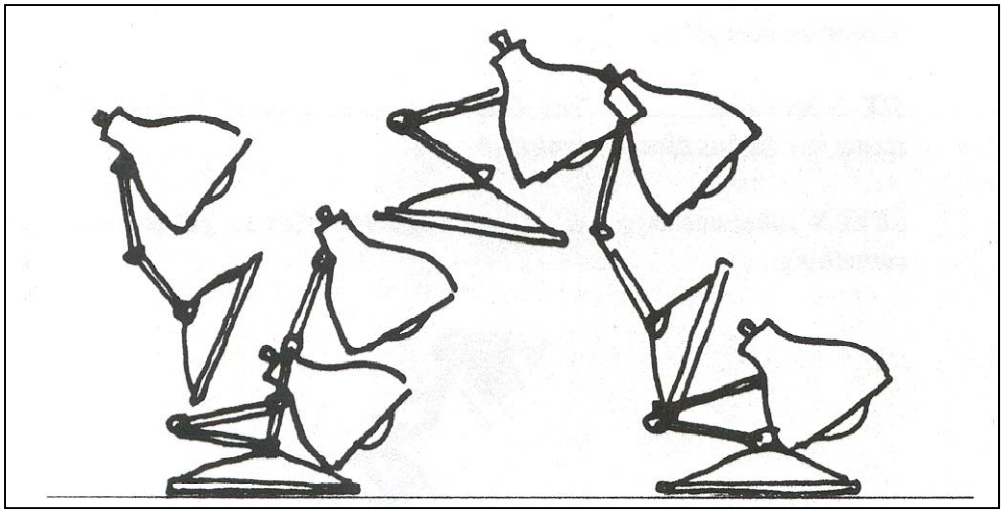
1. Use the laws of physics
2. Use functional (goal-driven) animation
3. Use keyframing

Keyframing

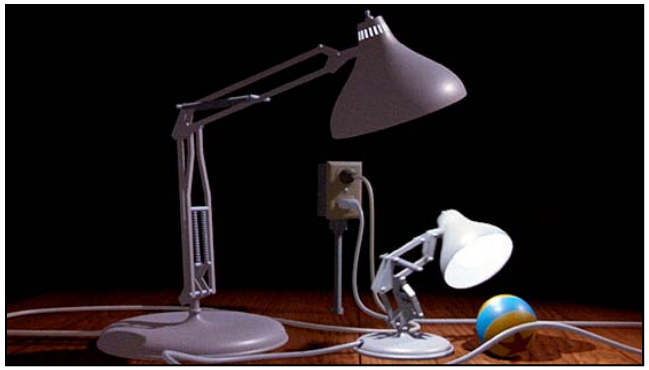
Keyframing involves creating certain key positions for the objects in the scene, and then later developing the animation frames in between the key frames.

In traditional animation, the key frames were developed by the senior animators, and the in-between frames were developed by the junior animators.

In our case, you are going to be the senior animator, and the computer will do the in-betweening.

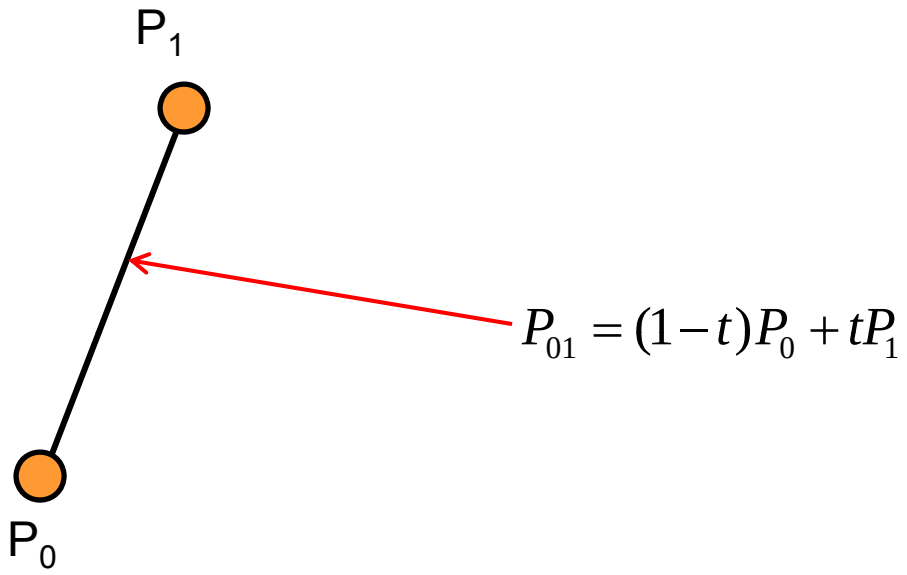


Pixar



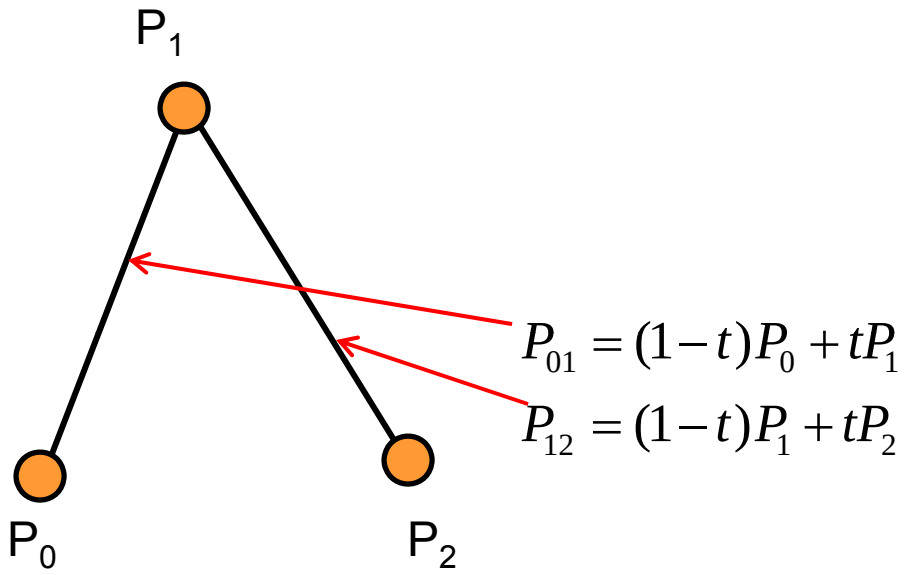
But, first we need to look into the mathematics of smooth curves . . .

Bézier Curves: the Derivation



1 1

Bézier Curves: the Derivation

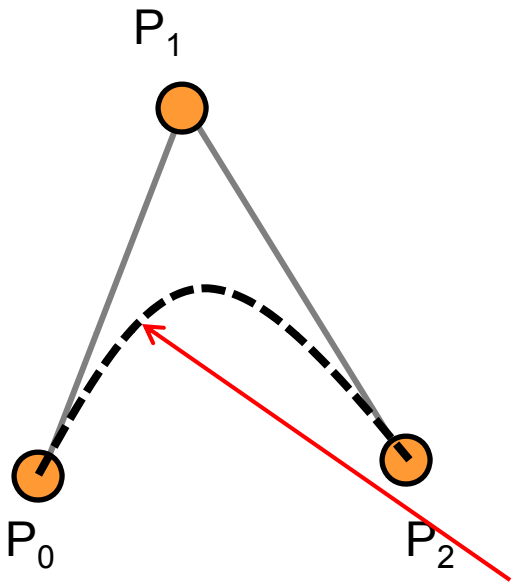


$$P_{01} = (1-t)P_0 + tP_1$$

$$P_{12} = (1-t)P_1 + tP_2$$

1 1

Bézier Curves: the Derivation



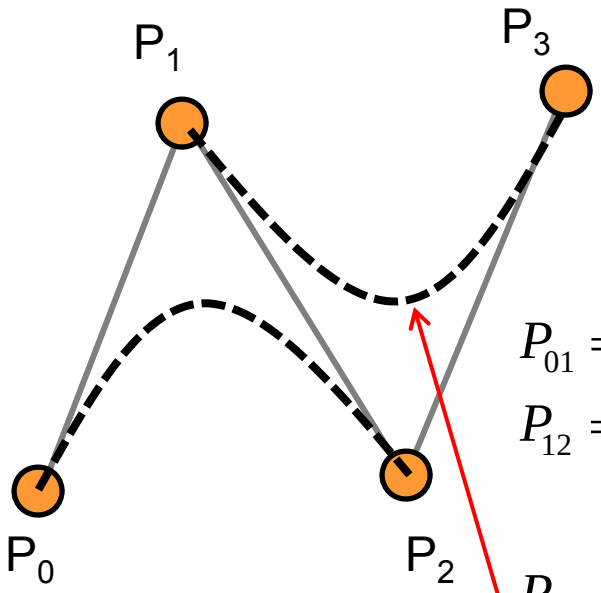
$$P_{01} = (1-t)P_0 + tP_1$$

$$P_{12} = (1-t)P_1 + tP_2$$

$$P_{012} = (1-t)P_{01} + tP_{12} = (1-t)^2 P_0 + 2t(1-t)P_1 + t^2 P_2$$

1 1
1 2 1

Bézier Curves: the Derivation



$$P_{01} = (1-t)P_0 + tP_1$$

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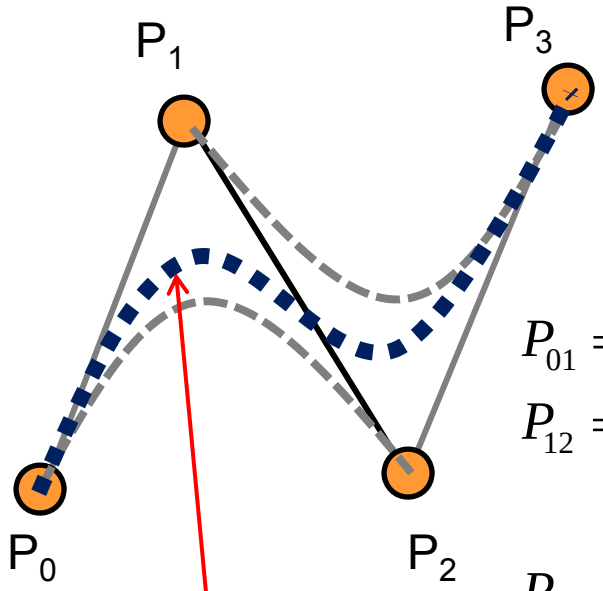
$$P_{012} = (1-t)P_{01} + tP_{12} = (1-t)^2 P_0 + 2t(1-t)P_1 + t^2 P_2$$

$$P_{123} = (1-t)P_{12} + tP_{23} = (1-t)^2 P_1 + 2t(1-t)P_2 + t^2 P_3$$

1 1
1 2 1

Bézier Curves: the Derivation



$$P_{01} = (1-t)P_0 + tP_1$$
$$P_{12} = (1-t)P_1 + tP_2$$

$$\begin{array}{ccccc} & & 1 & & 1 \\ & 1 & & 2 & & 1 \\ 1 & & 3 & & 3 & & 1 \end{array}$$

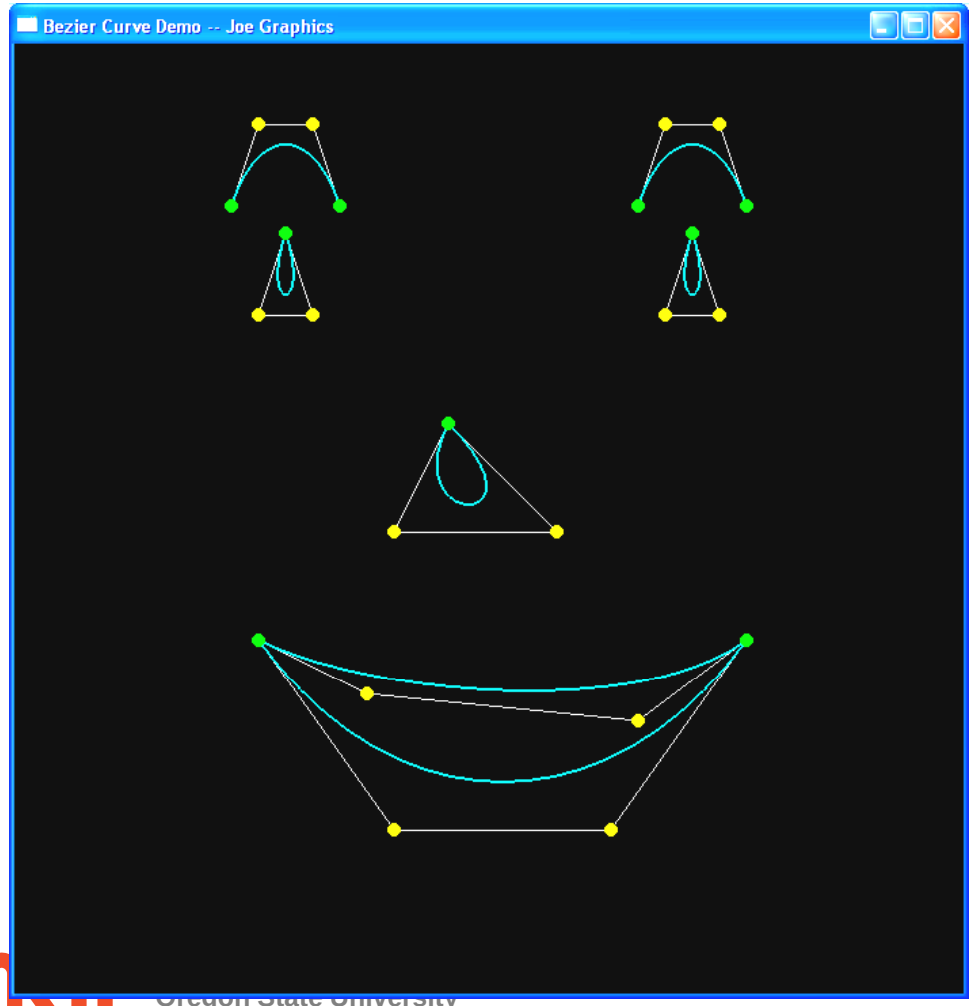
$$P_{012} = (1-t)P_{01} + tP_{12} = (1-t)^2 P_0 + 2t(1-t)P_1 + t^2 P_2$$
$$P_{123} = (1-t)P_{12} + tP_{23} = (1-t)^2 P_1 + 2t(1-t)P_2 + t^2 P_3$$

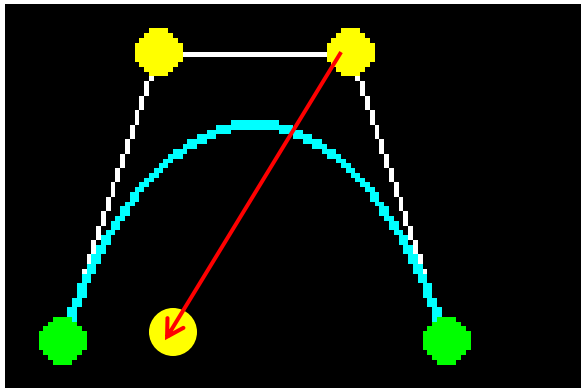
$$P_{0123} = (1-t)P_{012} + tP_{123} = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t)P_2 + t^3 P_3$$

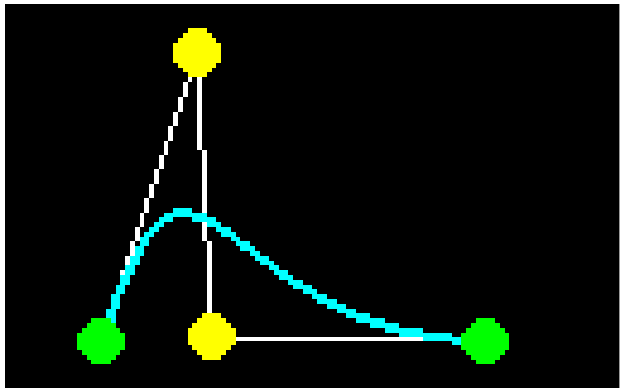
Bézier Curves: Drawing and Sculpting

$$P(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t)P_2 + t^3 P_3$$

t = 0., .02, .04, .06, …, .98, 1.0







The General Form of Cubic Curves

$$P(t) = A + Bt + Ct^2 + Dt^3$$

In this form, you need to determine 4 quantities (A, B, C, D) in order to use the equation. That means you have to provide 4 pieces of information. In the Bézier curve, this happens by specifying the 4 points.

$$P(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t)P_2 + t^3 P_3$$

Rearranging gives:

$$A = P_0$$

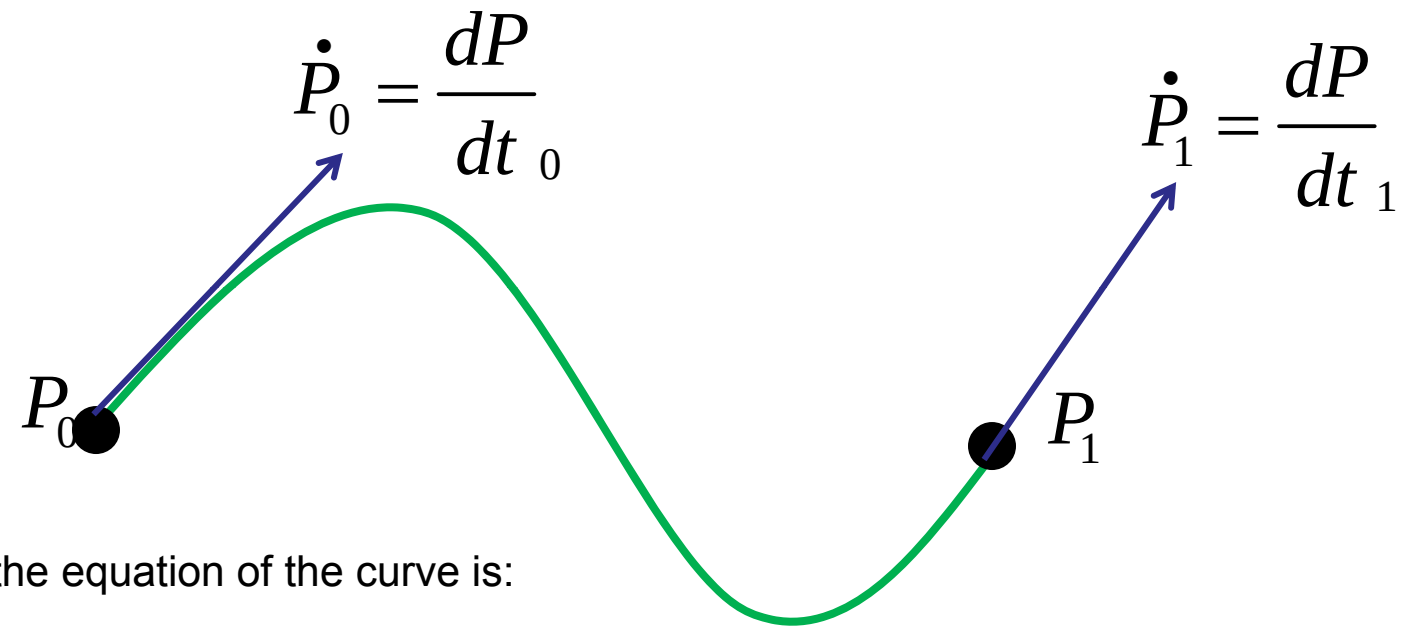
$$B = -3P_0 + 3P_1$$

$$C = 3P_0 - 6P_1 + 3P_2$$

$$D = -P_0 + 3P_1 - 3P_2 + P_3$$

Coons (also called Hermite) Cubic Curves

Another approach to specifying the 4 pieces of information would be to give a start point, an end point, a start slope, and an end slope.



If we do this, then the equation of the curve is:

$$P = A + Bt + Ct^2 + Dt^3$$

where:

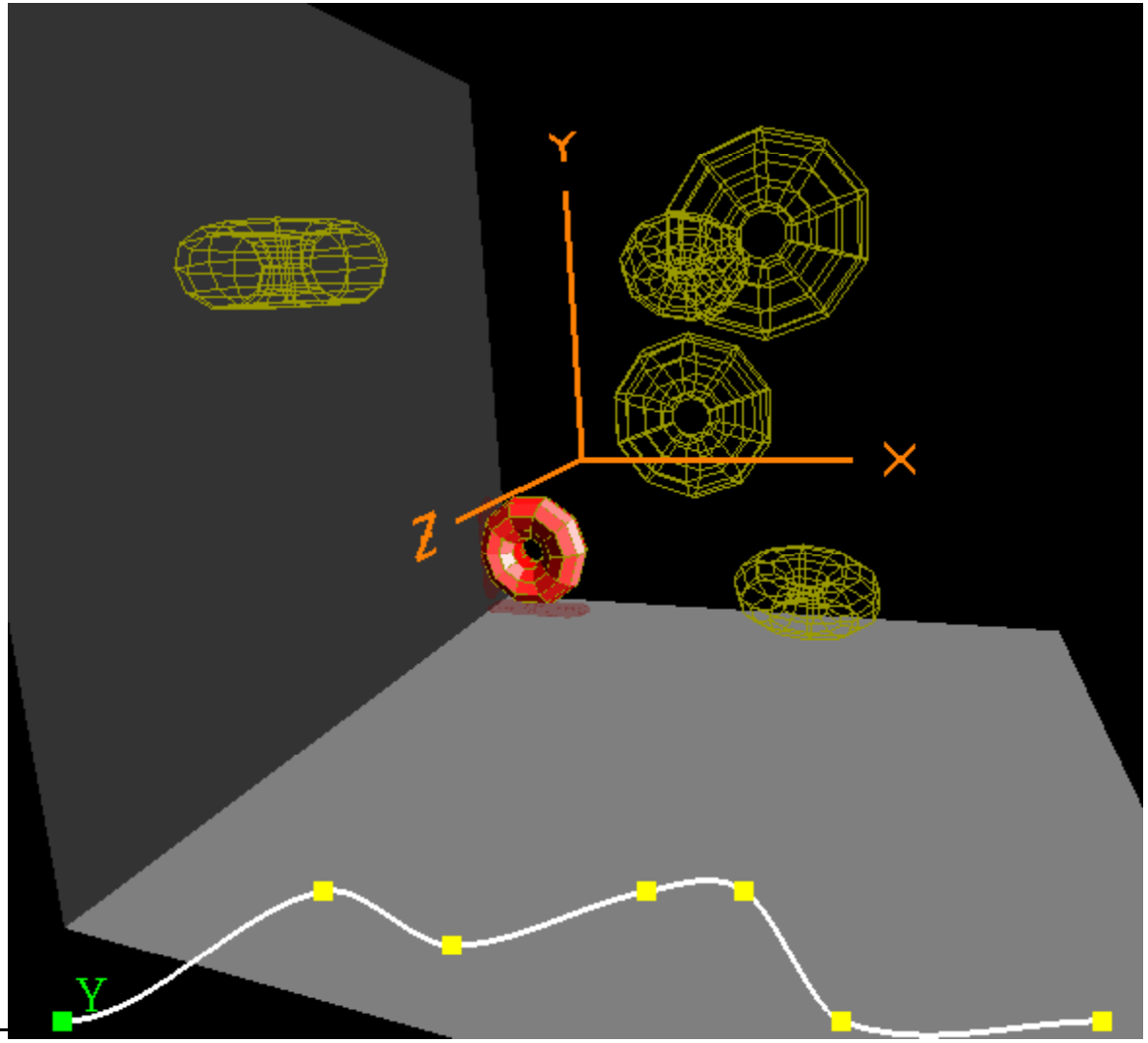
$$A = P_0$$

$$B = \dot{P}_0$$

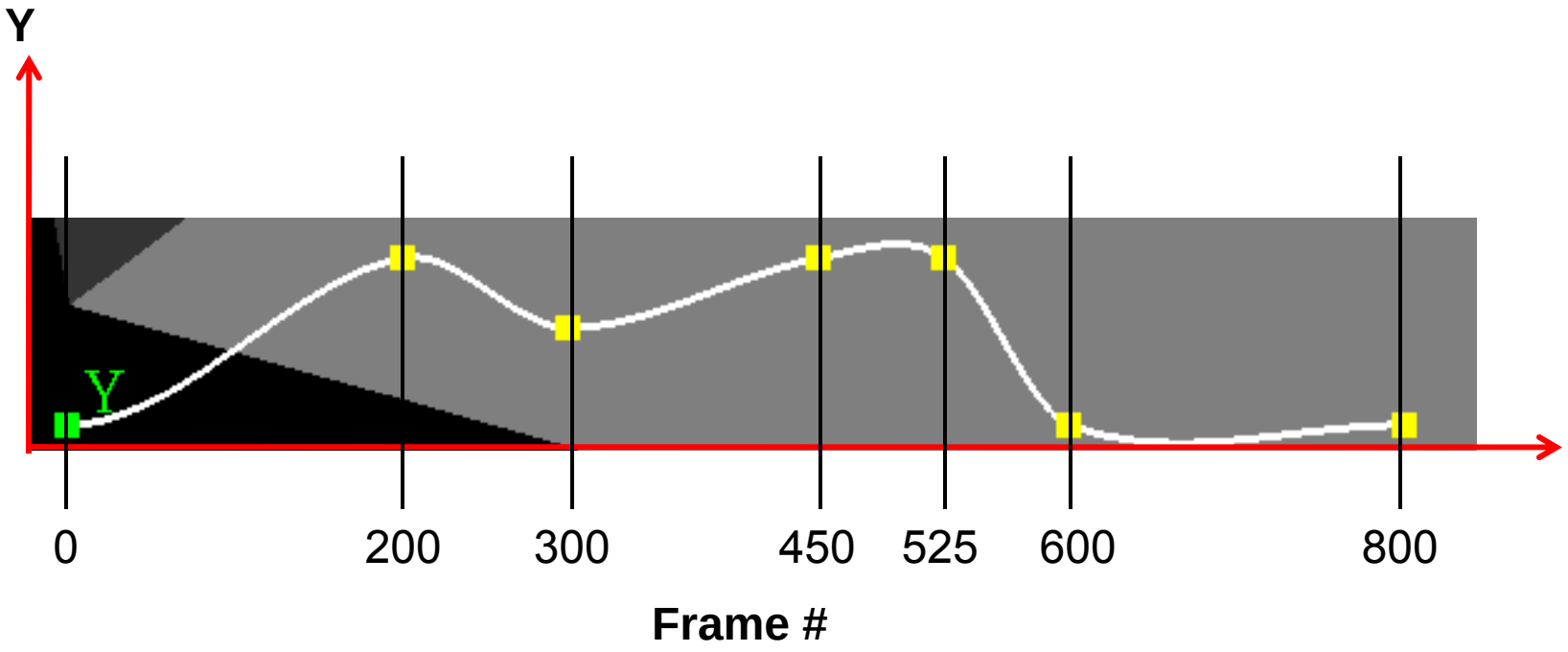
$$C = -3P_0 + 3P_1 - 2\dot{P}_0 - \dot{P}_1$$

$$D = 2P_0 - 2P_1 + \dot{P}_0 + \dot{P}_1$$

Now, Let's Apply this to the Y Translation of the Keyframe Animation



The Y vs. Frame Curve Looks Like This



Computing the End Slopes for the Y Translation

$$\frac{dY}{dF}_0 = \frac{Y_1 - Y_{-1}}{F_1 - F_{-1}}$$

$$\frac{dY}{dF}_1 = \frac{Y_2 - Y_0}{F_2 - F_0}$$

$$\frac{dF}{dt}_{0-1} = \frac{F_1 - F_0}{1. - 0.} = F_1 - F_0$$

$$\dot{Y}_0 = \frac{dY}{dt}_0 = \frac{dY}{dF}_0 \frac{dF}{dt}_{0-1}$$

$$\dot{Y}_1 = \frac{dY}{dt}_1 = \frac{dY}{dF}_1 \frac{dF}{dt}_{0-1}$$

$$A = Y_0$$

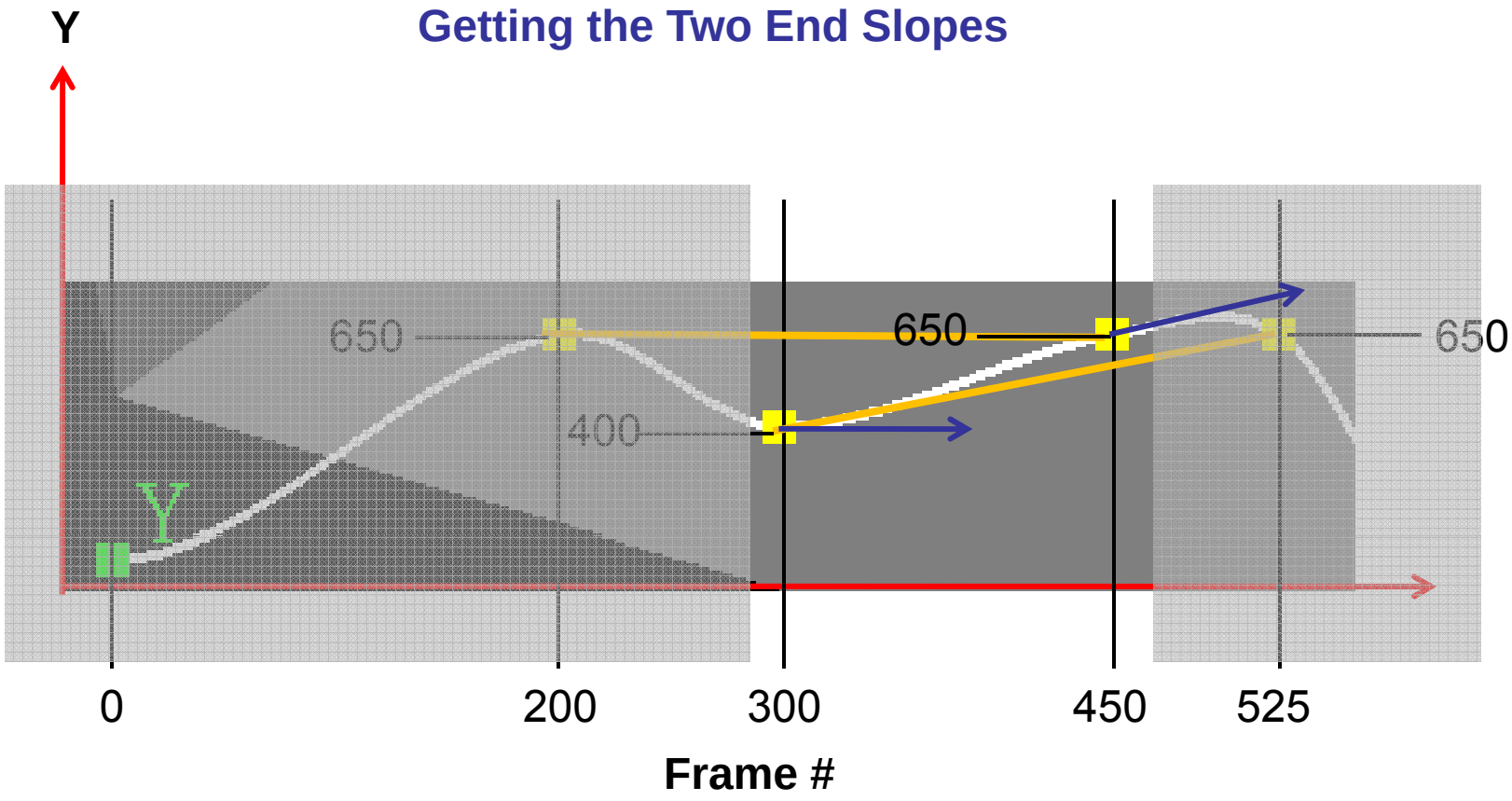
$$B = \dot{Y}_0$$

$$C = -3Y_0 + 3Y_1 - 2\dot{Y}_0 - \dot{Y}_1$$

$$D = 2Y_0 - 2Y_1 + \dot{Y}_0 + \dot{Y}_1$$

$$Y = A + Bt + Ct^2 + Dt^3$$

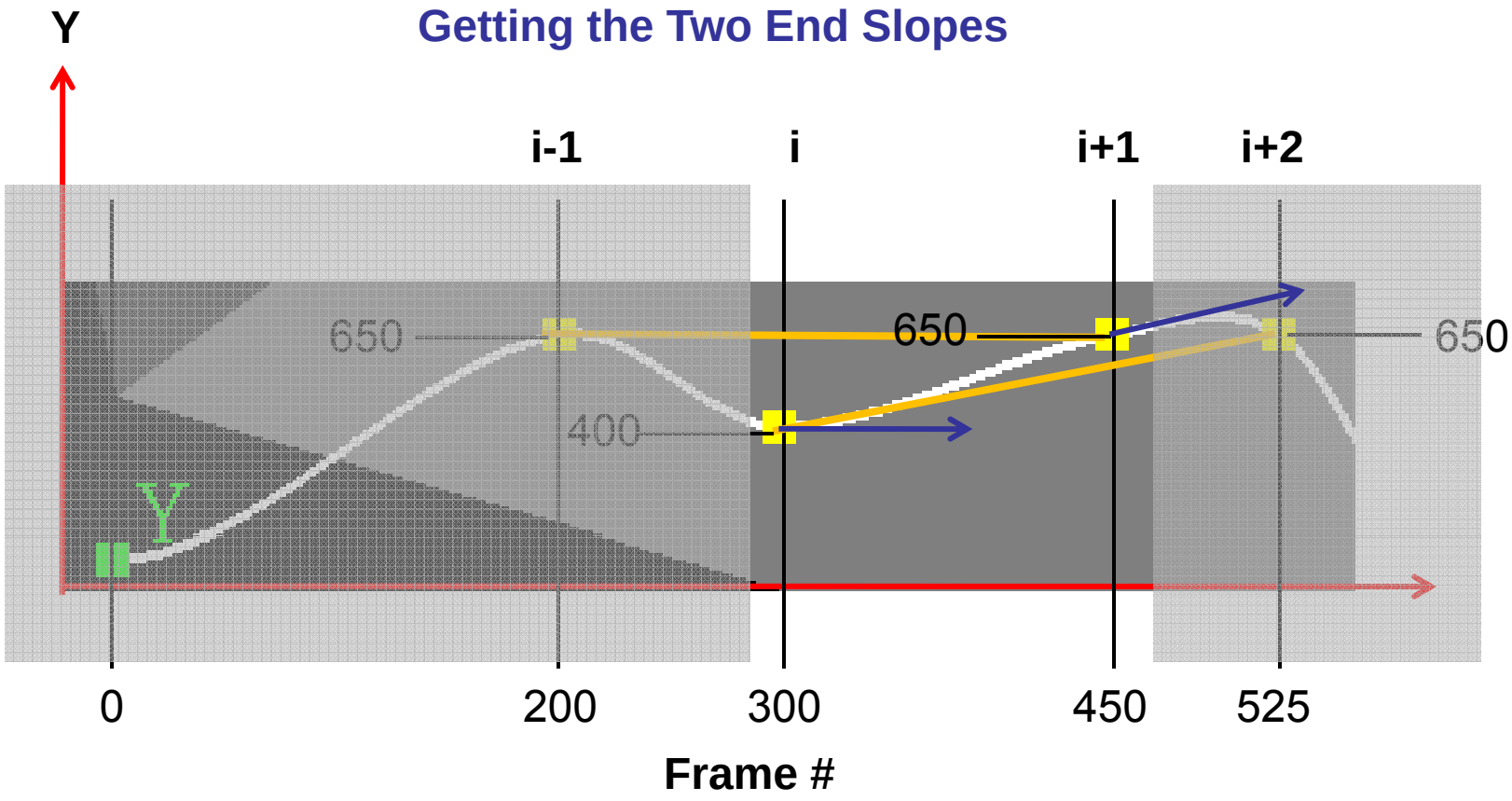
Getting the Two End Slopes



$$\frac{dY}{dt}_{i} = \frac{dY}{dF}_{i} \frac{dF}{dt}_{i \to i+1}$$

$$\frac{dY}{dt}_{i+1} = \frac{dY}{dF}_{i+1} \frac{dF}{dt}_{i \to i+1}$$

Getting the Two End Slopes

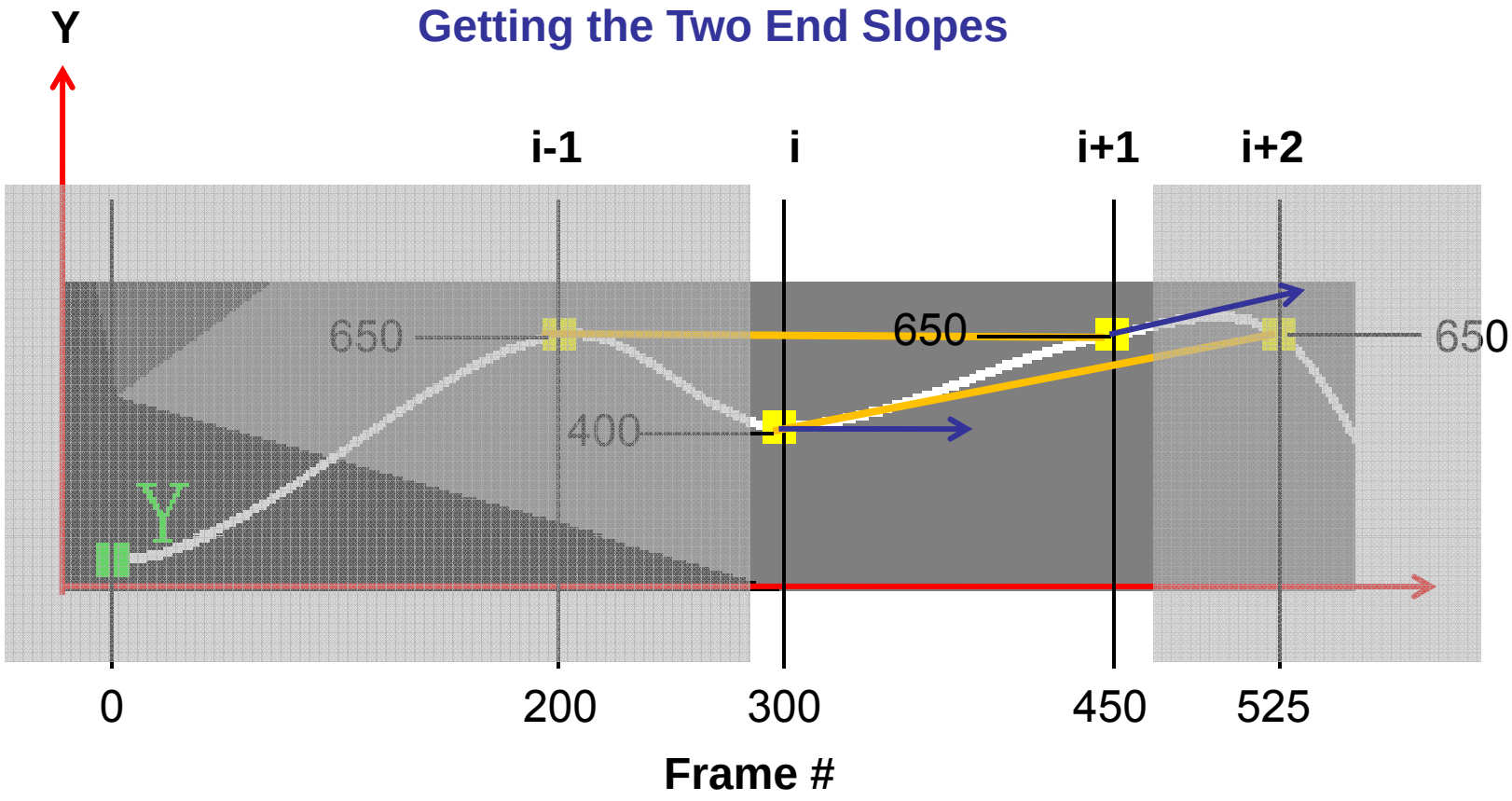


$$\frac{dY}{dF}_i = \frac{Y_{i+1} - Y_{i-1}}{F_{i+1} - F_{i-1}} =$$

$$\frac{dY}{dF}_{i+1} = \frac{Y_{i+2} - Y_i}{F_{i+2} - F_i} =$$

$$\frac{dF}{dt}_{i \to i+1} =$$

Getting the Two End Slopes

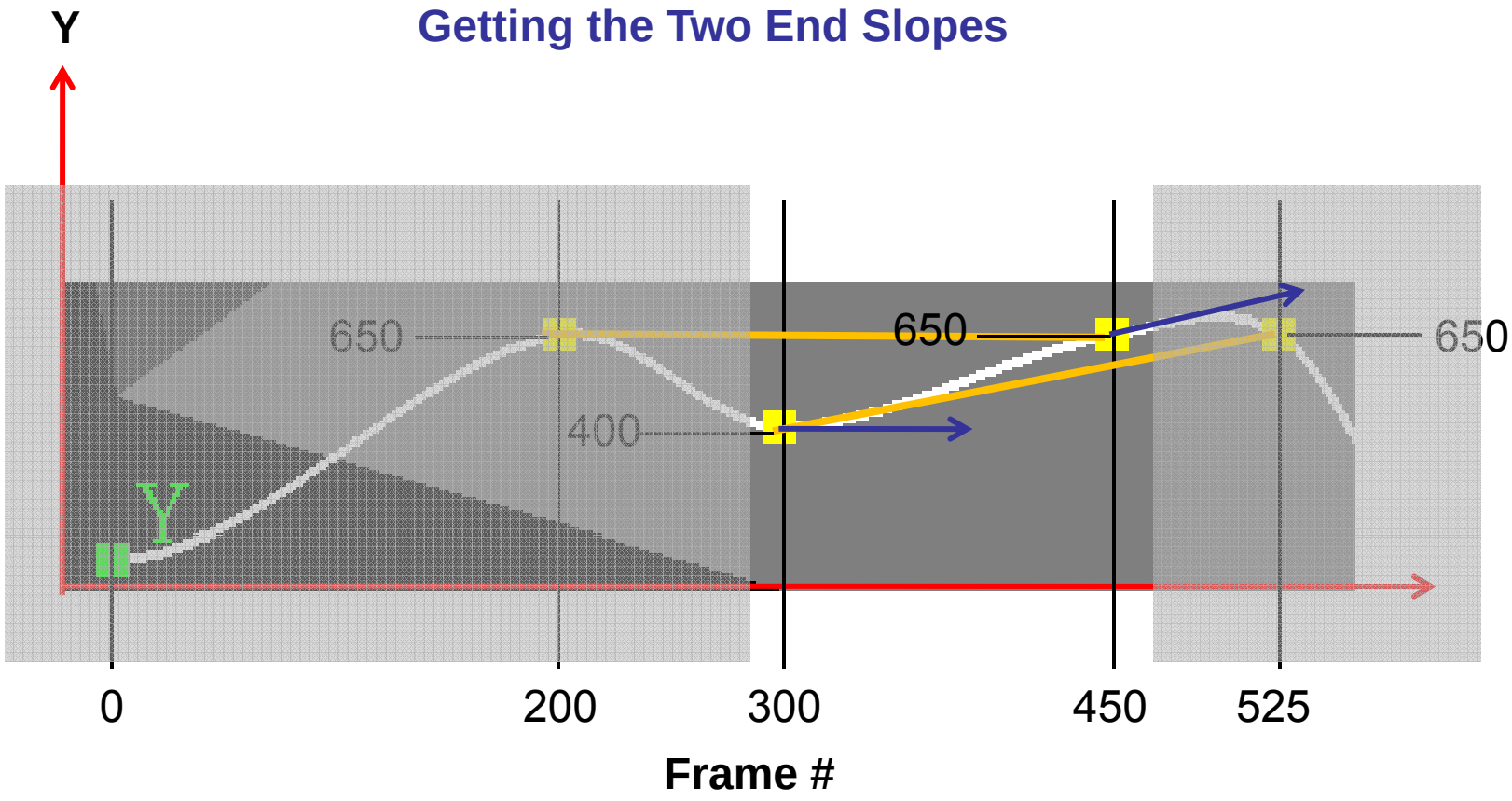


$$\frac{dY}{dF}_i = \frac{Y_{i+1} - Y_{i-1}}{F_{i+1} - F_{i-1}} = \frac{650 - 650}{450 - 200}$$

$$\frac{dY}{dF}_{i+1} = \frac{Y_{i+2} - Y_i}{F_{i+2} - F_i} = \frac{650 - 400}{525 - 300}$$

$$\frac{dF}{dt}_{i \to i+1} = \frac{F_{i+1} - F_i}{1. - 0.} = F_{i+1} - F_i = 450 - 300 = 150$$

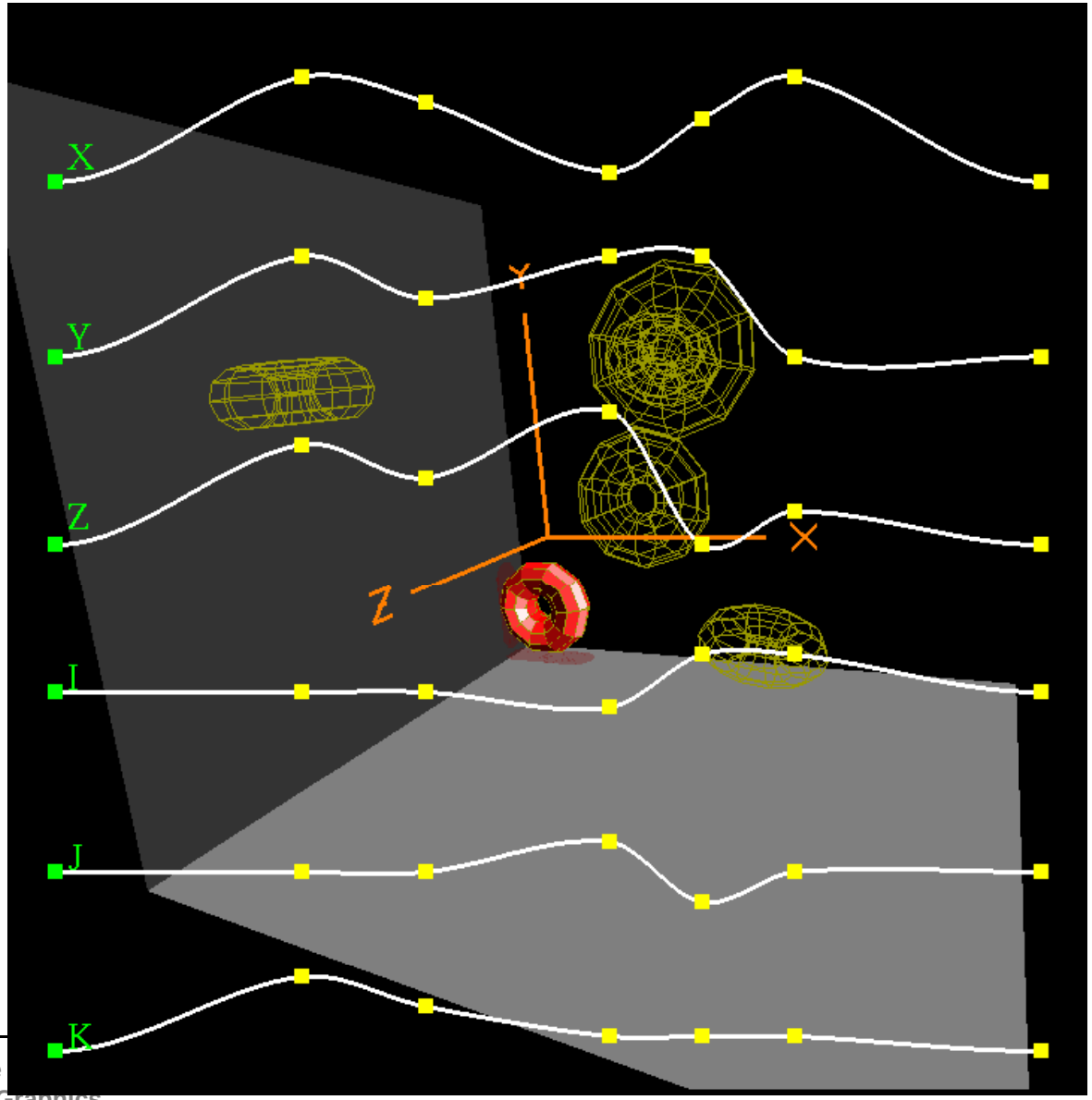
Getting the Two End Slopes



$$\frac{dY}{dt}_{i} = \frac{dY}{dF}_{i} \frac{dF}{dt}_{i \to i+1} = \frac{0}{250} \times 150 = 0$$

$$\frac{dY}{dt}_{i+1} = \frac{dY}{dF}_{i+1} \frac{dF}{dt}_{i \to i+1} = \frac{250}{225} \times 150 = 166.7$$

Do This Same Thing for the X, Y, and Z Translations and the X, Y, and Z Rotations



Using the System Clock in Animate( ) for Timing

```
if( AnimationIsOn )
{
	// # msec into the cycle ( 0 - MSEC-1 ):

	int msec = glutGet( GLUT_ELAPSED_TIME )  %  MSEC;

	// turn that into the current frame number:

	NowFrame = (int) (  (float)MAXFRAME * (float)msec / (float)MSEC  );

	// look through the keyframes and figure out which two keyframes this is between:

	for( int  i = 0; i < maxKeyframes - 1; i++ )
	{
		if( ????? )
		{
			KeyFrameBefore = ?????;
			KeyFrameAfter  = ?????;
			break;
		}
	}

	// get the t (0.-1.) for that frame in that interval:

	NowT = ?????
```

Using the System Clock in Animate( ) for Timing

```
        // determine the A, B, C, and D for all the interpolation curves in that interval:

        Ax, Bx, Cx, Dx = ?????
        Ay, By, Cy, Dy = ?????
        . . .

        // use the coefficients and t to compute the current transformation (and other) parameters:

        NowX = Ax + Bx*NowT + Cx*NowT*NowT + Dx*NowT*NowT*NowT ;
        NowY = Ay + By*NowT + Cy*NowT*NowT + Dy*NowT*NowT*NowT ;
        . . .

        // post a redisplay to use those parameters:
}
```

In Display( ):

```
        glPushMatrix( );
                glTranslatef( NowX, NowY, NowZ );
                 glRotatef( NowRx,    1., 0., 0. );
                 glRotatef( NowRy,    0., 1., 0. );
                 glRotatef( NowRz,    0., 0., 1. );

                 << Draw the moving object >>
        glPopMatrix( );
```

A Final Word

If you ever do this “for real”, ***quaternions*** are a better way to do the rotations.

The details of quaternions are beyond the scope of this class, but they do a smoother job of rotations because they deal with an angle and an axis of rotation, rather than 3 angles about the principle axes, which is somewhat arbitrary.