















Equation Setup

Triangle:
$$p = P0 + u^*(P1-P0) + v^*(P2-P0)$$

Ray: $p = S + tQ$

Re-arranging:

Computer Graphics

$$P0 + u*(P1-P0) + v*(P2-P0) = S + tQ$$

Re-arranging some more:

$$-tQ + u*(P1-P0) + v*(P2-P0) = S - P0$$

Then collecting terms, we get:

$$At + Bu + Cv = D$$

where:

A = -0

D = S - P0



B = P1-P0

C = P2-P0

Three Equations, Three Unknowns

Remembering that this equation is really 3 equations in (x,y,z):

$$At + Bu + Cv = D$$

we have 3 equations with 3 unknowns, which can be cast into a matrix form

$$\begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ A_z & B_z & C_z \end{bmatrix} \begin{pmatrix} t \\ u \\ v \end{pmatrix} = \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix}$$

Our goal is to solve this for t*, u*, and v*



Solve for (t*,u*,v*) using Cramer's Rule

$$\begin{bmatrix} A_x & B_x & C_x \\ A_z & B_y & C_y \\ A_z & B_z & C_z \end{bmatrix} \begin{pmatrix} t \\ u \\ v \end{pmatrix} = \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix}$$

$$D_0 = det \begin{bmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ A_z & B_z & C_z \end{bmatrix}$$

$$D_{t} = det \begin{bmatrix} D_{x} \\ D_{y} \\ D_{z} \end{bmatrix} \begin{bmatrix} B_{x} & C_{x} \\ B_{y} & C_{y} \\ B_{z} & C_{z} \end{bmatrix}$$

$$u = det \begin{bmatrix} A_x & D_x & C_x \\ A_y & D_y & C_y \\ A_z & D_z & C_z \end{bmatrix}$$





The Steps

- 1. Compute D₀
- 2. If $D_0 \approx 0$, then the ray is *parallel* to the plane of the triangle



- 3. Compute D_t
- 4. Compute t*
- 5. If t* < 0., the ray goes away from the triangle

- 6. Compute D_{...}
- 7. Compute u*
- 8. If $u^* < 0$. or $u^* > 1$., then the ray hits outside the triangle



- 9. Compute D_v
- 10. Compute v*
- 11. If $v^* < 0$. or $v^* > 1$.- u^* , then the ray hits outside the triangle



12. The intersection is at the point $p = S + Qt^*$

This is known as the Möller-Trumbore Triangle Intersection Algorithm



Oregon State University Computer Graphics







