

# Using Vertex Shaders for Hyperbolic Geometry



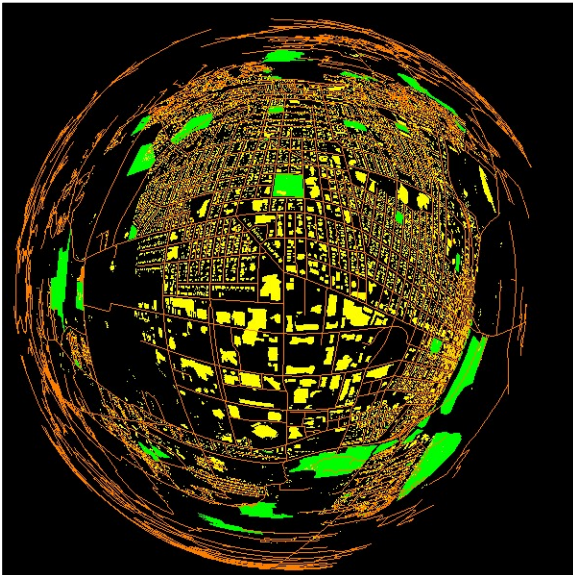
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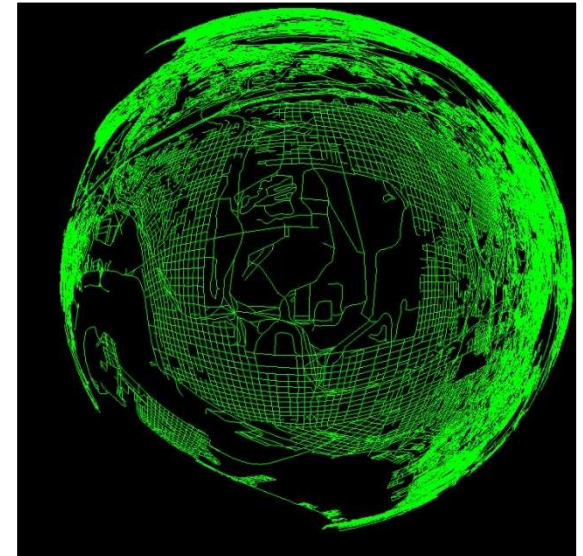
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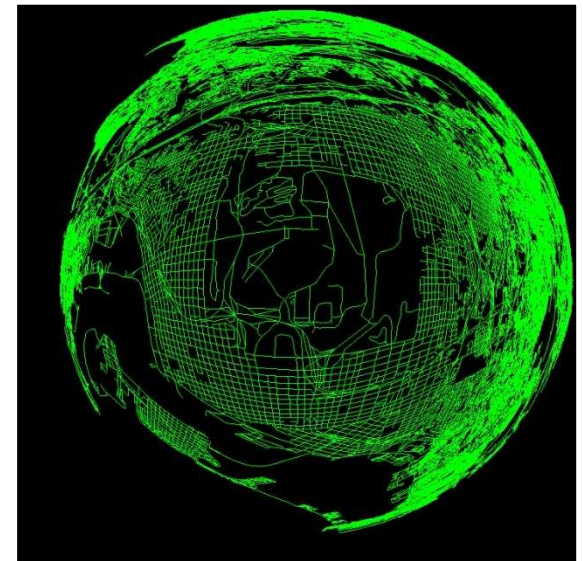


Computer Graphics



## Zooming and Panning Around a Complex 2D Display

- Standard (Euclidean) geometry zooming forces much of the information off the screen
- This eliminates the context from the zoomed-in display
- This problem can be solved with *hyperbolic methods* if we are willing to give up Euclidean geometry
- At one time, this would have also meant severely giving up graphics performance, but not now (thanks to shaders)



## Zooming in *Euclidean* Hyperbolic Space



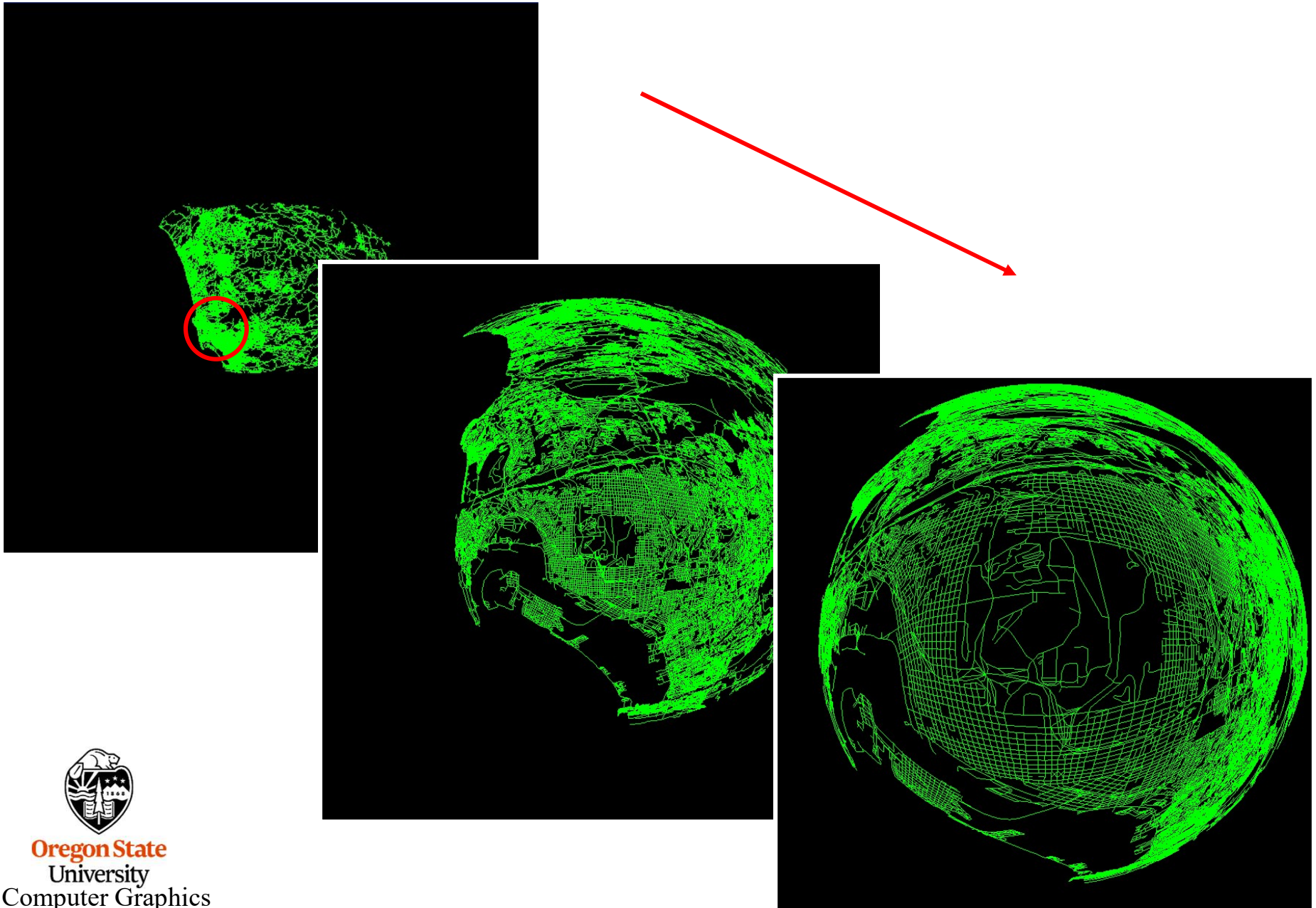
123,101 line strips  
446,585 points



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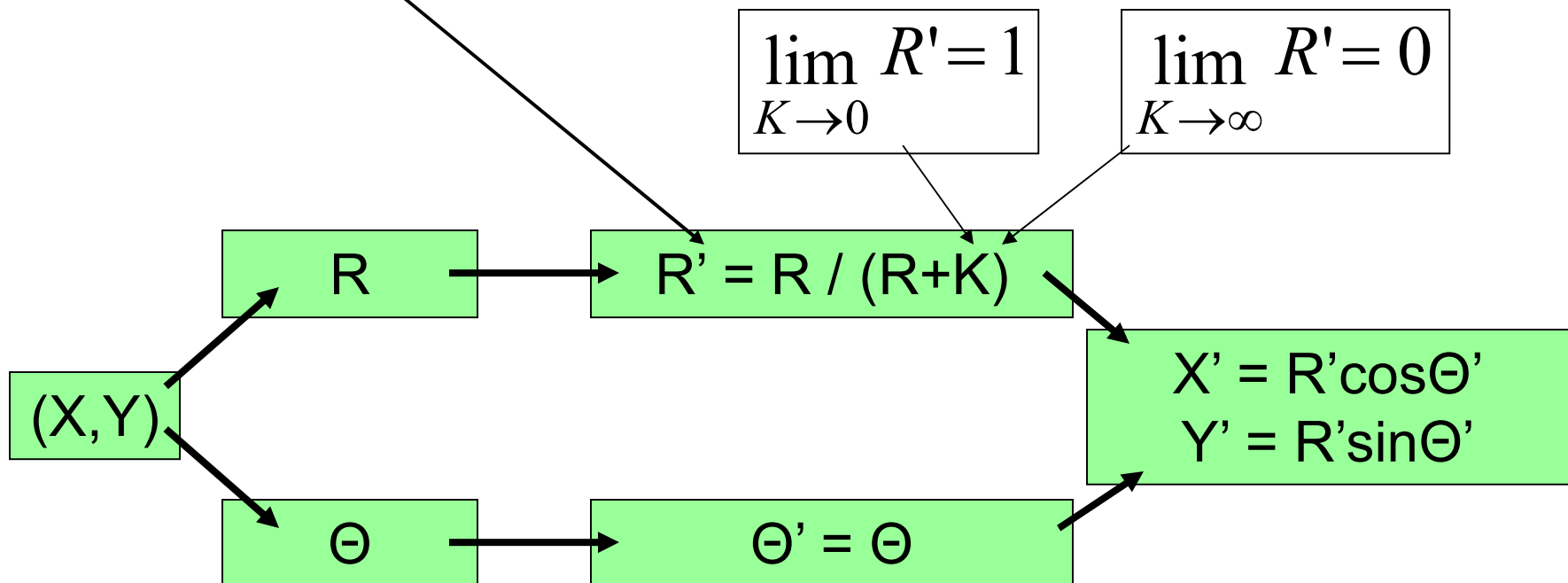


## Zooming in *Polar* Hyperbolic Space



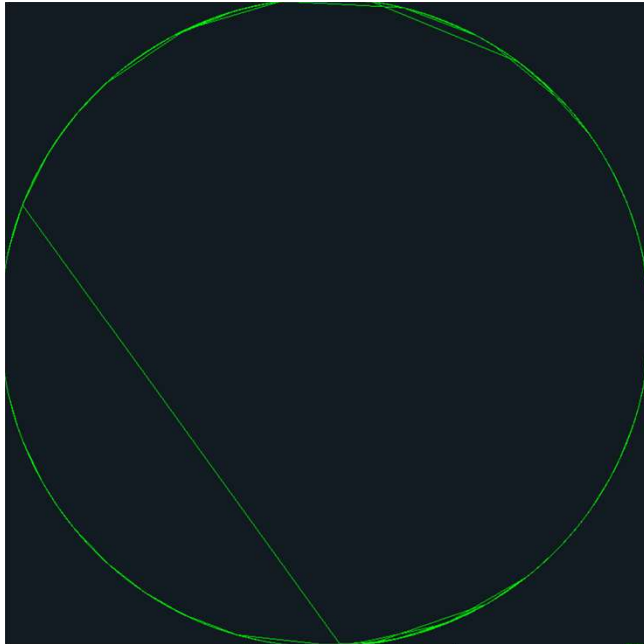
## Polar Hyperbolic Equations

*Overall theme: something divided by something a little bigger*



## The Effect of K

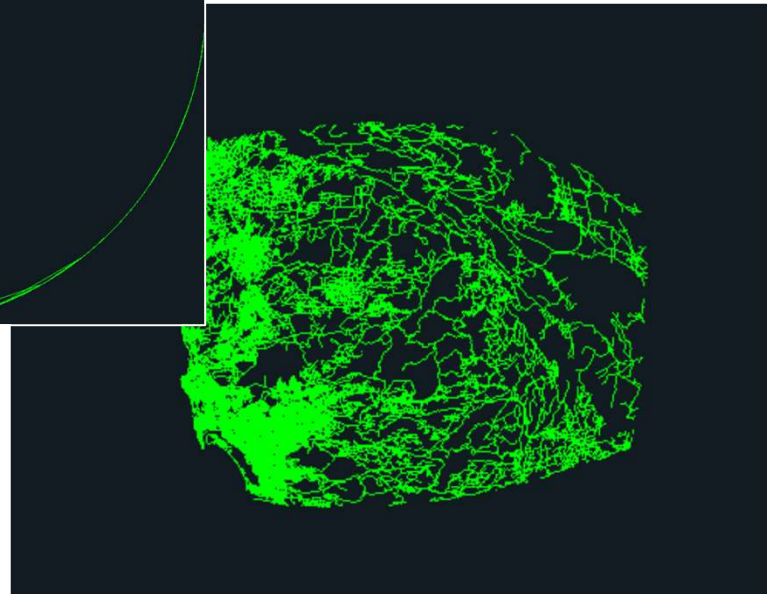
$K = 0.$



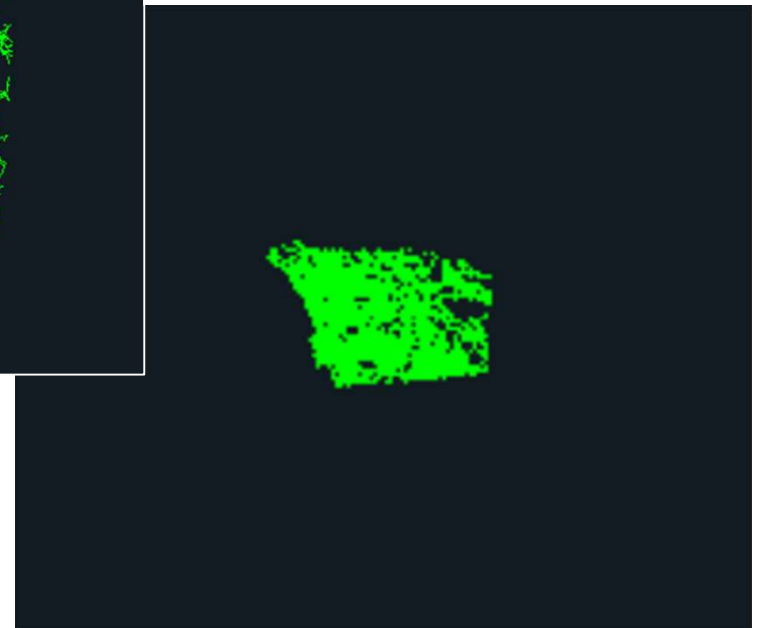
$$\lim_{K \rightarrow 0} R' = 1$$

$$\lim_{K \rightarrow \infty} R' = 0$$

$K = 1.$



$K = 10.$



## Polar Hyperbolic Equations

$$R = \sqrt{X^2 + Y^2}$$

$$\Theta = \tan^{-1}\left(\frac{Y}{X}\right)$$

But fortunately, we don't actually need to ever call the atan( ) function because there are shortcuts to get what we need:

$$\cos \Theta = \frac{X}{R}$$

$$\sin \Theta = \frac{Y}{R}$$

Coordinates moved to the outer edge when  $K = 0$

$$R' = \frac{R}{R + K}$$

Coordinates moved to the center when  $K = \infty$

$$X' = R' \cos \Theta = \frac{R}{R + K} \times \frac{X}{R} = \frac{X}{R + K}$$

$$Y' = R' \sin \Theta = \frac{R}{R + K} \times \frac{Y}{R} = \frac{Y}{R + K}$$

# Cartesian Hyperbolic Equations

Polar {

$$X' = \frac{X}{R + K}$$

Coordinates moved to the outer edge when  $K = 0$

Coordinates moved to the center when  $K = \infty$

$$Y' = \frac{Y}{R + K}$$

Cartesian {

$$X' = \frac{X}{\sqrt{X^2 + K^2}}$$

Coordinates moved to the outer edge when  $K = 0$

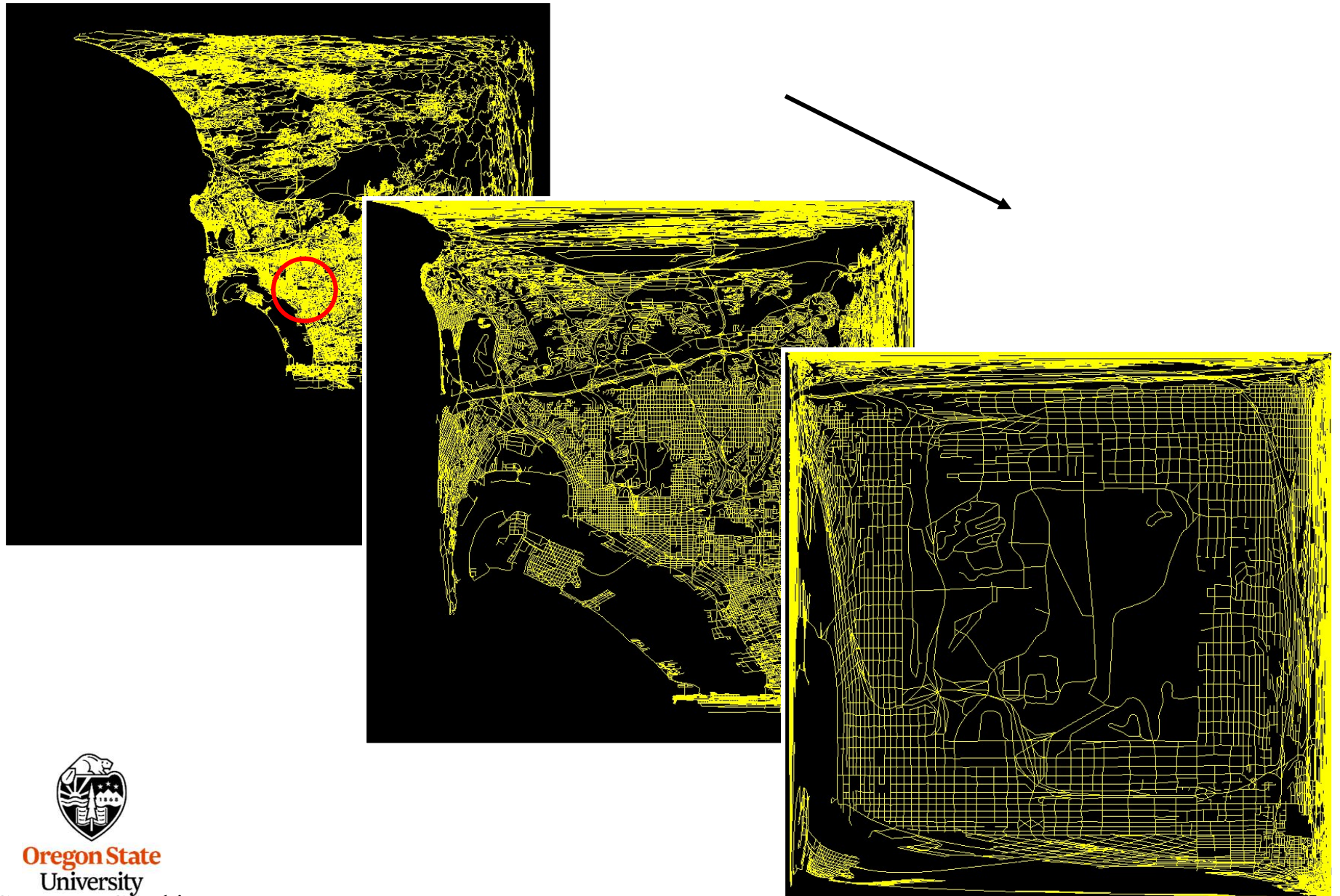
Coordinates moved to the center when  $K = \infty$

$$Y' = \frac{Y}{\sqrt{Y^2 + K^2}}$$





## Zooming in Cartesian Hyperbolic Space



```
#version 330 compatibility
uniform bool      uPolar;
uniform float     uK;
uniform float     uTransX;
uniform float     uTransY;
out vec3          vColor;

void
main( )
{
    vColor = gl_Color.rgb;

    vec2 pos = ( gl_ModelViewMatrix * gl_Vertex ).xy;
    pos += vec2( uTransX, uTransY );
    float r = length( pos );

    vec4 pos2 = vec4( 0., 0., -5., 1. );

    if( uPolar )
        pos2.xy = pos / ( r + uK );
    else
        pos2.xy = pos / ( pos*pos + uK*uK );

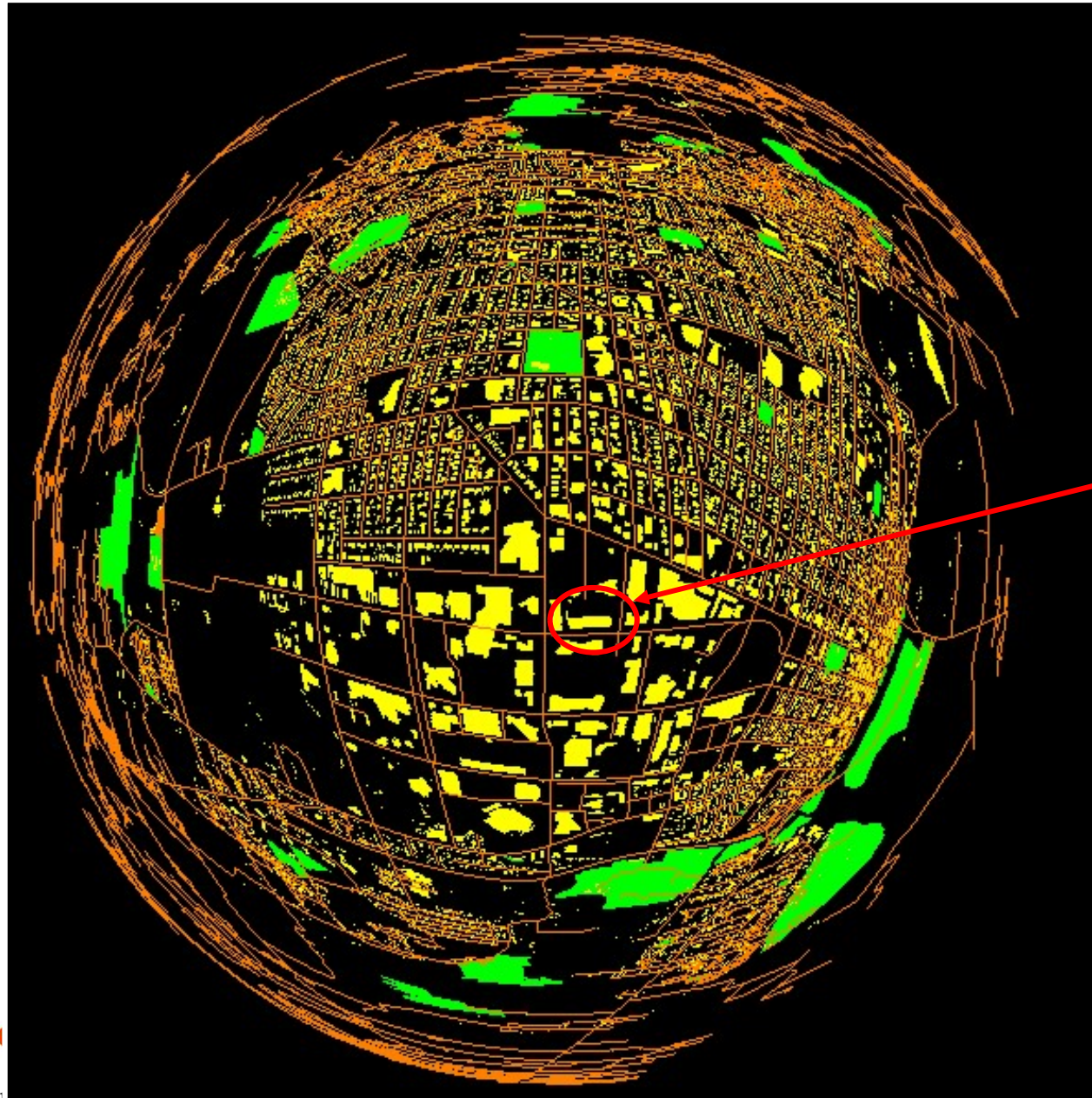
    gl_Position = gl_ProjectionMatrix * pos2;
}
```

## hyper.frag

```
#version 330 compatibility  
in vec3   vColor;  
  
void  
main( )  
{  
    gl_FragColor = vec4( vColor, 1. );  
}
```



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