Using Vertex Shaders for Hyperbolic Geometry

Zooming and Panning Around a Complex 2D Display

• Standard (Euclidean) geometry zooming forces much of the information off the screen
• This eliminates the context from the zoomed-in display
• This problem can be solved with hyperbolic methods if we are willing to give up Euclidean geometry
• At one time, this would have also meant severely giving up graphics performance, but not now (thanks to shaders)

Zooming in Euclidean Hyperbolic Space

Zooming in Polar Hyperbolic Space

123,101 line strips
446,585 points
Polar Hyperbolic Equations

Overall theme: something divided by something a little bigger

\[
\lim_{K \to 0} R' = 1 \\
\lim_{K \to \infty} R' = 0
\]

\[(X,Y) \rightarrow (X',Y') \]

\[R = R / (R+K)\]

\[\Theta' = \Theta\]

\[X' = R' \cos \Theta'\]

\[Y' = R' \sin \Theta'\]

The Effect of K

K = 0.

\[\lim_{K \to 0} R' = 1\]

K = 1.

\[\lim_{K \to 0} R' = 0\]

K = 10.

Polar Hyperbolic Equations

\[R = \sqrt{X^2 + Y^2}\]

\[\Theta = \tan^{-1} \left( \frac{Y}{X} \right)\]

\[\cos \Theta = \frac{X}{R}\]

\[\sin \Theta = \frac{Y}{R}\]

\[R' = \frac{R}{R+K}\]

\[X' = R' \cos \Theta = \frac{R}{R+K} \times \frac{X}{R} = \frac{X}{R+K}\]

\[Y' = R' \sin \Theta = \frac{R}{R+K} \times \frac{Y}{R} = \frac{Y}{R+K}\]

Cartesian Hyperbolic Equations

\[\left\{ \right. \]

\[X' = \frac{X}{R+K}\]

\[Y' = \frac{Y}{\sqrt{X^2 + Y^2}}\]

\[\left\} \right.\]

But, fortunately, we don’t actually need to ever call the \(\tan^{-1}\) function because there are shortcuts to get what we need:

Coordinates moved to outer edge when K = 0

Coordinates moved to center when K = ∞

Coordinates moved to outer edge when K = 0

Coordinates moved to center when K = ∞
### hyper.vert

```glsl
#version 330 compatibility
uniform bool uPolar;
uniform float uK;
uniform float uTransX;
uniform float uTransY;
out vec3 vColor;

void main( void )
{
    vColor = gl_Color.rgb;
    vec2 pos = ( gl_ModelViewMatrix * gl_Vertex ).xy;
    pos += vec2( uTransX, uTransY );
    float r = length( pos.xyz );
    vec4 pos2 = vec4( 0., 0., -5., 1.);
    if( uPolar )
        pos2.xy = pos / ( r + uK );
    else
        pos2.xy = pos / ( pos * pos + uK * uK );
    gl_Position = gl_ProjectionMatrix * pos2;
}
```

### hyper.frag

```glsl
#version 330 compatibility
in vec3 vColor;

void main( )
{
    gl_FragColor = vec4( vColor, 1. );
}
```
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Data courtesy of the Corvallis Fire Department