Using Vertex Shaders for Hyperbolic Geometry

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Zooming and Panning Around a Complex 2D Display

• Standard (Euclidean) geometry zooming forces much of the information off the screen
• This eliminates the context from the zoomed-in display
• This problem can be solved with hyperbolic methods if we are willing to give up Euclidean geometry
• At one time, this would have also meant severely giving up graphics performance, but now

2

Zooming in Euclidean Space

123,101 line strips
446,585 points

3

Zooming in Polar Hyperbolic Space

4

Polar Hyperbolic Equations

Overall theme: something divided by something a little bigger

\[
\begin{align*}
\lim_{K \to 0} R' &= 1 \\
\lim_{K \to \infty} R' &= 0 \\
X' &= R' \cos \theta' \\
Y' &= R' \sin \theta'
\end{align*}
\]

5

Polar Hyperbolic Equations

\[
\begin{align*}
R &= \sqrt{X^2 + Y^2} \\
\Theta &= \tan^{-1}\left(\frac{Y}{X}\right) \\
R' &= \frac{R}{R + K} \\
X' &= \frac{X}{R + K} \\
Y' &= \frac{Y}{R + K}
\end{align*}
\]

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Computer Graphics

Cartesian Hyperbolic Equations

\[
\begin{align*}
X' &= \frac{X}{\sqrt{X^2 + K^2}} \\
Y' &= \frac{Y}{\sqrt{Y^2 + K^2}}
\end{align*}
\]

Coordinates moved to outer edge when \( K = 0 \)
Coordinates moved to center when \( K = \infty \)

Zooming in Cartesian Hyperbolic Space

\begin{verbatim}
#version 330 compatibility
uniform bool uPolar;
uniform float uK;
uniform float uTransX;
uniform float uTransY;
out vec3 vColor;

void main( void )
{
  vec2 pos = ( gl_ModelViewMatrix * gl_Vertex ).xy;
  pos += vec2( uTransX, uTransY );
  float r = length( pos.xyz );
  vec4 pos2 = vec4( 0., 0., -5., 1. );
  if( uPolar )
    pos2.xy = pos / ( r + uK );
  else
    pos2.xy = pos / ( pos*pos + uK*uK );
  gl_Position = gl_ProjectionMatrix * pos2;
}
\end{verbatim}

hyper.vert

\begin{verbatim}
#version 330 compatibility
in vec3 vColor;

void main(  )
{
  gl_FragColor = vec4( vColor, 1. );
}
\end{verbatim}

hyper.frag

Corvallis Streets, Buildings, Parks

Kelley Engineering Center

Data courtesy of the Corvallis Fire Department