OpenMP Reduction Case Study:
Trapezoid Integration Example

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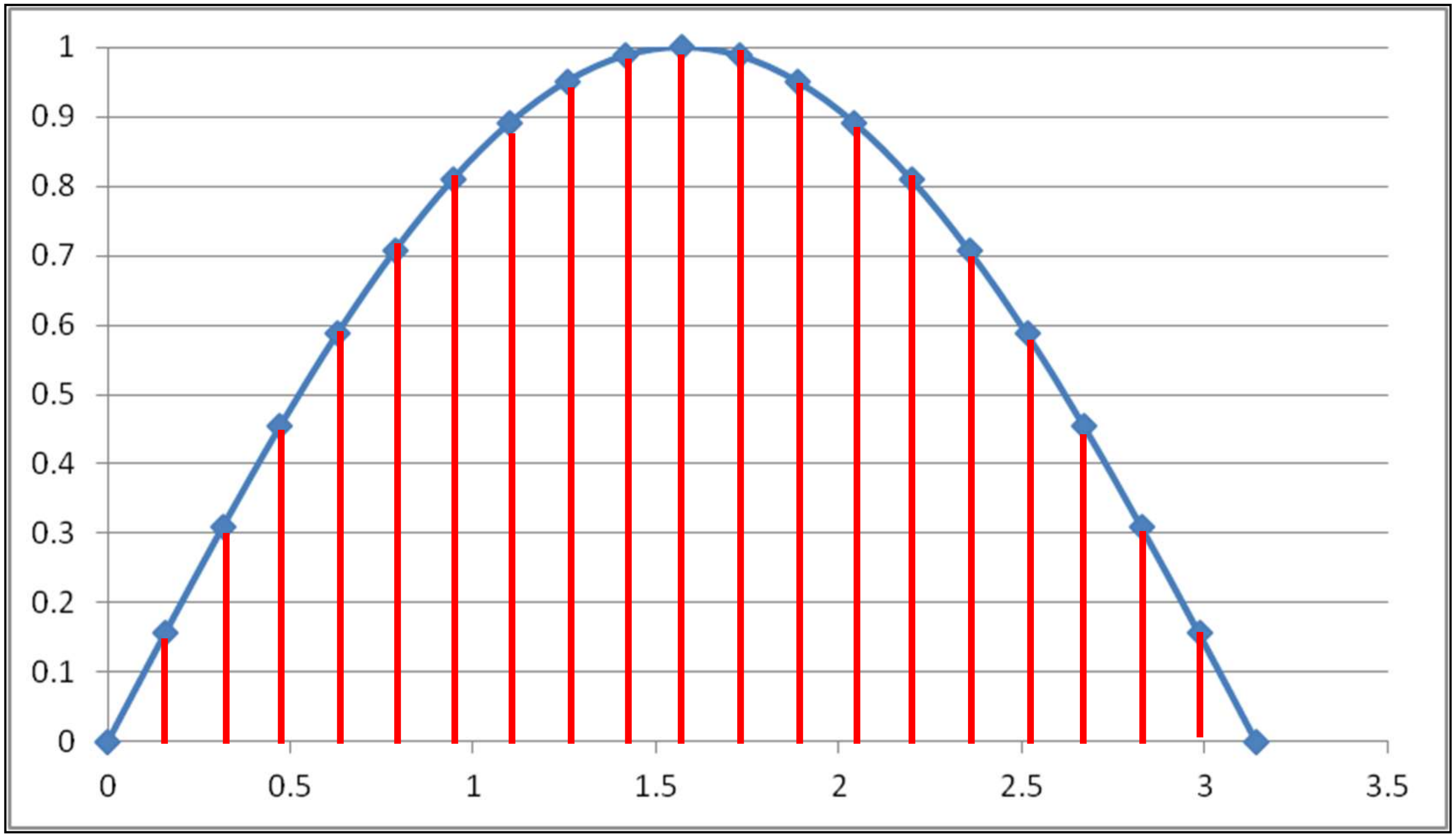
Find the area under the curve y = sin(x) for 0 ≤ x ≤ π using the Trapezoid Rule



Exact answer: $\int_0^{\pi} (\sin x) dx = -\cos x \,|_0^{\pi} = 2.0$

Oregon State University
Computer Graphics

Don’t do it this way !

```
const  double A  =  0.;
const  double B  =  M_PI;
double dx = ( B - A ) / (float) ( numSubdivisions – 1 );
double sum = (  Function( A ) + Function( B )   )  /  2.;

omp_set_num_threads(  numThreads );
#pragma  omp  parallel  for  default(none), shared(dx,sum)
for( int i = 1; i < numSubdivisions - 1; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        sum  +=  f;
}
sum *= dx;
```

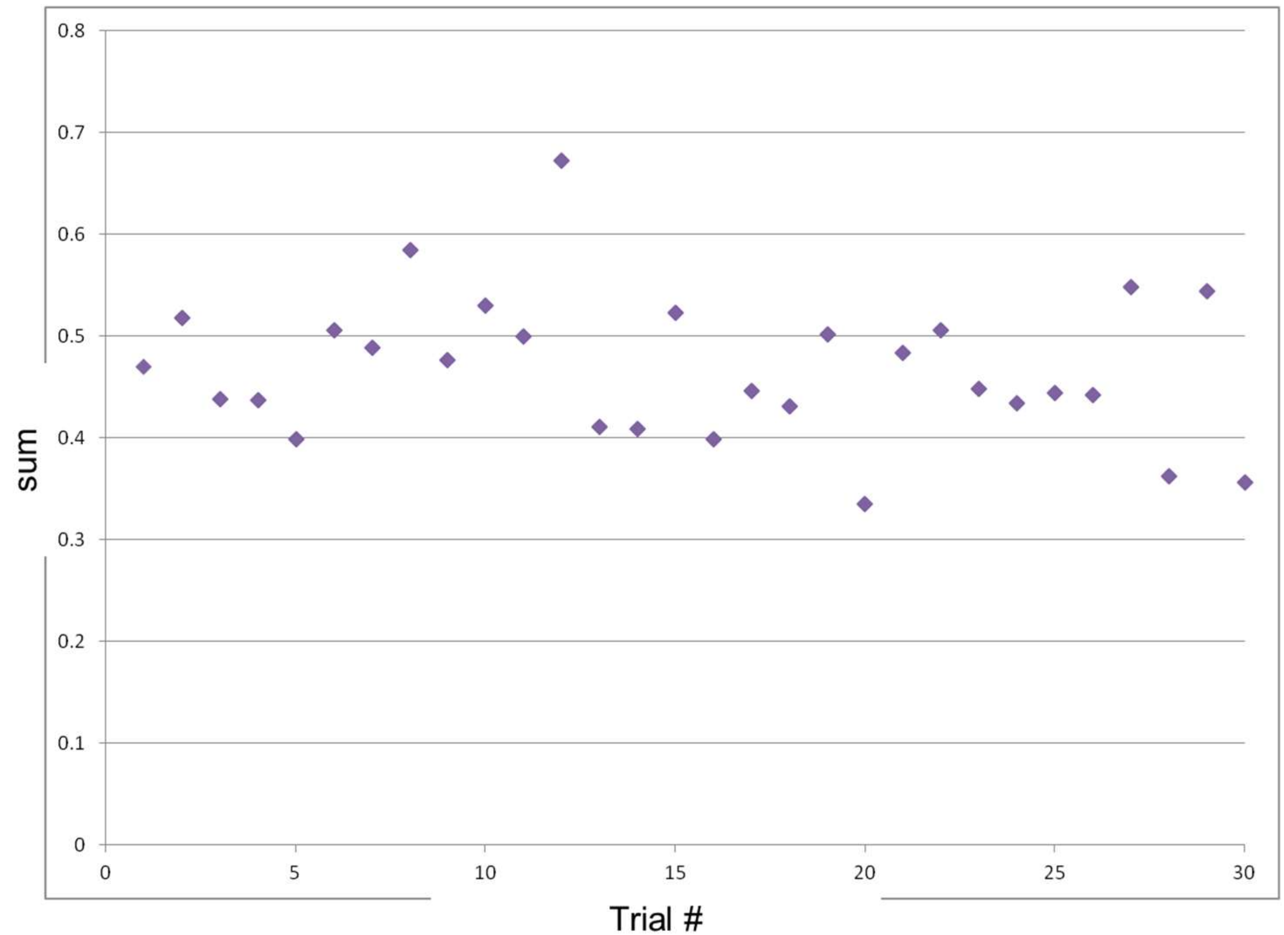
- There is no guarantee when each thread will execute this line
- There is not even a guarantee that each thread will finish this line before some other thread interrupts it.

Assembly code:
Load sum Add f Store sum

What if the scheduler decides to switch threads right here?

The answer should be 2.0 *exactly*, but in 30 trials, it's not even close. And the answers aren't even consistent. How do we fix this?

0.469635	0.398893
0.517984	0.446419
0.438868	0.431204
0.437553	0.501783
0.398761	0.334996
0.506564	0.484124
0.489211	0.506362
0.584810	0.448226
0.476670	0.434737
0.530668	0.444919
0.500062	0.442432
0.672593	0.548837
0.411158	0.363092
0.408718	0.544778
0.523448	0.356299



There are Three Ways to Make the Summing Work Correctly: #1: Atomic

1

```
#pragma omp parallel for shared(dx)
for( int i = 0; i < numSubdivisions; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        #pragma omp atomic
        sum += f;
}
```

- More lightweight than *critical* (#2)
- Uses a hardware instruction CMPXCHG (compare-and-exchange)
- Can only handle these operations:

  x++, ++x, x--, --x
  x op= expr , x = x op expr , x = expr op x
  where op is one of: +, -, *, /, &, |, ^, <<, >>

There are Three Ways to Make the Summing Work Correctly: #2: Critical

2

```
#pragma omp parallel for shared(dx)
for( int i = 0; i < numSubdivisions; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        #pragma omp critical
        sum += f;
}
```

- More heavyweight than *atomic* (#1)
- Allows only one thread at a time to enter this block of code (similar to a mutex)
- Can have any operations you want in this block of code

There are Three Ways to Make the Summing Work Correctly: #3: Reduction

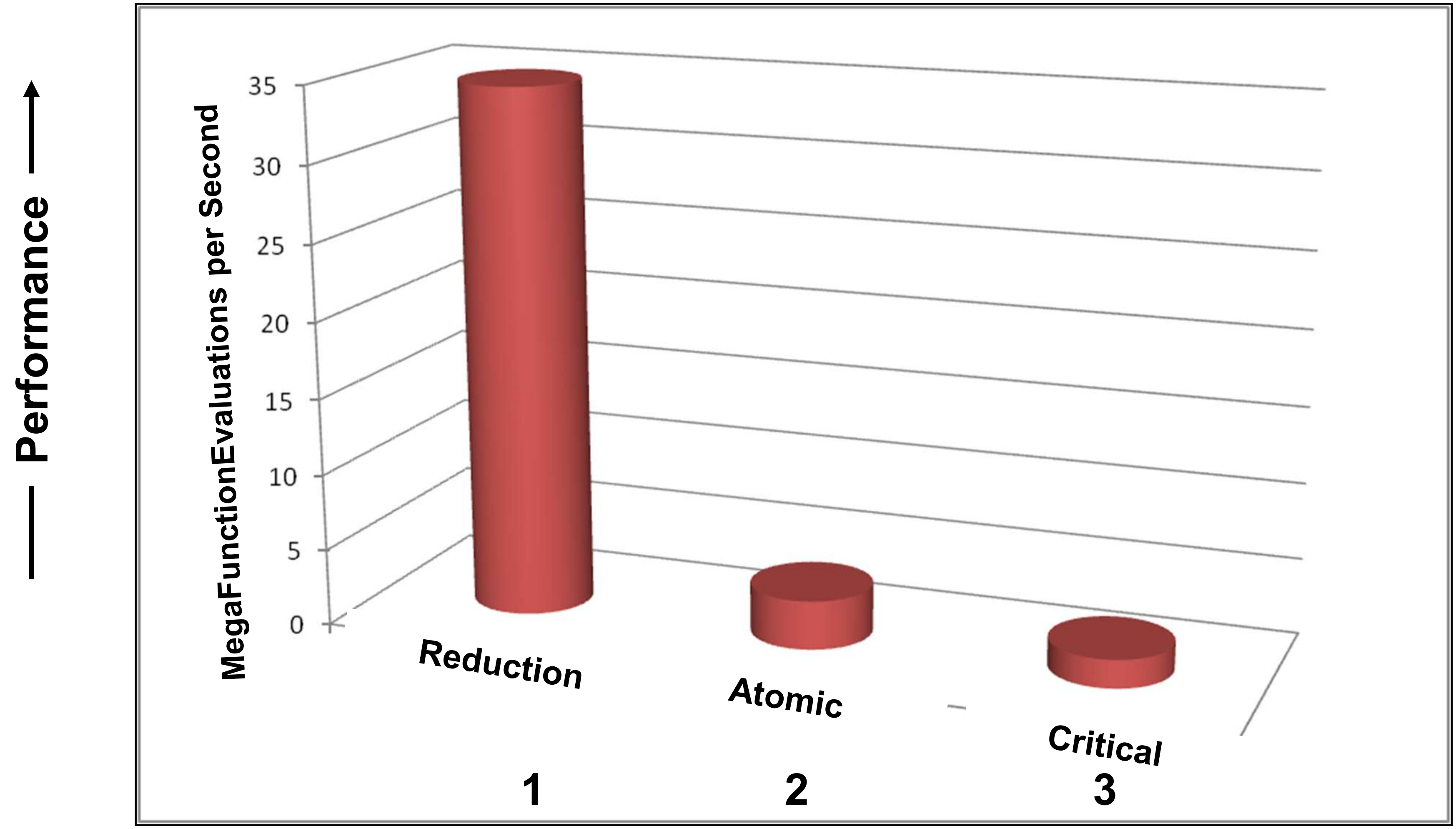
3

```
#pragma omp parallel for shared(dx),reduction(+:sum)
for( int i = 0; i < numSubdivisions; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        sum += f;
}
```

- OpenMP creates code to make this as fast as possible
- Reduction operators can be: + , - , * , & , | , ^ , && , || , max , min

Speed of Reduction vs. Atomic vs. Critical

(*up* = faster)

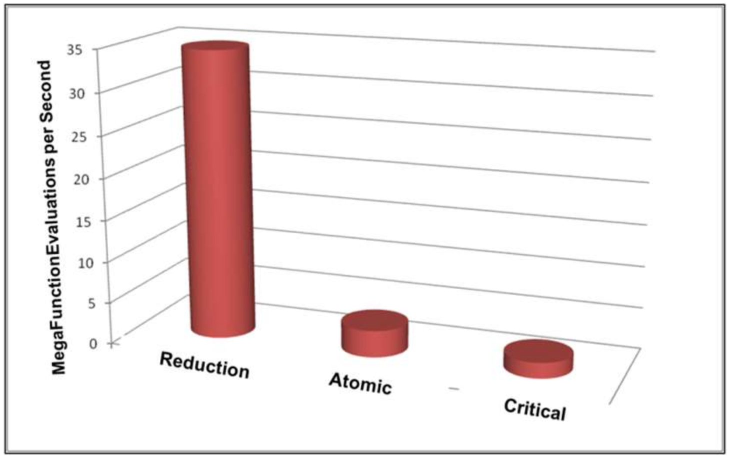




So, do it this way !

```
const  double  A  =  0.;
const  double  B  =  M_PI;

double dx = ( B - A ) / (float) ( numSubdivisions – 1 );

omp_set_num_threads(  numThreads );

double sum = (  Function( A ) + Function( B )   )  /  2.;

#pragma  omp  parallel  for  default(none),shared(dx),reduction(+:sum)
for( int i = 1; i < numSubdivisions - 1; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        sum  +=  f;
}

sum *= dx;
```

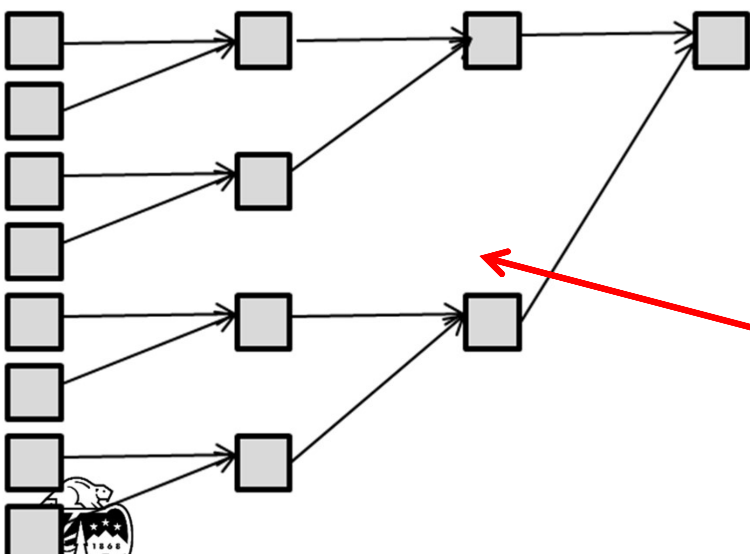
Two Reasons Why Reduction is so Much Better in this Case




```
#pragma omp parallel for shared(dx),reduction(+:sum)
for( int i = 0; i < numSubdivisions; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        sum += f;
}
```

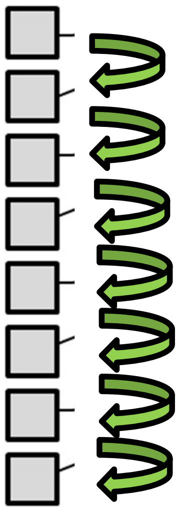
1. Reduction secretly creates a temporary private variable for each thread's running ***sum***. Each thread adding into its own running ***sum*** doesn't interfere with any other thread adding into *its* own running ***sum***, and so threads don't need to slow down to get out of the way of each other.

2. Reduction automatically creates a binary tree structure, like this, to add the N running sums in $\mathbf{log_2N}$ time instead **N** time.

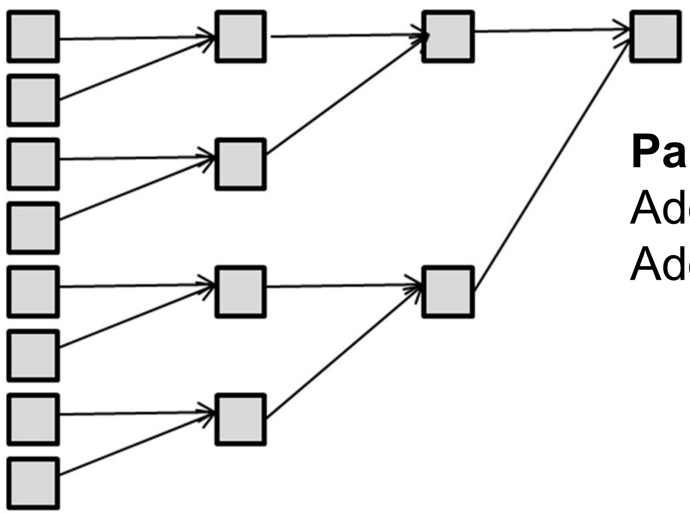


O(N) vs. O($\log_2$N)

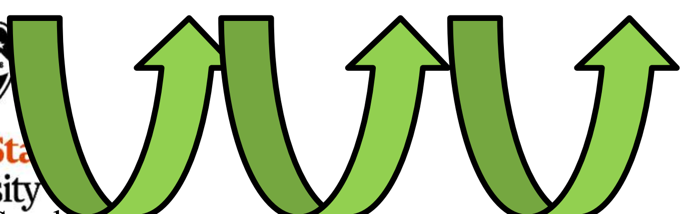
2. Reduction automatically creates a binary tree structure, like this, to add the N running sums in **$\log_2$N** time instead **N** time.

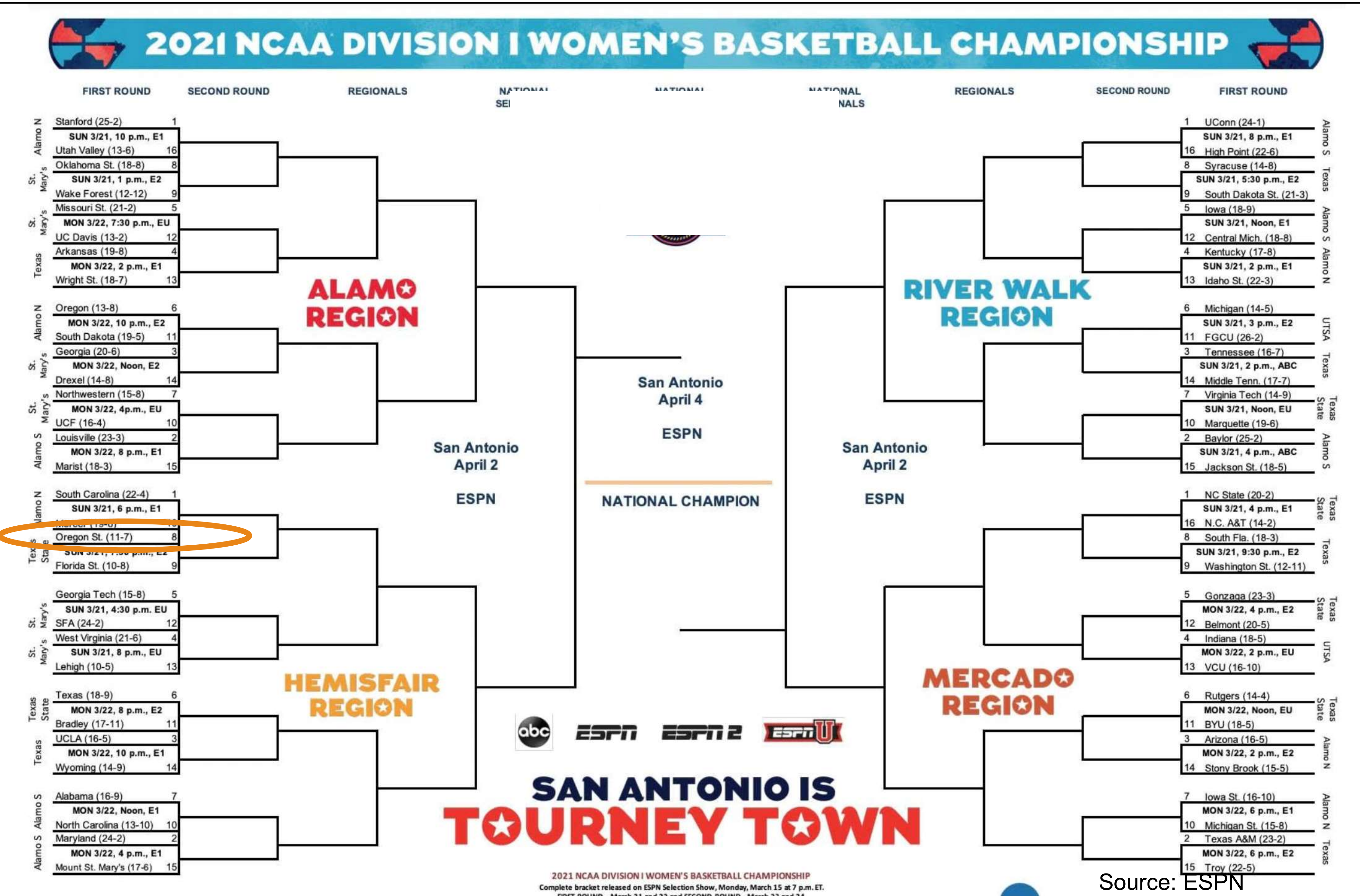


Serial addition:
Adding 8 numbers requires 7 steps
Adding 1,048,576 (1M) numbers requires 1,048,575 steps



Parallel addition:
Adding 8 numbers requires 3 steps
Adding 1,048,576 (1M) numbers requires 20 steps




2021 NCAA DIVISION I WOMEN'S BASKETBALL CHAMPIONSHIP
FIRST ROUND
SECOND ROUND
REGIONALS
REGIONALS
SECOND ROUND
FIRST ROUND
Stanford (25-2) 1
SUN 3/21, 10 p.m., E1
Utah Valley (13-6) 16
Oklahoma St. (18-8) 8
SUN 3/21, 1 p.m., E2
Wake Forest (12-12) 9
Missouri St. (21-2) 5
MON 3/22, 7:30 p.m., EU
UC Davis (13-2) 12
Arkansas (19-8) 4
MON 3/22, 2 p.m., E1
Wright St. (18-7) 13
Oregon (13-8) 6
MON 3/22, 10 p.m., E2
South Dakota (19-5) 11
Georgia (20-6) 3
MON 3/22, Noon, E2
Drexel (14-8) 14
Northwestern (15-8) 7
MON 3/22, 4p.m., EU
UCF (16-4) 10
Louisville (23-3) 2
MON 3/22, 8 p.m., E1
Marist (18-3) 15
South Carolina (22-4) 1
SUN 3/21, 6 p.m., E1
Oregon St. (11-7) 8
Florida St. (10-8) 9
Georgia Tech (15-8) 5
SUN 3/21, 4:30 p.m. EU
SFA (24-2) 12
West Virginia (21-6) 4
SUN 3/21, 8 p.m., EU
Lehigh (10-5) 13
Texas (18-9) 6
MON 3/22, 8 p.m., E2
Bradley (17-11) 11
UCLA (16-5) 3
MON 3/22, 10 p.m., E1
Wyoming (14-9) 14
Alabama (16-9) 7
MON 3/22, Noon, E1
North Carolina (13-10) 10
Maryland (24-2) 2
MON 3/22, 4 p.m., E1
Mount St. Mary's (17-6) 15
ALAMO REGION
HEMISFAIR REGION
San Antonio
April 2
ESPN
San Antonio
April 4
ESPN
NATIONAL CHAMPION
San Antonio
April 2
ESPN
RIVER WALK REGION
MERCADO REGION
1 UConn (24-1)
SUN 3/21, 8 p.m., E1
16 High Point (22-6)
8 Syracuse (14-8)
SUN 3/21, 5:30 p.m., E2
9 South Dakota St. (21-3)
5 Iowa (18-9)
SUN 3/21, Noon, E1
12 Central Mich. (18-8)
4 Kentucky (17-8)
SUN 3/21, 2 p.m., E1
13 Idaho St. (22-3)
6 Michigan (14-5)
SUN 3/21, 3 p.m., E2
11 FGCU (26-2)
3 Tennessee (16-7)
SUN 3/21, 2 p.m., ABC
14 Middle Tenn. (17-7)
7 Virginia Tech (14-9)
SUN 3/21, Noon, EU
10 Marquette (19-6)
2 Baylor (25-2)
SUN 3/21, 4 p.m., ABC
15 Jackson St. (18-5)
1 NC State (20-2)
SUN 3/21, 4 p.m., E1
16 N.C. A&T (14-2)
8 South Fla. (18-3)
SUN 3/21, 9:30 p.m., E2
9 Washington St. (12-11)
5 Gonzaga (23-3)
MON 3/22, 4 p.m., E2
12 Belmont (20-5)
4 Indiana (18-5)
MON 3/22, 2 p.m., EU
13 VCU (16-10)
6 Rutgers (14-4)
MON 3/22, Noon, EU
11 BYU (18-5)
3 Arizona (16-5)
MON 3/22, 2 p.m., E2
14 Stony Brook (15-5)
7 Iowa St. (16-10)
MON 3/22, 6 p.m., E1
10 Michigan St. (15-8)
2 Texas A&M (23-2)
MON 3/22, 6 p.m., E2
15 Troy (22-5)
SAN ANTONIO IS
TOURNEY TOWN
2021 NCAA DIVISION I WOMEN'S BASKETBALL CHAMPIONSHIP
Complete bracket released on ESPN Selection Show, Monday, March 15 at 7 p.m. ET.
FIRST-ROUND - March 21 and 22 and SECOND-ROUND - March 23 and 24.

Source: ESPN

Why Not Do Reduction by Creating Your Own *sums* Array, one for each Thread, Like This?

```
float *sums = new float [  omp_get_num_threads( )  ];
for( int i = 0; i < omp_get_num_threads( ); i++ )
                sums[ i ] = 0.;

#pragma omp parallel for private(myPartialSum),shared(sums)
 for( int i = 0; i < N; i++ )
{
        myPartialSum = …

        sums[  omp_get_thread_num( )  ]    +=    myPartialSum;
}

float sum = 0.;
for( int i= 0; i < omp_get_num_threads( ); i++ )
                sum += sums[ i ];

delete [ ] sums;
```

- This seems perfectly reasonable, it works, and it gets rid of the problem of multiple threads trying to write into the same reduction variable.

- But the reason we don't do this is that this method provokes a problem called **False Sharing**. We will get to that when we discuss caching.