OpenMP Reduction Case Study: Trapezoid Integration Example

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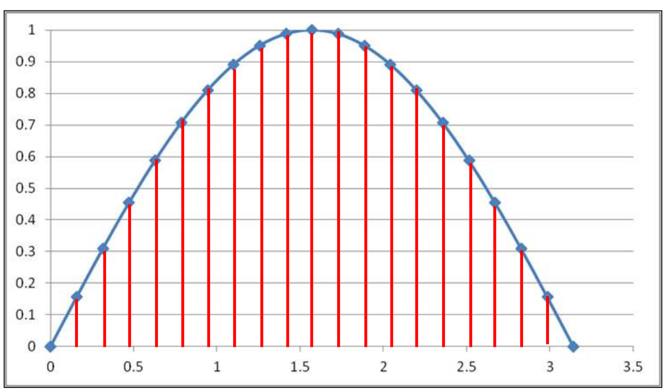
Find the area under the curve y = sin(x) for $0 \le x \le \pi$ using the Trapezoid Rule



Exact answer: $\int_0^{\pi} (\sin x) dx = -\cos x \,|_0^{\pi} = 2.0$

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Don't do it this way !

3

```
const  double A  =  0.;
const  double B  =  M_PI;
double dx = ( B - A ) / (float) ( numSubdivisions – 1 );
double sum = (  Function( A ) + Function( B )   )  /  2.;

omp_set_num_threads(  numThreads );
#pragma  omp  parallel  for  default(none), shared(dx,sum)
for( int i = 1; i < numSubdivisions - 1; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        sum  +=  f;
}
sum *= dx;
```

- There is no guarantee when each thread will execute this line
- There is not even a guarantee that each thread will finish this line before some other thread interrupts it.

Assembly code:

```
Load sum
Add   f
Store sum
```

What if the scheduler decides to switch threads right here?

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The answer should be 2.0 *exactly*, but in 30 trials, it's not even close. And the answers aren't even consistent. How do we fix this?

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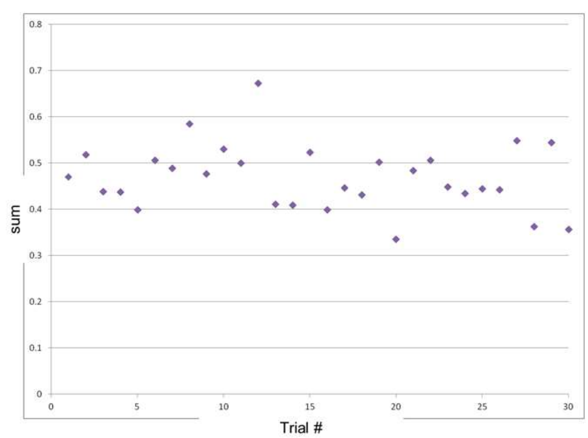
0.469635	0.398893
0.517984	0.446419
0.438868	0.431204
0.437553	0.501783
0.398761	0.334996
0.506564	0.484124
0.489211	0.506362
0.584810	0.448226
0.476670	0.434737
0.530668	0.444919
0.500062	0.442432
0.672593	0.548837
0.411158	0.363092
0.408718	0.544778
0.523448	0.356299



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There are Three Ways to Make the Summing Work Correctly: #1: Atomic

1

```
#pragma omp parallel for shared(dx)
for( int i = 0; i < numSubdivisions; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        #pragma omp atomic
        sum += f;
}
```

- More lightweight than *critical* (#2)
- Uses a hardware instruction CMPXCHG (compare-and-exchange)
- Can only handle these operations:

    x++, ++x, x--, --x
    x op= expr , x = x op expr , x = expr op x
    where op is one of: +, -, *, /, &, |, ^, <<, >>

There are Three Ways to Make the Summing Work Correctly: #2: Critical

2

```
#pragma omp parallel for shared(dx)
for( int i = 0; i < numSubdivisions; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        #pragma omp critical
        sum += f;
}
```

- More heavyweight than *atomic* (#1)
- Allows only one thread at a time to enter this block of code (similar to a mutex)
- Can have any operations you want in this block of code

There are Three Ways to Make the Summing Work Correctly: #3: Reduction

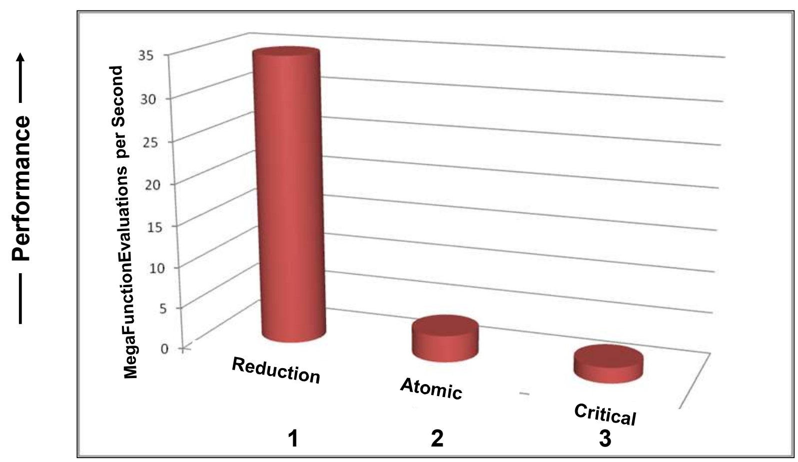
3

```
#pragma omp parallel for shared(dx),reduction(+:sum)
for( int i = 0; i < numSubdivisions; i++ )
{
      double x = A + dx * (float) i;
      double f = Function( x );
      sum += f;
}
```

- OpenMP creates code to make this as fast as possible
- Reduction operators can be: + , - , * , & , | , ^ , && , || , max , min

Speed of Reduction vs. Atomic vs. Critical

(*up* = faster)



So, do it this way !


```
const  double  A  =  0.;
const  double  B  =  M_PI;

double dx = ( B - A ) / (float) ( numSubdivisions – 1 );

omp_set_num_threads(  numThreads );

double sum = (  Function( A ) + Function( B )   )  /  2.;

#pragma  omp  parallel  for  default(none),shared(dx),reduction(+:sum)
for( int i = 1; i < numSubdivisions - 1; i++ )
{
            double x = A + dx * (float) i;
            double f = Function( x );
            sum  +=  f;
}

sum *= dx;
```

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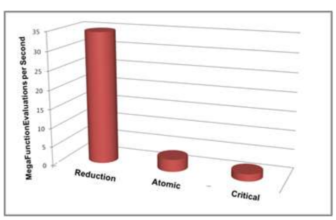
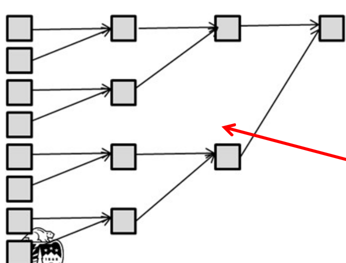
Two Reasons Why Reduction is so Much Better in this Case




```
#pragma omp parallel for shared(dx),reduction(+:sum)
for( int i = 0; i < numSubdivisions; i++ )
{
        double x = A + dx * (float) i;
        double f = Function( x );
        sum += f;
}
```



1. Reduction secretly creates a temporary private variable for each thread's running ***sum***. Each thread adding into its own running ***sum*** doesn't interfere with any other thread adding into *its* own running ***sum***, and so threads don't need to slow down to get out of the way of each other.

2. Reduction automatically creates a binary tree structure, like this, to add the N running sums in **$\log_2$N** time instead **N** time.

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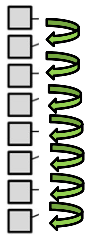
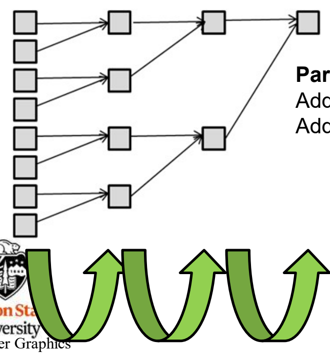
O(N) vs. O($\log_2$N)

2. Reduction automatically creates a binary tree structure, like this, to add the N running sums in **$\log_2$N** time instead **N** time.



Serial addition:
Adding 8 numbers requires 7 steps
Adding 1,048,576 (1M) numbers requires 1,048,575 steps



Parallel addition:
Adding 8 numbers requires 3 steps
Adding 1,048,576 (1M) numbers requires 20 steps

If You Understand NCAA Basketball Brackets, You Understand Power-of-Two Reduction

Source: CBS Sports

Why Not Do Reduction by Creating Your Own *sums* Array, one for each Thread, Like This?

```
float *sums = new float [  omp_get_num_threads( )  ];
for( int i = 0; i < omp_get_num_threads( ); i++ )
            sums[ i ] = 0.;

#pragma omp parallel for private(myPartialSum),shared(sums)
 for( int i = 0; i < N; i++ )
{
      myPartialSum = …

      sums[  omp_get_thread_num( )  ]   +=   myPartialSum;
}

float sum = 0.;
for( int i= 0; i < omp_get_num_threads( ); i++ )
            sum += sums[ i ];

delete [ ] sums;
```

• This seems perfectly reasonable, it works, and it gets rid of the problem of multiple threads trying to write into the same reduction variable.

• But the reason we don't do this is that this method provokes a problem called **False Sharing**. We will get to that when we discuss caching.