Chapter 24

Performing Pole/Zero Analysis

Pole/zero analysis is a useful method for studying the behavior of linear, time-invariant networks, and may be applied to the design of analog circuits, such as amplifiers and filters. It may be used for determining the stability of a design, and it may also be used to calculate the poles and zeroes for specification in a POLE statement as "Using Pole/Zero Analysis" on page 24-3 describes.

Pole/zero analysis is characterized by the use of the .PZ statement, as opposed to pole/zero and Laplace transfer function modeling, which employ the LAPLACE and POLE functions respectively. These are described in "Using Pole/Zero Analysis" on page 24-3.

This chapter covers these topics:

- Understanding Pole/Zero Analysis
- Using Pole/Zero Analysis

Understanding Pole/Zero Analysis

In pole/zero analysis, a network is described by its network transfer function which, for any linear time-invariant network, can be written in the general form:

$$H(s) = \frac{N(s)}{D(s)} = \frac{a_0 s^m + a_1 \cdot s^{(m-1)} + \ldots + a_m}{b_0 s^n + b_1 \cdot s^{(n-1)} + \ldots + b_n}$$

In the factorized form, the general function is:

$$H(s) = \frac{a_0}{b_0} \cdot \frac{(s+z_1)(s+z_2)\ldots(s+z_i)\ldots(s+z_m)}{(s+p_1)(s+p_2)\ldots(s+p_j)\ldots(s+p_m)}$$

The roots of the numerator N(s) (that is, z_i) are called the zeros of the network function, and the roots of the denominator D(s) (that is, p_j) are called the poles of the network function. S is a complex frequency[1].

The dynamic behavior of the network depends upon the location of the poles and zeros on the network function curve. The poles are called the natural frequencies of the network. In general, you can graphically deduce the magnitude and phase curve of any network function from the location of its poles and zeros[2].

The section “References”at the end of this chapter lists a variety of source material addressing transfer functions of physical systems[3], design of systems and physical modeling[4], and interconnect transfer function modeling[5,6].

Using Pole/Zero Analysis

Star-Hspice uses the Muller method[7] to calculate the roots of polynomials N(s) and D(s). This method approximates the polynomial with a quadratic equation that fits through three points in the vicinity of a root. Successive iterations toward a particular root are obtained by finding the nearer root of a quadratic whose curve passes through the last three points.

In Muller's method, the selection of the three initial points affects the convergence of the process and accuracy of the roots obtained. If the poles or zeros are spread over a wide frequency range, choose (X0R, X0I) close to the origin to find poles or zeros at zero frequency first. Then find the remaining poles or zeros in increasing order. The values (X1R, X1I) and (X2R, X2I) may be orders of magnitude larger than (X0R, X0I). If there are poles or zeros at high frequencies, X1I and X2I should be adjusted accordingly.

Pole/zero analysis results are based on the circuit's DC operating point, so the operating point solution must be accurate. Consequently, the .NODESET statement (not .IC) is recommended for initialization to avoid DC convergence problems.

.PZ (Pole/Zero) Statement

The syntax is:

```
.PZ output input
```

PZ	invokes the pole/zero analysis
input	input source, which may be any independent voltage or current source name
output	output variables, which may be any node voltage, V(n), or any branch current, I(element name)

Examples

```
.PZ   V(10)   VIN
.PZ   I(RL)   ISORC
.PZ   I1(M1)  VSRC
```

Pole/Zero Control Options

CSCAL	sets the capacitance scale. Capacitances are multiplied by CSCAL. Default=1e+12.
FMAX	sets the maximum pole and zero frequency value. Default=1.0e+12 · FSCAL.
FSCAL	sets the frequency scale. Frequency is multiplied by FSCAL. Default=1e-9.
GSCAL	sets the conductance scale. Conductances are multiplied by GSCAL, and resistances are divided by GSCAL. Default=1e+3.
ITLPZ	sets the pole/zero analysis iteration limit. Default=100.
LSCAL	sets the inductance scale. Inductances are multiplied by LSCAL. Default=1e+6. ***Note:*** The scale factors must satisfy the following relations. $GSCAL = CSCAL \cdot FSCAL$ $GSCAL = \frac{1}{LSCAL \cdot FSCAL}$ If scale factors are changed, the initial Muller points, (X0R, X0I), (X1R, X1I) and (X2R, X2I), may have to be modified, even though internally the program multiplies the initial values by (1e-9/GSCAL).
PZABS	sets absolute tolerances for poles and zeros. This option affects the low frequency poles or zeros. It is used as follows: If $(\lvert X_{real}\rvert + \lvert X_{imag}\rvert < PZABS)$, then $X_{real} = 0$ and $X_{imag} = 0$. This option is also used for convergence tests. Default=1e-2.
PZTOL	sets the relative error tolerance for poles or zeros. Default=1.0e-6.

RITOL	sets the minimum ratio value for (real/imaginary) or (imaginary/real) parts of the poles or zeros. Default1.0e-6. RITOL is used as follows: If $\lvert X_{imag} \rvert \le RITOL \cdot \lvert X_{real} \rvert$, then $X_{imag} = 0$ If $\lvert X_{real} \rvert \le RITOL \cdot \lvert X_{imag} \rvert$, then $X_{real} = 0$
(X0R,X0I) *(x1R,X1I)* *(X2R,X21)*	the three complex starting trial points in the Muller algorithm for pole/zero analysis. Defaults: X0R=-1.23456e6 X0I=0.0 X1R=1.23456e5 X1I=0.0 X2R=+1.23456e6 X21=0.0 These initial points are multiplied by FSCAL.

Pole/Zero Analysis Examples

Pole/Zero Example 1 – Low-Pass Filter

The following is an HSPICE input file for a low-pass prototype filter for pole/zero and AC analysis[8]. This file can be found in *$installdir/demo/hspice/filters/flp5th.sp*.

Fifth-Order Low-Pass Filter HSPICE File

```
*FILE: FLP5TH.SP
5TH-ORDER LOW_PASS FILTER
****
* T = I(R2) / IIN
*   = 0.113*(S**2 + 1.6543)*(S**2 + 0.2632) /
*     (S**5 + 0.9206*S**4 + 1.26123*S**3 +
*      0.74556*S**2 + 0.2705*S  + 0.09836)
*****
.OPTIONS POST
.PZ  I(R2) IN
.AC  DEC  100  .001HZ  10HZ
.PLOT AC  IDB(R2)  IP(R2)

IN 0 1  1.00  AC  1
R1   1  0  1.0
```

```
C3   1  0  1.52
C4   2  0  1.50
C5   3  0  0.83
C1   1  2  0.93
L1   1  2  0.65
C2   2  3  3.80
L2   2  3  1.00
R2   3  0  1.00
.END
```

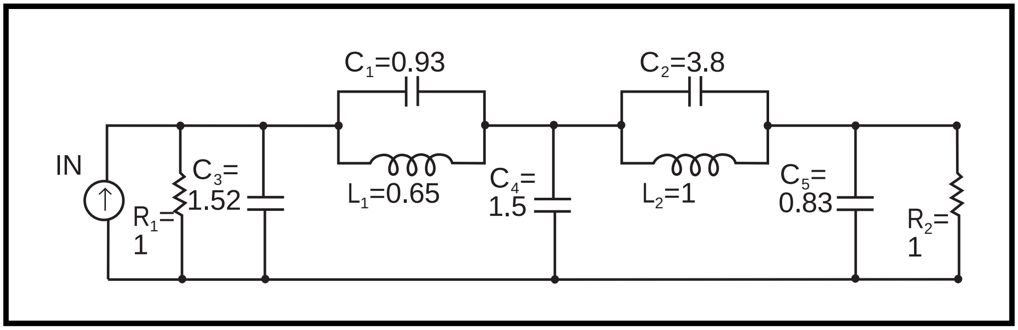

Figure 24-1: Low-Pass Prototype Filter

Table 24-1: shows the magnitude and phase variation of the current output resulting from AC analysis. These results are consistent with the pole/zero analysis. The pole/zero unit is radians per second or hertz. The X-axis unit in the plot is in hertz.

Table 24-1: Pole/Zero Analysis Results for Low-Pass Filter

Poles (rad/sec)		**Poles (hertz)**	
Real	**Imag**	**Real**	**Imag**
-6.948473e-02	-4.671778e-01	-1.105884e-02	-7.435365e-02
-6.948473e-02	4.671778e-01	-1.105884e-02	7.435365e-02
-1.182742e-01	-8.914907e-01	-1.882392e-02	-1.418852e-01
-1.182742e-01	8.914907e-01	-1.882392e-02	1.418852e-01
-5.450890e-01	0.000000e+00	-8.675361e-02	0.000000e+00

Table 24-1: Pole/Zero Analysis Results for Low-Pass Filter

Zeros (rad/sec		Zeros (hertz)	
Real	**Imag**	**Real**	**Imag**
0.000000e+00	-1.286180e+00	0.000000e+00	-2.047019e-01
0.000000e+00	-5.129892e-01	0.000000e+00	-8.164476e-02
0.000000e+00	5.129892e-01	0.000000e+00	8.164476e-02
0.000000e+00	1.286180e+00	0.000000e+00	2.047019e-01
Constant Factor = 1.129524e-01			

‘

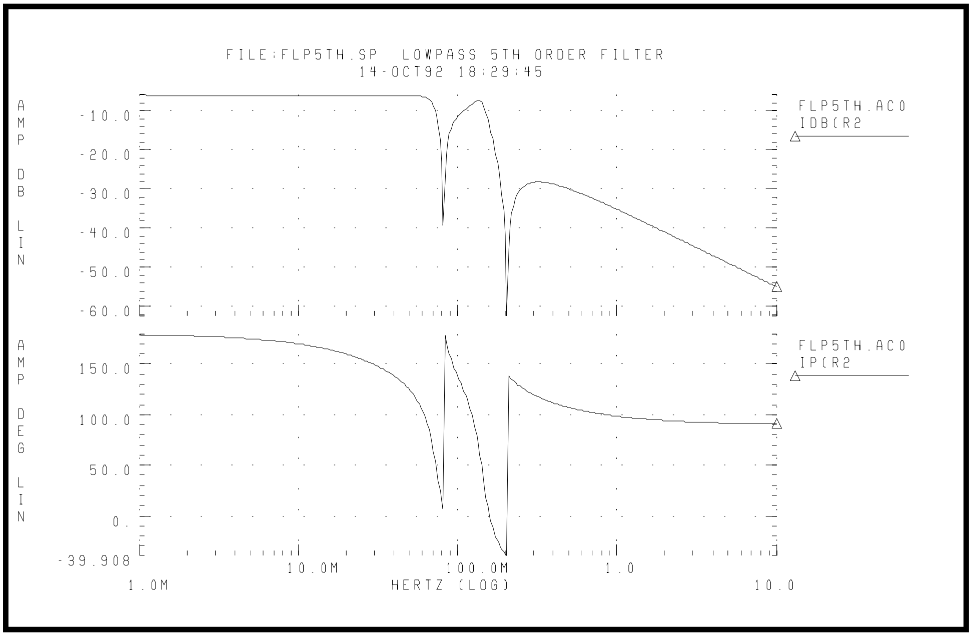

Figure 24-2: Fifth-Order Low-Pass Filter Response

Pole/Zero Example 2 – Kerwin’s Circuit

The following is an HSPICE input file for pole/zero analysis of Kerwin’s circuit[9]. This file can be found in *$installdir/demo/hspice/filters/fkerwin.sp*. Table 24-2: lists the results of the analysis.

Kerwin’s Circuit HSPICE File

```
*FILE: FKERWIN.SP
KERWIN'S CIRCUIT   HAVING JW-AXIS TRANSMISSION ZEROS.
**
* T = V(5) / VIN
*   = 1.2146 (S**2 + 2) / (S**2 + 0.1*S + 1)
* POLES = (–0.05004, +0.9987), (–0.05004, –0.9987)
* ZEROS = (0.0, +1.4142), (0.0, –1.4142)
*****
.PZ  V(5)  VIN
VIN  1  0  1
C1   1  2  0.7071
C2   2  4  0.7071
C3   3  0  1.4142
C4   4  0  0.3536
R1   1  3  1.0
R2   3  4  1.0
R3   2  5  0.5
E1   5  0  4  0  2.4293
.END
```

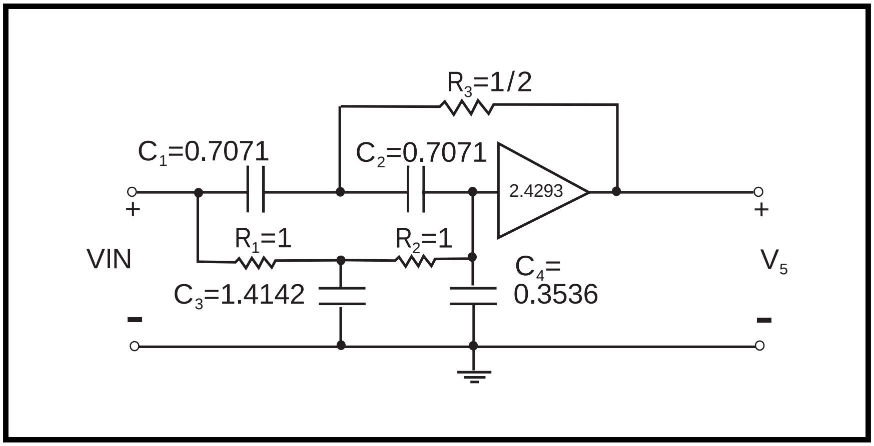

Figure 24-3: Design Example for Kerwin’s Circuit

Table 24-2: Pole/Zero Analysis Results for Kerwin’s Circuit

Poles (rad/sec)		**Poles (hertz)**	
Real	**Imag**	**Real**	**Imag**
-5.003939e-02	9.987214e-01	-7.964016e-03	1.589515e-01
-5.003939e-02	-9.987214e-01	-7.964016e-03	-1.589515e-01
-1.414227e+00	0.000000e+00	-2.250812e-01	0.000000e+00
Zeros (rad/sec		**Zeros (hertz)**	
Real	**Imag**	**Real**	**Imag**
0.000000e+00	-1.414227e+00	0.000000e+00	-2.250812e-01
0.000000e+00	1.414227e+00	0.000000e+00	2.250812e-01
-1.414227e+00	0.000000e+00	-2.250812e-01	0.000000e+00
Constant Factor = 1.214564e+00			

Pole/Zero Example 3 – High-Pass Butterworth Filter

The following is an HSPICE input file for pole/zero analysis of a high-pass Butterworth filter.[10] This file can be found in *$installdir/demo/hspice/filters/fhp4th.sp*. The analysis results are shown in Table 24-3:.

Fourth-Order High-Pass Butterworth Filter HSPICE File

```
*FILE: FHP4TH.SP
*****

* T = V(10) / VIN
*  = (S**4) / ((S**2 + 0.7653*S + 1) * (S**2 + 1.8477*S + 1))
*

* POLES, (-0.38265, +0.923895), (-0.38265, -0.923895)
*          (-0.9239, +0.3827),   (-0.9239, -0.3827)
* ZEROS, FOUR ZEROS AT (0.0, 0.0)
*****
.OPTIONS  ITLPZ=200
.PZ   V(10)   VIN
VIN  1  0  1
```

```
C1   1  2  1
C2   2  3  1
R1   3  0  2.613
R2   2  4  0.3826
E1   4  0  3  0  1
C3   4  5  1
C4   5  6  1
R3   6  0  1.0825
R4   5  10 0.9238
E2   10 0  6  0  1
RL   10 0  1E20
.END
```

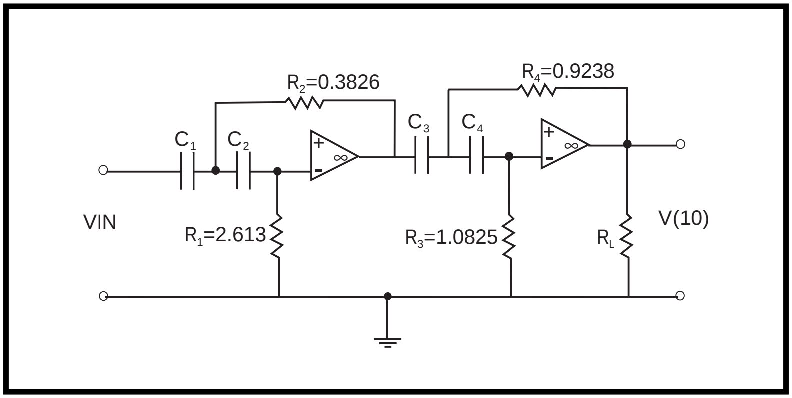

Figure 24-4: Fourth-Order High-Pass Butterworth Filter

Table 24-3: Pole/Zero Analysis Results for High-Pass Butterworth Filter

Poles (rad/sec)		Poles (hertz)	
Real	**Imag**	**Real**	**Imag**
-3.827019e-01	-9.240160e-01	-6.090889e-02	1.470617e-01
-3.827019e-01	9.240160e-01	-6.090890e-02	-1.470617e-01
-9.237875e-01	3.828878e-01	-1.470254e-01	6.093849e-02
-9.237875e-01	-3.828878e-01	-1.470254e-01	-6.093849e-02
Zeros (rad/sec		**Zeros (hertz)**	
Real	**Imag**	**Real**	**Imag**
0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
0.000000e+00	0.000000e+00	0.000000e+00	0.000000e+00
Constant Factor = 1.000000e+00			

Pole/Zero Example 4 – CMOS Differential Amplifier

The following is an HSPICE input file for pole/zero analysis of a CMOS differential amplifier for pole/zero and AC analysis. The file can be found in *$installdir/demo/hspice/apps/mcdiff.sp*. The analysis results are shown in Table 24-4:.

CMOS Differential Amplifier HSPICE File

```
FILE: MCDIFF.SP
CMOS DIFFERENTIAL AMPLIFIER
.OPTIONS PIVOT SCALE=1E-6 SCALM=1E-6  WL
.PZ V(5) VIN
VIN 7 0 0 AC 1
.AC DEC 10 20K 500MEG
.PRINT AC VDB(5) VP(5)
M1  4  0  6  6  MN  100  10  2  2
M2  5  7  6  6  MN  100  10  2  2
```

```
M3  4  4  1  1  MP  60  10  1.5  1.5
M4  5  4  1  1  MP  60  10  1.5  1.5
M5  6  3  2  2  MN  50  10  1.0  1.0
VDD  1  0  5
VSS  2  0  –5
VGG  3  0  –3
RIN  7  0   1
.MODEL  MN  NMOS  LEVEL=5  VT=1  UB=700  FRC=0.05  DNB=1.6E16
+  XJ=1.2  LATD=0.7  CJ=0.13  PHI=1.2  TCV=0.003  TOX=800
$
.MODEL  MP  PMOS  LEVEL=5  VT=-1  UB=245  FRC=0.25  TOX=800
+  DNB=1.3E15  XJ=1.2  LATD=0.9  CJ=0.09  PHI=0.5  TCV=0.002
.END
```

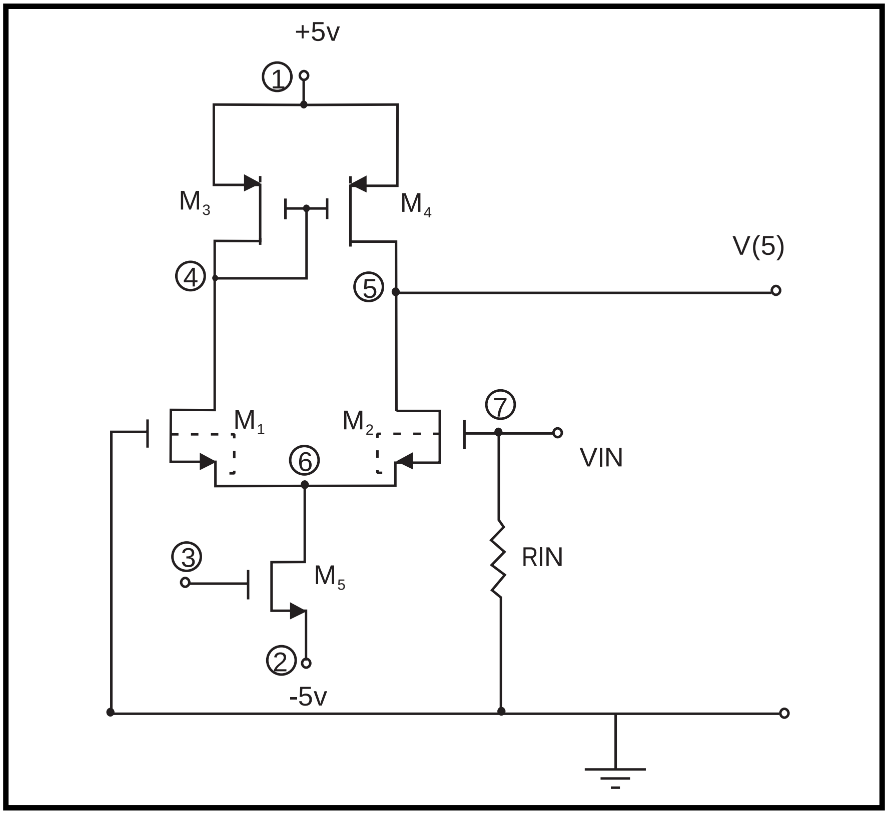

Figure 24-5: CMOS Differential Amplifier

Table 24-4: Pole/Zero Analysis Results for CMOS Differential Amplifier

Poles (rad/sec)		Poles (hertz)	
Real	**Imag**	**Real**	**Imag**
-1.798766e+06	0.000000e+00	-2.862825e+05	0.000000e+00
-1.126313e+08	-6.822910e+07	-1.792583e+07	-1.085900e+07
-1.126313e+08	6.822910e+07	-1.792583e+07	1.085900e+07
Zeros (rad/sec		**Zeros (hertz)**	
Real	**Imag**	**Real**	**Imag**
-1.315386e+08	7.679633e+07	-2.093502e+07	1.222251e+07
-1.315386e+08	-7.679633e+07	-2.093502e+07	-1.222251e+07
7.999613e+08	0.000000e+00	1.273178e+08	0.000000e+00
Constant Factor = 3.103553e-01			

Pole/Zero Example 5 – Simple Amplifier

The following is an HSPICE input file for pole/zero analysis of an equivalent circuit of a simple amplifier with RS =RPI=RL=1000 ohms, gm=0.04 mho, CMU=1.0e-11 farad, and CPI =1.0e-9 farad[1]. The file can be found in *$installdir/demo/hspice/apps/ampg.sp*. The analysis results are shown in Table 24-5:.

Amplifier HSPICE File

```
FILE: AMPG.SP
A SIMPLE AMPLIFIER.
* T = V(3) / VIN
* T = 1.0D6*(S - 4.0D9) / (S**2 + 1.43D8*S + 2.0D14)
* POLES =  (-0.14D7, 0.0), (-14.16D7, 0.0)
* ZEROS =  (+4.00D9, 0.0)

.PZ   V(3)   VIN
RS   1  2  1K
RPI  2  0  1K
RL   3  0  1K
```

```
GMU  3  0  2  0  0.04
CPI  2  0  1NF
CMU  2  3  10PF
VIN  1  0  1
.END
```

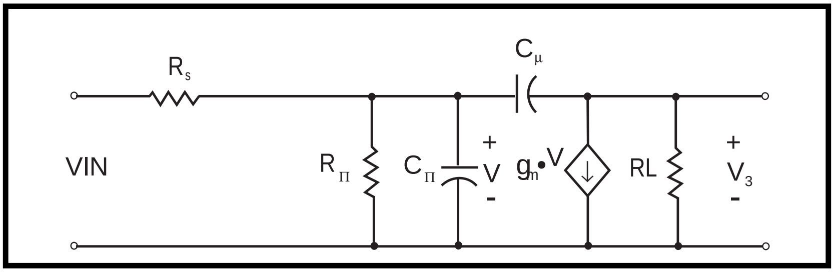

Figure 24-6: Simple Amplifier

<table>
<tr><th colspan="2">Poles (rad/sec)</th><th colspan="2">Poles (hertz)</th></tr>
<tr><th>Real</th><th>Imag</th><th>Real</th><th>Imag</th></tr>
<tr><td>-1.412555+06</td><td>0.000000e+00</td><td>-2.248151e+05</td><td>0.000000e+00</td></tr>
<tr><td>-1.415874+08</td><td>0.000000e+00</td><td>-2.253434e+07</td><td>0.000000e+00</td></tr>
<tr><th colspan="2">Zeros (rad/sec</th><th colspan="2">Zeros (hertz)</th></tr>
<tr><th>Real</th><th>Imag</th><th>Real</th><th>Imag</th></tr>
<tr><td>4.000000e+09</td><td>0.000000e+00</td><td>6.366198e+08</td><td>0.000000e+00</td></tr>
<tr><td colspan="4">Constant Factor = 1.000000e+06</td></tr>
</table>

Table 24-5: Pole/Zero Analysis Results for Amplifier

Pole/Zero Example 6— Active Low-Pass Filter

The following is an HSPICE input file for pole/zero analysis of an active ninth-order low-pass filter[12] using the ideal op-amp element. AC analysis is performed. The file can be found in *$installdir/demo/hspice/filters/flp9th.sp*. The analysis results are shown in Table 24-6:.

Ninth Order Low-Pass Filter HSPICE File

```
FILE: FLP9TH.SP
******
VIN  IN  0  AC 1
.PZ  V(OUT) VIN
.AC  DEC  50  .1K  100K
.OPTIONS  POST  DCSTEP=1E3  X0R=-1.23456E+3  X1R=-1.23456E+2
+  X2R=1.23456E+3  FSCAL=1E-6  GSCAL=1E3  CSCAL=1E9  LSCAL=1E3
.PLOT  AC  VDB(OUT)
.SUBCKT OPAMP IN+ IN- OUT GM1=2 RI=1K CI=26.6U GM2=1.33333
RL=75
RII IN+ IN- 2MEG
RI1 IN+ 0 500MEG
RI2 IN- 0 500MEG
G1  1  0  IN+  IN-  GM1
C1  1  0  CI
R1  1  0  RI
G2   OUT  0  1  0  GM2
RLD  OUT  0  RL
.ENDS
.SUBCKT FDNR 1 R1=2K C1=12N R4=4.5K
RLX=75
R1  1  2  R1
C1  2  3  C1
R2  3  4  3.3K
R3  4  5  3.3K
R4  5  6  R4
C2  6  0  10N
XOP1  2  4  5  OPAMP
XOP2  6  4  3  OPAMP
.ENDS
*
RS IN 1 5.4779K
R12 1 2 4.44K
R23 2 3 3.2201K
R34 3 4 3.63678K
R45 4 OUT 1.2201K
C5 OUT 0 10N
X1 1 FDNR R1=2.0076K  C1=12N    R4=4.5898K
X2 2 FDNR R1=5.9999K  C1=6.8N   R4=4.25725K
X3 3 FDNR R1=5.88327K C1=4.7N   R4=5.62599K
X4 4 FDNR R1=1.0301K  C1=6.8N   R4=5.808498K
.END
```

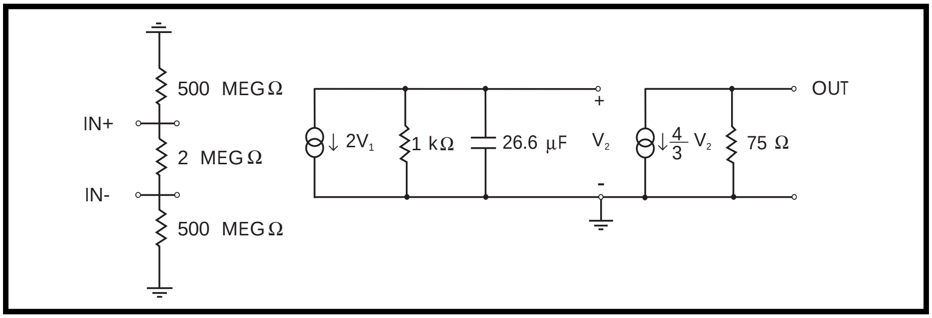

Figure 24-7: Linear Model of the 741C Op-Amp

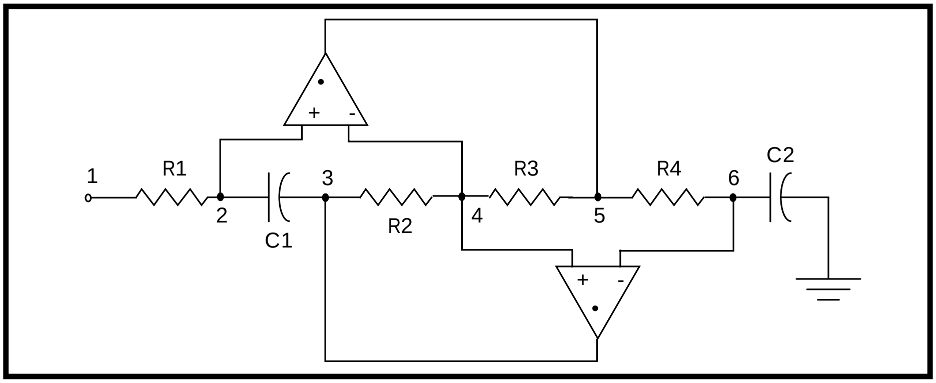

Figure 24-8: The FDNR Subcircuit

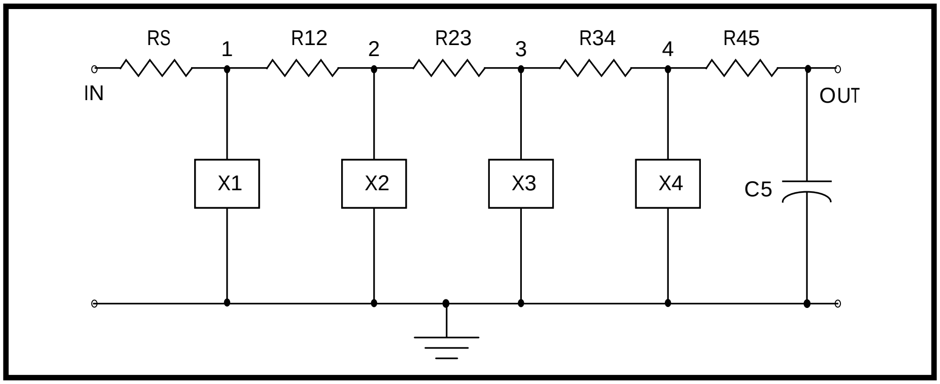

Figure 24-9: Active Realization of the Low-Pass Filter

Table 24-6: Pole/Zero Analysis Results for the Active Low-Pass Filter

Poles (rad/sec)		**Poles (hertz)**	
Real	**Imag**	**Real**	**Imag**
-4.505616e+02	-2.210451e+04	-7.170911e+01	-3.518042e+03
-4.505616e+02	2.210451e+04	-7.170911e+01	3.518042e+03
-1.835284e+03	2.148369e+04	-2.920944e+02	3.419236e+03
-1.835284e+03	-2.148369e+04	-2.920944e+02	-3.419236e+03
-4.580172e+03	1.944579e+04	-7.289571e+02	3.094894e+03
-4.580172e+03	-1.944579e+04	-7.289571e+02	-3.094894e+03
-9.701962e+03	1.304893e+04	-1.544115e+03	2.076802e+03
-9.701962e+03	-1.304893e+04	-1.544115e+03	-2.076802e+03
-1.353908e+04	0.000000e+00	-2.154811e+03	0.000000e+00
-3.668995e+06	-3.669793e+06	-5.839386e+05	-5.840657e+05
-3.668995e+06	3.669793e+06	-5.839386e+05	5.840657e+05
-3.676439e+06	-3.676184e+06	-5.851234e+05	-5.850828e+05
-3.676439e+06	3.676184e+06	-5.851234e+05	5.850828e+05
-3.687870e+06	3.687391e+06	-5.869428e+05	5.868665e+05
-3.687870e+06	-3.687391e+06	-5.869428e+05	-5.868665e+05
-3.695817e+06	-3.695434e+06	-5.882075e+05	-5.881466e+05
-3.695817e+06	+3.695434e+06	-5.882075e+05	5.881466e+05

Zeroes (rad/sec)		**Zeroes (hertz)**	
Real	**Imag**	**Real**	**Imag**
-3.220467e-02	-2.516970e+04	-5.125532e-03	-4.005882e+03
-3.220467e-02	2.516970e+04	-5.125533e-03	4.005882e+03
2.524420e-01	-2.383956e+04	4.017739e-02	-3.794184e+03
2.524420e-01	2.383956e+04	4.017739e-02	3.794184e+03
1.637164e+00	2.981593e+04	2.605627e-01	4.745353e+03
1.637164e+00	-2.981593e+04	2.605627e-01	-4.745353e+03

Table 24-6: Pole/Zero Analysis Results for the Active Low-Pass Filter

4.888484e+00	4.852376e+04	7.780265e-01	7.722796e+03
4.888484e+00	-4.852376e+04	7.780265e-01	-7.722796e+03
-3.641366e+06	-3.642634e+06	-5.795413e+05	-5.797432e+05
-3.641366e+06	3.642634e+06	-5.795413e+05	5.797432e+05
-3.649508e+06	-3.649610e+06	-5.808372e+05	-5.808535e+05
-3.649508e+06	3.649610e+06	-5.808372e+05	5.808535e+05
-3.683700e+06	3.683412e+06	-5.862790e+05	5.862333e+05
-3.683700e+06	-3.683412e+06	-5.862790e+05	-5.862333e+05
-3.693882e+06	3.693739e+06	5.878995e+05	5.878768e+05
-3.693882e+06	-3.693739e+06	-5.878995e+05	-5.878768e+05
Constant Factor = 4.451586e+02			

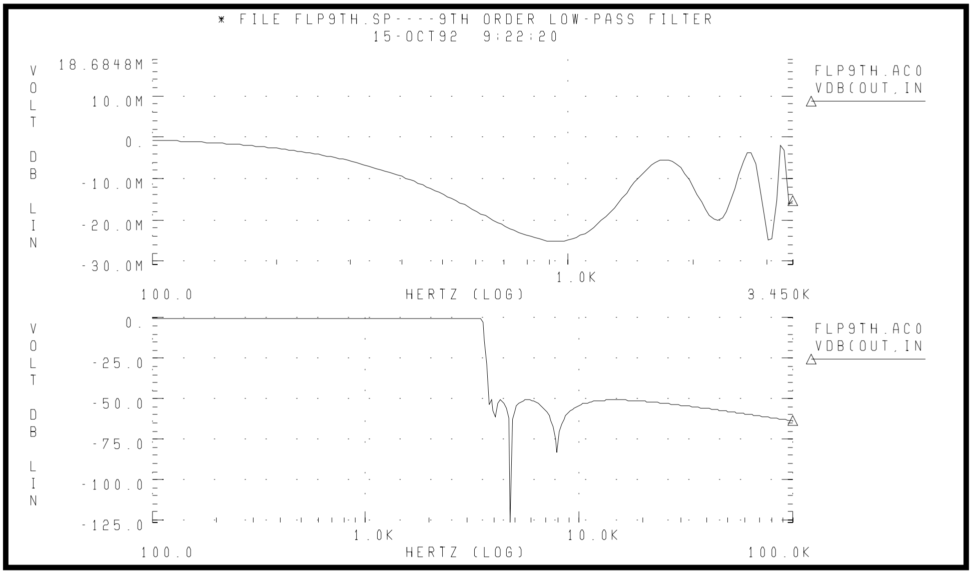

Figure 24-10: 9th Order Low-Pass Filter Response

The top graph in Figure 24-10: plots the bandpass response of the Pole/Zero Example 6 low-pass filter. The bottom graph shows the overall response of the low-pass filter.

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