Avant!

Chapter 25

Performing FFT Spectrum Analysis

Spectrum analysis is the process of determining the frequency domain representation of a time domain signal and most commonly employs the Fourier transform. The Discrete Fourier Transform (DFT) is used to determine the frequency content of analog signals encountered in circuit simulation, which deals with sequences of time values. The Fast Fourier Transform (FFT) is an efficient method for calculating the DFT, and Star-Hspice uses it to provide a highly accurate spectrum analysis tool.

The .FFT statement in Star-Hspice uses the internal time point values and, through a second order interpolation, obtains waveform samples based on the user-specified number of points. This accounts for the high degree of accuracy of the FFT results in Star-Hspice. Moreover, by using one of the windowing functions, you can reduce the effects of truncation of the waveform on the spectral content. The .FFT command also allows you to specify the desired output format, to specify a frequency of interest, and to obtain any number of harmonics, as well as the total harmonic distortion (THD).

This chapter covers the following topics:

Using Windows In FFT Analysis

One problem with spectrum analysis in circuit simulators is that the duration of the signals is finite, although adjustable. Applying the FFT method to finite-duration sequences can produce inadequate results because of "spectral leakage", due primarily to the periodic extension assumption underlying DFT.

The effect occurs when the finite duration of the signal does not result in a sequence that contains a whole number of periods. This is especially true when FFT is used for signal detection or estimation – that is, for detecting weak signals in the presence of strong signals or resolving a cluster of equal strength frequencies.

In FFT analysis, "windows" are frequency weighting functions applied to the time domain data to reduce the spectral leakage associated with finite-duration time signals. Windows are smoothing functions that peak in the middle frequencies and decrease to zero at the edges, thus reducing the effects of the discontinuities as a result of finite duration. Figure 25-1: shows the windows available in Star-Hspice. Table 25-1: lists the common performance parameters for FFT windows available in Star-Hspice.

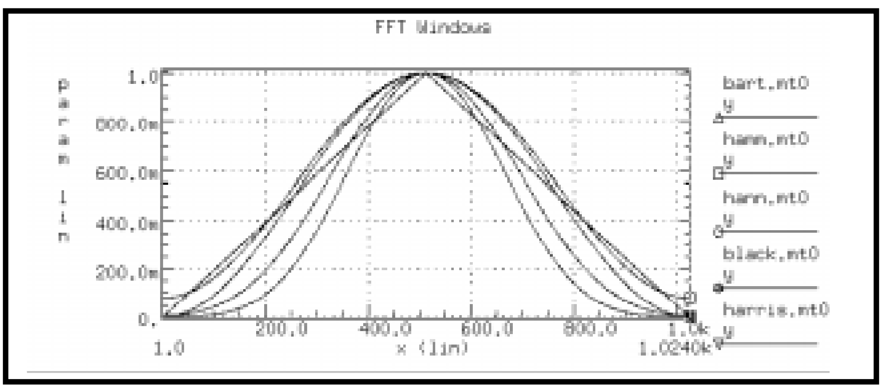

Figure 25-1: FFT Windows

Table 25-1: Window Weighting Characteristics in FFT Analysis

Window	Equation	Highest Side-Lobe (dB)	Side-Lobe Roll-Off (dB/octave)	3.0-dB Bandwidth (1.0/T)	Worst Case Process Loss (dB)
Rectangular	W(n)=1, $0 \le n < \mathrm{NP}$ †	-13	-6	0.89	3.92
Bartlett	W(n)=2n/(NP–1), $0 \le n \le$ (NP/2)–1 W(n)=2–2n/(NP–1), NP/2 $\le n <$ NP	-27	-12	1.28	3.07
Hanning	W(n)=0.5–0.5[cos(2πn/(NP–1))], $0 \le n <$ NP	-32	-18	1.44	3.18
Hamming	W(n)=0.54–0.46[cos(2πn/(NP–1))], $0 \le n <$ NP	-43	-6	1.30	3.10
Blackman	W(n)=0.42323 –0.49755[cos(2πn/(NP–1))] +0.07922cos[cos(4πn/(NP–1))], $0 \le n <$ NP	-58	-18	1.68	3.47
Blackman-Harris	W(n)=0.35875 –0.48829[cos(2πn/(NP–1))] +0.14128[cos(4πn/(NP–1))] –0.01168[cos(6πn/(NP–1))], $0 \le n <$ NP	-92	-6	1.90	3.85
Gaussian a=2.5 a=3.0 a=3.5	W(n)=exp[-0.5a2(NP/2-1-n)2/(NP)2], 0 ≤ n ≤ (NP/2)-1 W(n)=exp[-0.5a2(n-NP/2)2/(NP)2], NP/2 ≤ n < NP	 -42 -55 -69	 -6 -6 -6	 1.33 1.55 1.79	 3.14 3.40 3.73
Kaiser-Bessel a=2.0 a=2.5 a=3.0 a=3.5	W(n)=I0(x2)/I0(x1) x1=pa x2=x1*sqrt[1-(2(NP/2-1-n)/NP)2], 0 ≤ n ≤ (NP/2)-1 x2=x1*sqrt[1-(2(n-NP/2)/NP)2], NP/2 ≤ n < NP I0 is the zero-order modified Bessel function	 -46 -57 -69 -82	 -6 -6 -6 -6	 1.43 1.57 1.71 0.89	 3.20 3.38 3.56 3.74

†NP is the number of points used for the FFT analysis.

The most important parameters in Table 25-1: are the highest side-lobe level (to reduce bias, the lower the better) and the worst-case processing loss (to increase detectability, the lower the better). Some compromise usually is necessary to find a suitable window filtering for each application. As a rule, the window performance improves with functions of higher complexity (those listed lower in the table). The Kaiser window has an ALFA parameter that allows adjustment of the compromise between different figures of merit for the window.

The simple rectangular window produces a simple bandpass truncation in the classical Gibbs phenomenon. The Bartlett or triangular window has good processing loss and good side-lobe roll-off, but lacks sufficient bias reduction. The Hanning, Hamming, Blackman, and Blackman-Harris windows use progressively more complicated cosine functions that provide smooth truncation and a wide range of side-lobe level and processing loss. The last two windows in the table are parameterized windows that allow you to adjust the side-lobe level, the 3 dB bandwidth, and the processing loss.[1]

The characteristics of two typical windows are shown in Figures 25-2 and 25-3.

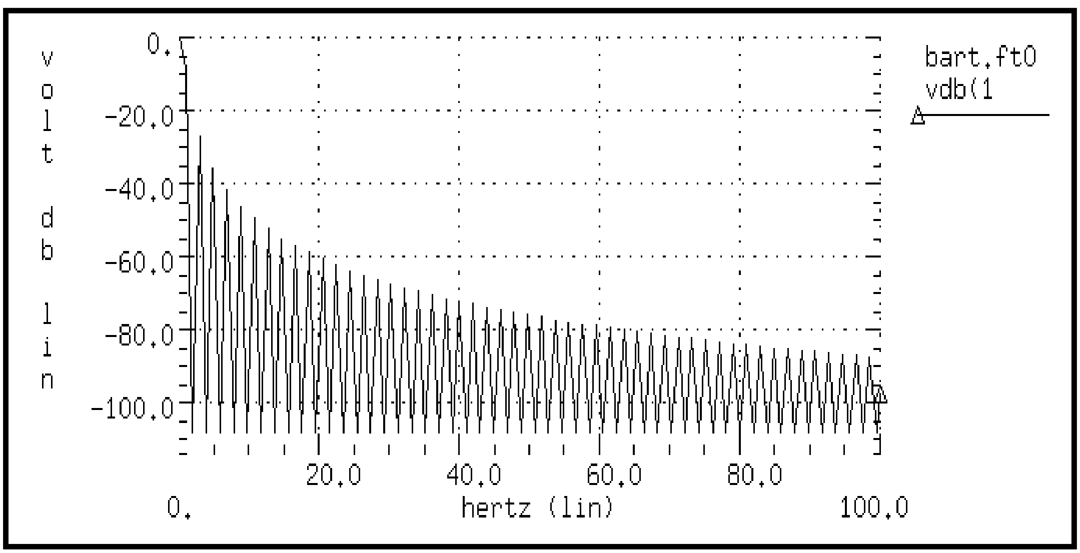

Figure 25-2: Bartlett Window Characteristics

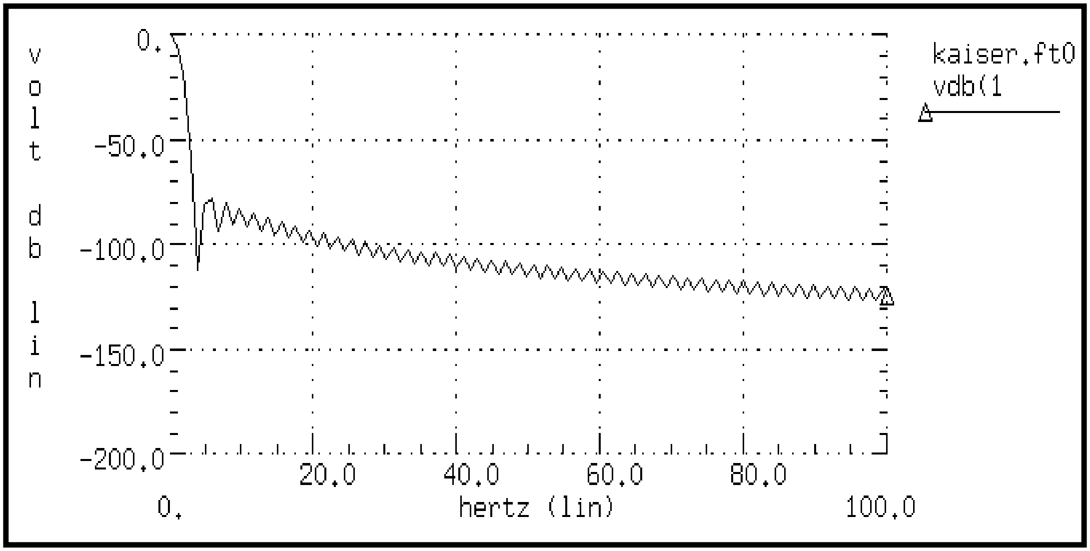

Figure 25-3: Kaiser-Bessel Window Characteristics, ALFA=3.0

Using the .FFT Statement

The general form of the .FFT statement is shown below. The parameters are described in Table 25-2:.

```
.FFT <output_var> <START=value> <STOP=value> <NP=value>
<FORMAT=keyword> <WINDOW=keyword> <ALFA=value> <FREQ=value>
<FMIN=value> <FMAX=value>
```

Table 25-2: .FFT Statement Parameters

Parameter	Default	Description
output_var	—	can be any valid output variable, such as voltage, current, or power
START	see Description	specifies the beginning of the output variable waveform to be analyzed – Defaults to the START value in the .TRAN statement, which defaults to 0 s.
FROM	see START	an alias for START in .FFT statements
STOP	see Description	specifies the end of the output variable waveform to be analyzed. Defaults to the TSTOP value in the .TRAN statement.
TO	see STOP	an alias for STOP in .FFT statements
NP	1024	specifies the number of points used in the FFT analysis. NP must be a power of 2; if NP is not a power of 2, Star-Hspice automatically adjusts it to the closest higher number that is a power of 2.
FORMAT	NORM	specifies the output format: NORM= normalized magnitude UNORM=unnormalized magnitude

Table 25-2: .FFT Statement Parameters

Parameter	Default	Description
WINDOW	RECT	specifies the window type to be used: RECT=simple rectangular truncation window BART=Bartlett (triangular) window HANN=Hanning window HAMM=Hamming window BLACK=Blackman window HARRIS=Blackman-Harris window GAUSS=Gaussian window KAISER=Kaiser-Bessel window
ALFA	3.0	specifies the parameter used in GAUSS and KAISER windows to control the highest side-lobe level, bandwidth, and so on 1.0 <= ALFA <= 20.0
FREQ	0.0 (Hz)	specifies a frequency of interest. If FREQ is nonzero, the output listing is limited to the harmonics of this frequency, based on FMIN and FMAX. The THD for these harmonics also is printed.
FMIN	1.0/T (Hz)	specifies the minimum frequency for which FFT output is printed in the listing file or which is used in THD calculations. T = (STOP–START)
FMAX	0.5*NP*FMIN (Hz)	specifies the maximum frequency for which FFT output is printed in the listing file or which is used in THD calculations.

Syntax Examples

Below are four examples of valid .FFT statements.

```
.fft v(1)
```

```
.fft v(1,2) np=1024 start=0.3m stop=0.5m freq=5.0k
window=kaiser alfa=2.5
```

```
.fft I(rload) start=0m to=2.0m fmin=100k fmax=120k
format=unorm
```

```
.fft 'v(1) + v(2)' from=0.2u stop=1.2u window=harris
```

Only one output variable is allowed in an .FFT command. The following is an *incorrect* use of the command.

```
.fft v(1) v(2) np=1024
```

The correct use of the command is shown in the example below. In this case, an *.ft0* and an *.ft1* file are generated for the FFT of v(1) and v(2), respectively.

```
.fft v(1) np=1024
.fft v(2) np=1024
```

Examining the FFT Output

Star-Hspice prints the results of the FFT analysis in a tabular format in the *.lis* file, based on the parameters in the .FFT statement. The normalized magnitude values are printed unless you specify FORMAT= UNORM, in which case unnormalized magnitude values are printed. The number of printed frequencies is half the number of points (NP) specified in the .FFT statement.

If you specify a minimum or a maximum frequency using FMIN or FMAX, the printed information is limited to the specified frequency range. Moreover, if you specify a frequency of interest using FREQ, then the output is limited to the harmonics of this frequency, along with the percent of total harmonic distortion.

In the sample output below, notice that all the parameters used in the FFT analysis are defined in the header.

```
****** Sample FFT output extracted from the .lis file

fft test ... sine
 ******  fft analysis                     tnom=  25.000
temp=  25.000
******
fft components of transient response v(1)

Window: Rectangular
First Harmonic:    1.0000k
Start Freq:    1.0000k
Stop  Freq:   10.0000k

dc component: mag(db)=  -1.132D+02  mag=   2.191D-06
phase=   1.800D+02

frequency    frequency    fft_mag     fft_mag    fft_phase
  index        (hz)        (db)                    (deg)
    2         1.0000k      0.          1.0000      -3.8093m
    4         2.0000k   -125.5914    525.3264n     -5.2406
    6         3.0000k   -106.3740      4.8007u    -98.5448
    8         4.0000k   -113.5753      2.0952u     -5.5966
   10         5.0000k   -112.6689      2.3257u   -103.4041
   12         6.0000k   -118.3365      1.2111u    167.2651
   14         7.0000k   -109.8888      3.2030u   -100.7151
   16         8.0000k   -117.4413      1.3426u    161.1255
```

```
18          9.0000k    -97.5293     13.2903u     70.0515
20         10.0000k   -114.3693      1.9122u    -12.5492
total harmonic distortion =     1.5065m  percent
```

The preceding example specifies a frequency of 1 kHz and THD up to 10 kHz, which corresponds to the first ten harmonics.

Note: The highest frequency shown in the Star-Hspice FFT output might not be exactly the same as the specified FMAX, due to adjustments made by Star-Hspice.

Table 25-3: describes the output of the Star-Hspice FFT analysis.

Table 25-3: .FFT Output Description

Column Heading	Description
Frequency Index	runs from 1 to NP/2, or the corresponding index for FMIN and FMAX. Note that the DC component corresponding to the index 0 is displayed independently.
Frequency	the actual frequency associated with the index
fft_mag (dB), fft_mag	There are two FFT magnitude columns, the first in dB and the second in the units of the output variable. The magnitude is normalized unless UNORM format is specified.
fft_phase	the associated phase, in degrees

A *.ft#* file is generated, in addition to the listing file, for each FFT output variable. The *.ft#* file contains the graphical data needed to display the FFT analysis results in MetaWaves. The magnitude in dB and the phase in degrees are available for display.

Notes:

1. The following formula should be used as a guideline when specifying a frequency range for FFT output:

   $$\text{frequency increment} = 1.0/(\text{STOP} - \text{START})$$

   Each frequency index corresponds to a multiple of this increment. Hence, to obtain a finer frequency resolution you should maximize the duration of the time window.

2. FMIN and FMAX have no effect on the *.ft0*, *.ft1*, ..., *.ftn* files.

Example 1 – AM Modulation

This example input listing on the following page shows a 1 kHz carrier (FC) that is modulated by a 100 Hz signal (FM). The voltage at node 1, which is an AM signal, can be described by

$$1) = sa \cdot (offset + \sin(\omega_m(Time - td))) \cdot \sin(\omega_c(Time - td$$

The preceding equation can be expanded as follows.

$$v(1) = (sa \cdot offset \cdot \sin(\omega_c(Time - td)) + 0.5 \cdot sa \cdot \cos((\omega_c - \omega_m)(Time - td)))$$

$$- 0.5 \cdot sa \cdot \cos((\omega_c + \omega_m)(Time - td))$$

where

$$\omega_c = 2\pi f_c$$

$$\omega_f = 2\pi f_m$$

The preceding equations indicate that v(1) is a summation of three signals with frequency f_c , $(f_c - f_m)$, and $(f_c + f_m)$ — namely, the carrier frequency and the two sidebands.

Input Listing

```
AM Modulation

.OPTION post
.PARAM sa=10 offset=1 fm=100 fc=1k td=1m
VX 1 0 AM(sa offset fm fc td)
Rx 1 0 1

.TRAN 0.01m 52m
.FFT V(1) START=10m STOP=40m FMIN=833 FMAX=1.16K
.END
```

Output Listing

The relevant portion of the listing file is shown below.

```
*********
am modulation
 ******  fft analysis                     tnom=  25.000
temp=  25.000
******
fft components of transient response v(1)
Window: Rectangular
Start Freq:  833.3333
Stop  Freq:    1.1667k

dc component: mag(db)=  -1.480D+02  mag=   3.964D-08
phase=   0.000D+00

frequency    frequency    fft_mag     fft_mag     fft_phase
  index        (hz)        (db)                    (deg)
   25        833.3333    -129.4536   336.7584n   -113.0047
   26        866.6667    -143.7912    64.6308n     45.6195
   27        900.0000      -6.0206   500.0008m     35.9963
   28        933.3333    -125.4909   531.4428n    112.6012
   29        966.6667    -142.7650    72.7360n    -32.3152
   30          1.0000k      0.          1.0000    -90.0050
   31          1.0333k   -132.4062   239.7125n     -9.0718
   32          1.0667k   -152.0156    25.0738n      3.4251
   33          1.1000k     -6.0206   499.9989m    143.9933
   34          1.1333k   -147.0134    44.5997n     -3.0046
   35          1.1667k   -147.7864    40.8021n     -4.7543
         ***** job concluded
```

Graphical Output

Figures 25-4 and 25-5 display the results. Figure 25-4: shows the time domain curve of node 1. Figure 25-5: shows the frequency domain components of the magnitude of node 1. Note the carrier frequency at 1 kHz, with two sideband frequencies 100 Hz apart. The third, fifth and seventh harmonics are more than 100 dB below the fundamental, indicating excellent numerical accuracy. Since the time domain data contains an integer multiple of the period, no windowing is needed.

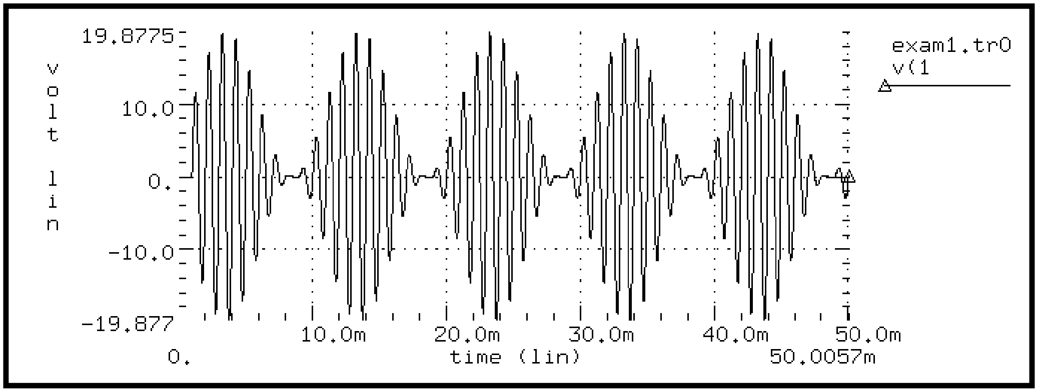

Figure 25-4: AM Modulation

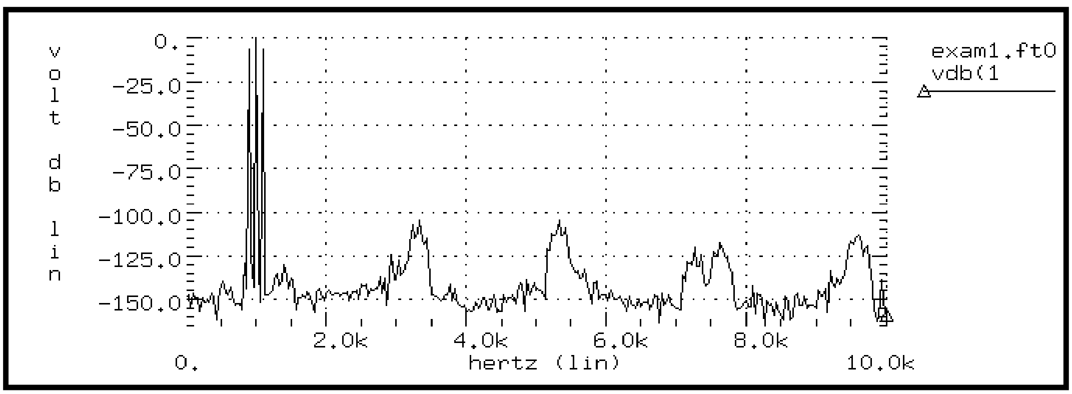

Figure 25-5: AM Modulation Spectrum

Example 2 – Balanced Modulator and Demodulator

Demodulation, or detection, is the process of recovering a modulating signal from the modulated output voltage. The netlist below illustrates this process, using Star-Hspice behavioral models and FFT analysis to confirm the validity of the process in the frequency domain. The Laplace element is used in the low-pass filter. This filter introduces some delay in the output signal, which causes spectral leakage if no windowing is used in FFT. However, when window weighting is used to perform FFT, the spectral leakage is virtually eliminated. This can be verified from the THD of the two outputs shown in the output listing that follows. Since a 1 kHz output signal is expected, a frequency of 1 kHz is specified in the .FFT command. Additionally, specifying the desired FMAX provides the first few harmonics in the output listing for THD calculations.

Input Listing

```
Balanced Modulator & Demodulator Circuit

V1 mod1 GND sin(0 5 1K 0 0 0) $ modulating signal
r1 mod1 2 10k
r2 2 3 10k
r3 2 GND 10K
E1 3 GND OPAMP 2 GND $ buffered output of modulating signal
V2 mod2 GND sin(0 5 10K 0 0 0) $ modulated signal
E2 modout GND  vol='(v(3)*v(mod2))/10.0' $ multiply to
modulate
V3 8 GND sin(0 5 10K 0 0 0)
E3 demod GND  vol='(v(modout)*v(8))/10.0' $ multiply to
demodulate
* use a laplace element for filtering
E_filter lpout 0 laplace demod 0 67.11e6 / 66.64e6 6.258e3
1.0 $ filter out +the modulating signal
*
.tran 0.2u 4m
.fft v(mod1)
.fft v(mod2)
.fft v(modout)
.fft v(demod)
```

```
.fft v(lpout) freq=1.0k fmax=10k $ ask to see the first few
harmonics
.fft v(lpout) window=harris freq=1.0k fmax=10k $ window
should reduce +spectral leakage
.probe tran v(mod1) V(mod2) v(modout) v(demod) v(lpout)
.option acct post probe
.end
```

Output Listing

The relevant portion of the output listing is shown below to illustrate the effect of windowing in reducing spectral leakage and consequently, reducing the THD.

```
balanced modulator & demodulator circuit
 ******  fft analysis                     tnom=  25.000
temp=  25.000
******

fft components of transient response v(lpout)

Window: Rectangular
First Harmonic:    1.0000k
Start Freq:    1.0000k
Stop  Freq:   10.0000k

dc component: mag(db)=  -3.738D+01  mag=   1.353D-02
phase=   1.800D+02
frequency    frequency    fft_mag     fft_mag    fft_phase
  index        (hz)        (db)                    (deg)
    4         1.0000k      0.          1.0000      35.6762
    8         2.0000k    -26.6737     46.3781m    122.8647
   12         3.0000k    -31.4745     26.6856m    108.1100
   16         4.0000k    -34.4833     18.8728m    103.6867
   20         5.0000k    -36.6608     14.6880m    101.8227
   24         6.0000k    -38.3737     12.0591m    100.9676
   28         7.0000k    -39.7894     10.2455m    100.6167
   32         8.0000k    -40.9976      8.9150m    100.5559
   36         9.0000k    -42.0524      7.8955m    100.6783
   40        10.0000k    -42.9888      7.0886m    100.9240
    total harmonic distortion =     6.2269   percent
******

balanced modulator & demodulator circuit
```

```
 ******  fft analysis                     tnom=  25.000
temp=  25.000
******
fft components of transient response v(lpout)

Window: Blackman-Harris
First Harmonic:    1.0000k
Start Freq:    1.0000k
Stop  Freq:   10.0000k

dc component: mag(db)=  -8.809D+01  mag=   3.938D-05
phase=   1.800D+02
frequency    frequency    fft_mag     fft_mag    fft_phase

  index        (hz)        (db)                    (deg)
    4         1.0000k      0.          1.0000      34.3715
    8         2.0000k    -66.5109    472.5569u    -78.8512
   12         3.0000k    -97.5914     13.1956u    -55.7167
   16         4.0000k   -107.8004      4.0736u    -41.6389
   20         5.0000k   -117.9984      1.2592u    -23.9325
   24         6.0000k   -125.0965    556.1309n     33.3195
   28         7.0000k   -123.6795    654.6722n     74.0461
   32         8.0000k   -122.4362    755.4258n     86.5049
   36         9.0000k   -122.0336    791.2570n     91.6976
   40        10.0000k   -122.0388    790.7840n     94.5380

    total harmonic distortion =    47.2763m  percent
*******
```

The signals and their spectral content are shown in Figures 25-6 through 25-14. The modulated signal contains only the sum and the difference of the carrier frequency and the modulating signal (1 kHz and 10 kHz). At the receiver end the carrier frequency is recovered in the demodulated signal, which also shows a 10 kHz frequency shift in the above signals (to 19 kHz and 21 kHz).

A low-pass filter is used to extract the carrier frequency using a second order Butterworth filter. Use of a Harris window significantly improves the noise floor in the filtered output spectrum and reduces THD in the output listing (from 9.23% to 0.047%). However, it appears that a filter with a steeper transition region and better delay characteristics is needed to suppress the modulating frequencies below the -60 dB level. The “Filtered Output Signal” waveform in Figure 25-9: is normalized.

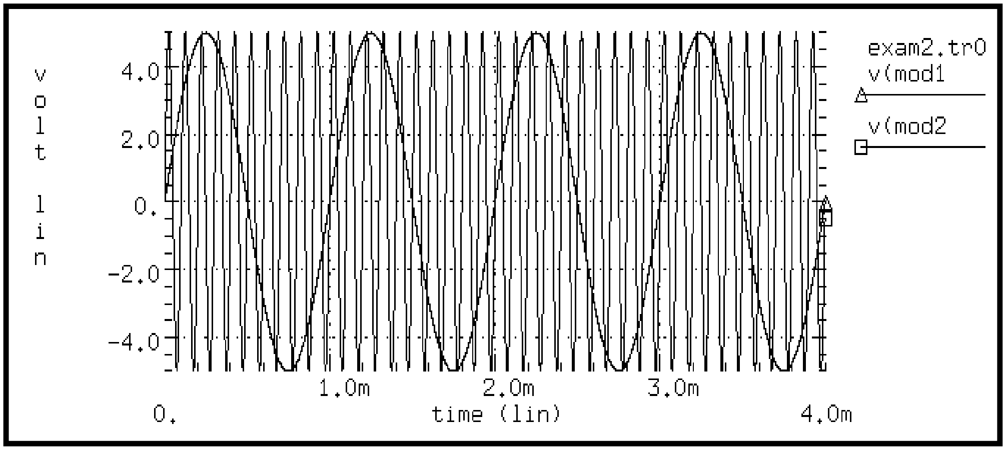

Figure 25-6: Modulating and Modulated Signals

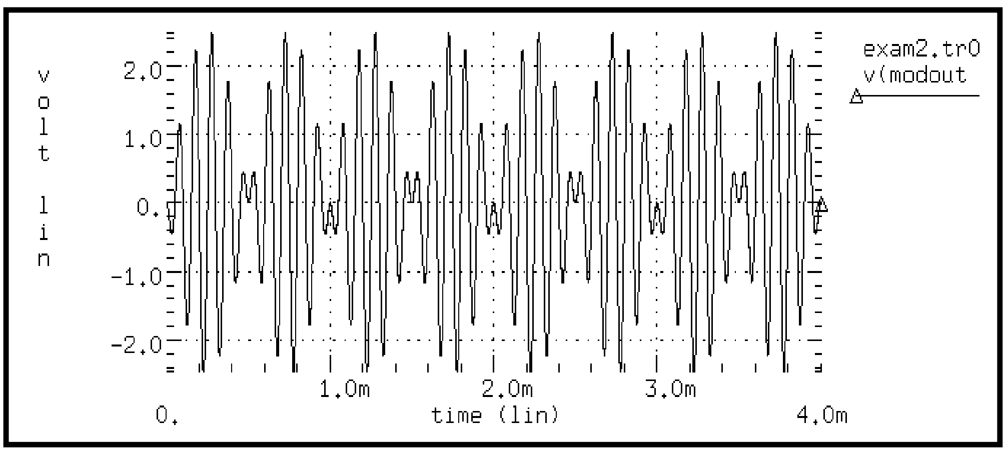

Figure 25-7: Modulated Signal

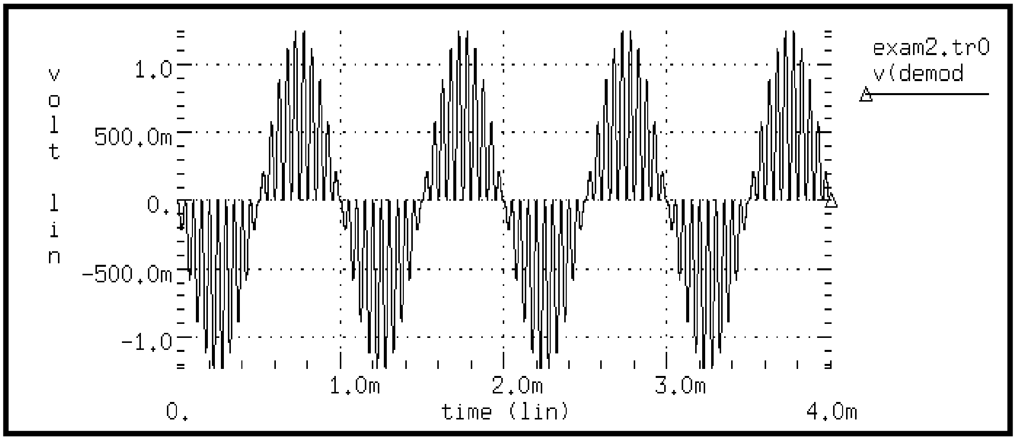

Figure 25-8: Demodulated Signal

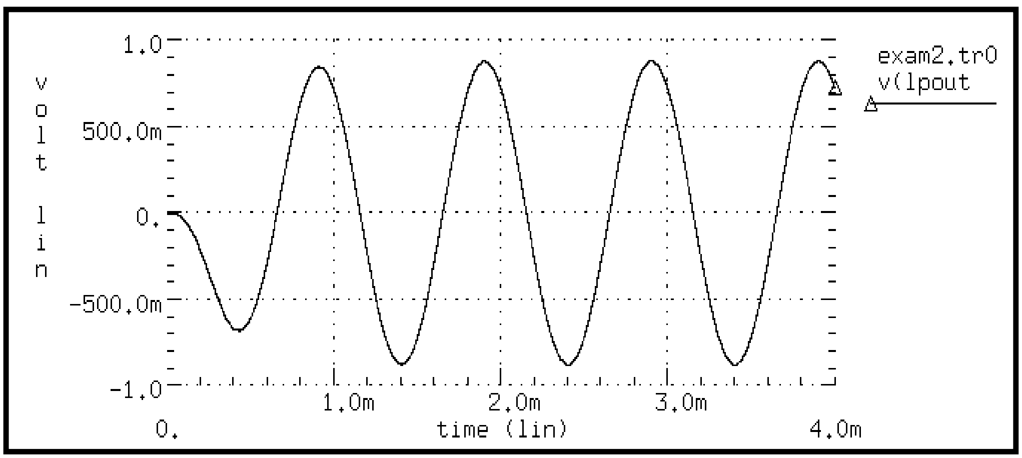

Figure 25-9: Filtered Output Signal

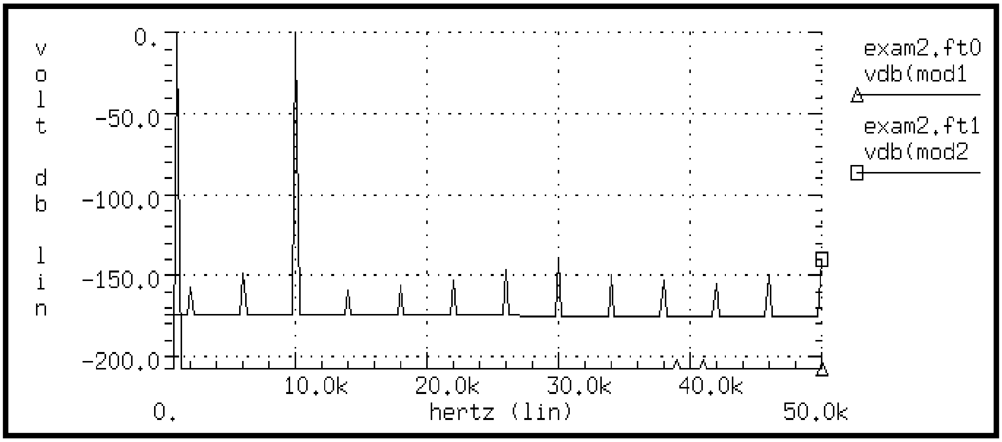

Figure 25-10: Modulating and Modulated Signal Spectrum

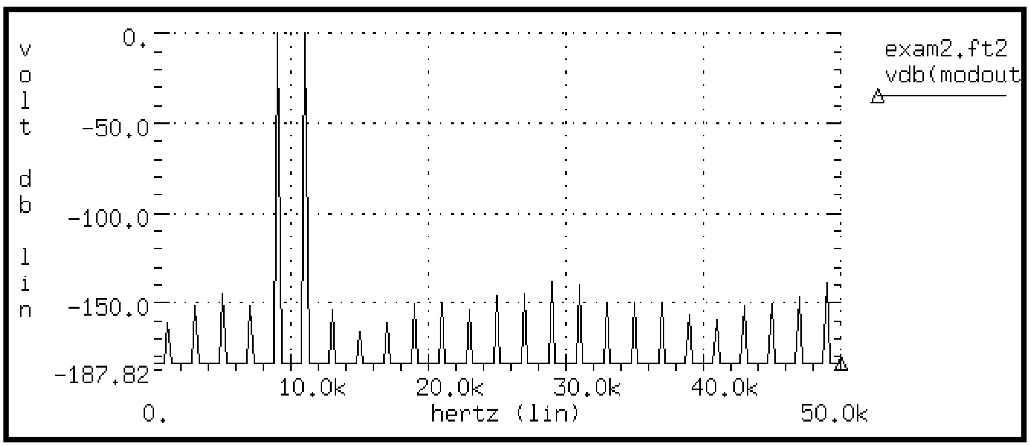

Figure 25-11: Modulated Signal Spectrum

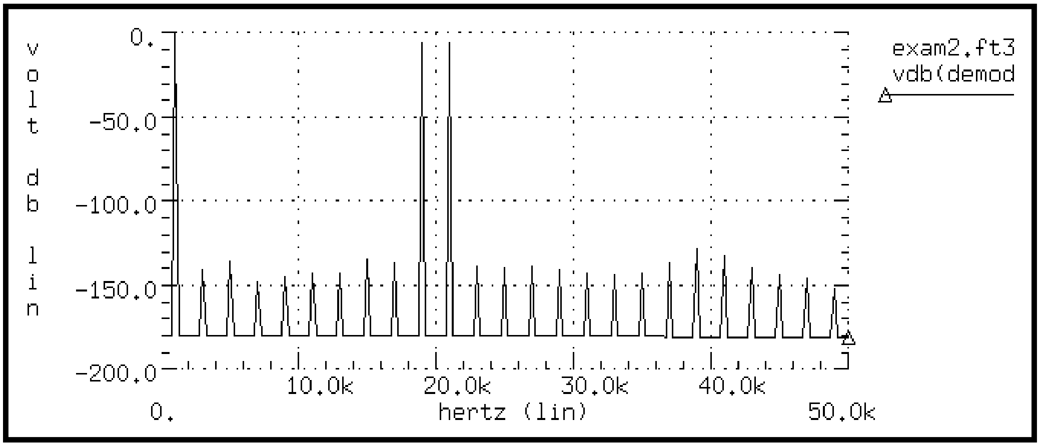

Figure 25-12: Demodulated Signal Spectrum

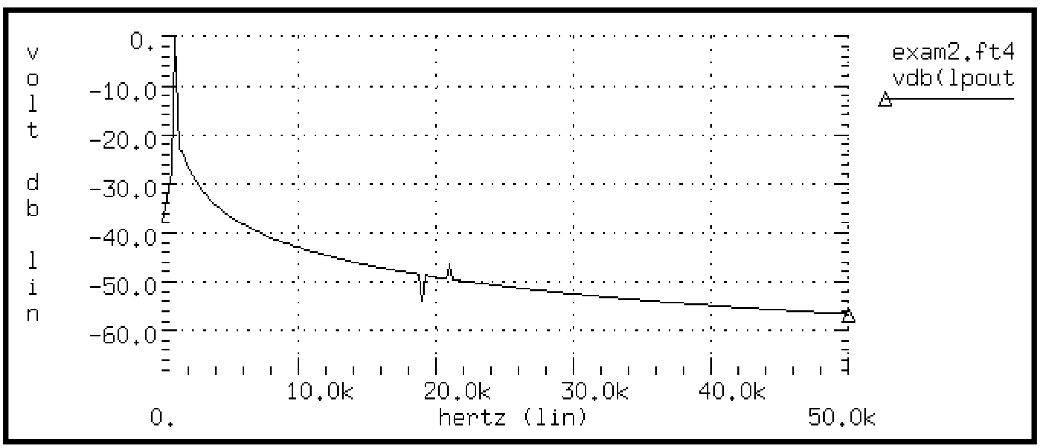

Figure 25-13: Filtered Output Signal (no window)

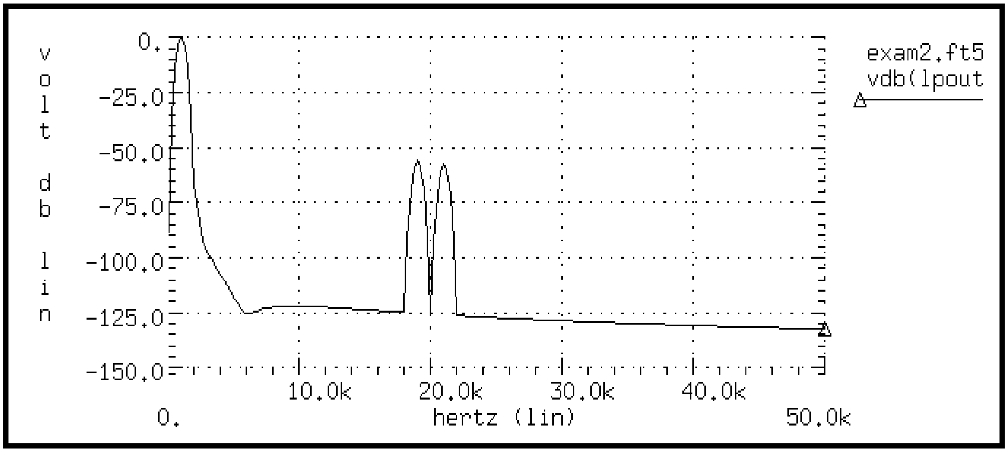

Figure 25-14: Filtered Output Signal (Blackman-Harris window)

Example 3 – Signal Detection Test Circuit

This example is a high frequency mixer test circuit, illustrating the effect of using a window to detect a weak signal in the presence of a strong signal at a nearby frequency. Two high frequency signals are added that have a 40 dB separation (that is, amplitudes are 1.0 and 0.01).

Input Listing

```
Signal Detection Test Circuit For FFT
v1 1 0 sin(0 1 1470.2Meg 0 0 90)
r1 1 0 1
v2 2 0 sin(0 0.01 1560.25Meg 0 0 90)
r2 2 0 1
E1 3 0 vol='v(1)+v(2)'
r3 3 0 1

.tran 0.1n 102.4n
.option post probe
.fft v(3)
.fft v(3) window=Bartlett fmin=1.2g fmax=2.2g
.fft v(3) window=hanning fmin=1.2g fmax=2.2g
.fft v(3) window=hamminn fmin=1.2g fmax=2.2g
.fft v(3) window=blackman fmin=1.2g fmax=2.2g
```

```
.fft v(3) window=harris fmin=1.2g fmax=2.2g
.fft v(3) window=gaussian fmin=1.2g fmax=2.2g
.fft v(3) window=kaiser fmin=1.2g fmax=2.2g
.end
```

For comparison with the rectangular window in Figure 25-15:, the spectra of the output for all of the FFT window types are shown in Figures 25-16 through 25-22. Without windowing, the weak signal is essentially undetectable due to spectral leakage.

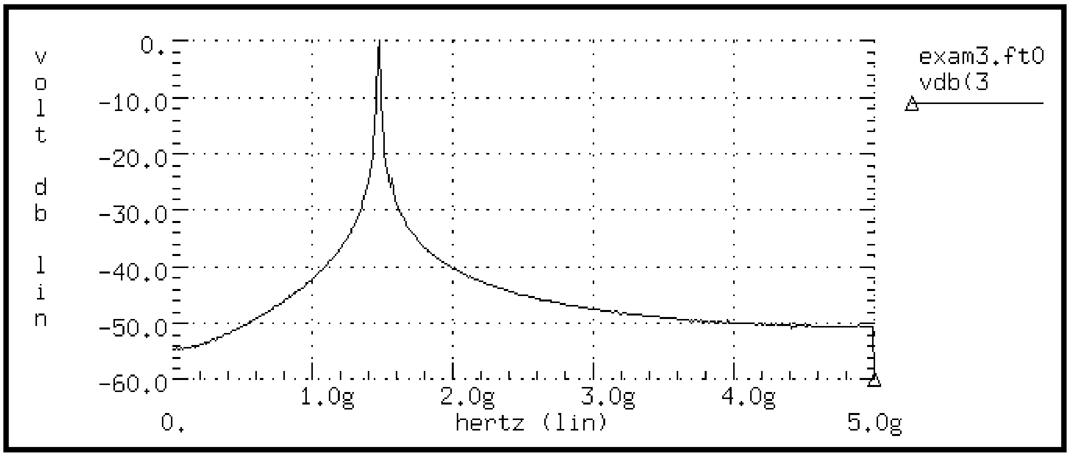

Figure 25-15: Mixer Output Spectrum, Rectangular Window

In the Bartlett window in Figure 25-16:, notice the dramatic decrease in the noise floor over the rectangular window (from -55 to more than -90 dB). The cosine windows (Hanning, Hamming, Blackman, and Blackman-Harris) all produce better results than the Bartlett window. However, the degree of separation of the two tones and the noise floor is best with the Blackman-Harris window. The final two windows (Figures 25-21 and 25-22) are parameterized with ALFA=3.0, which is the default value in Star-Hspice. These two windows also produce acceptable results, especially the Kaiser-Bessel window, which gives sharp separation of the two tones and almost a -100-dB noise floor.

Such processing of high frequencies, as demonstrated in this example, shows the numerical stability and accuracy of the FFT spectrum analysis algorithms in Star-Hspice.

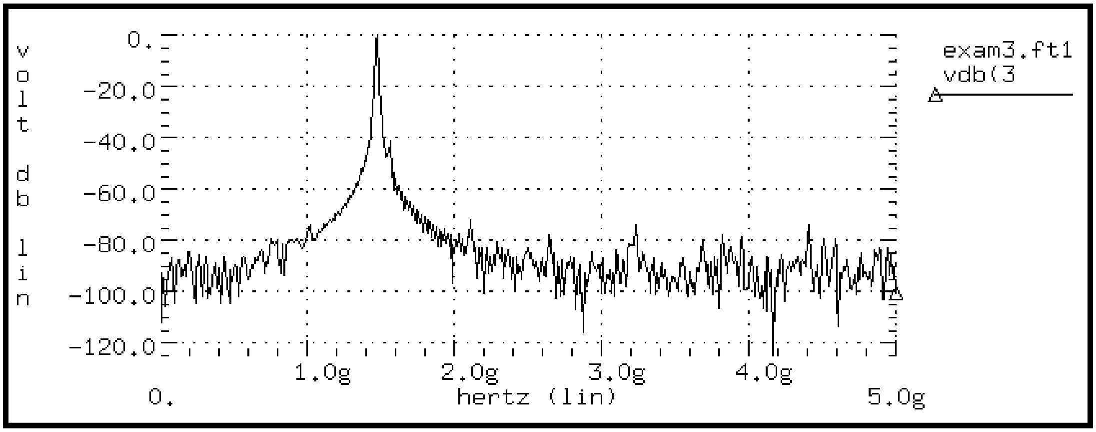

Figure 25-16: Mixer Output Spectrum, Bartlett Window

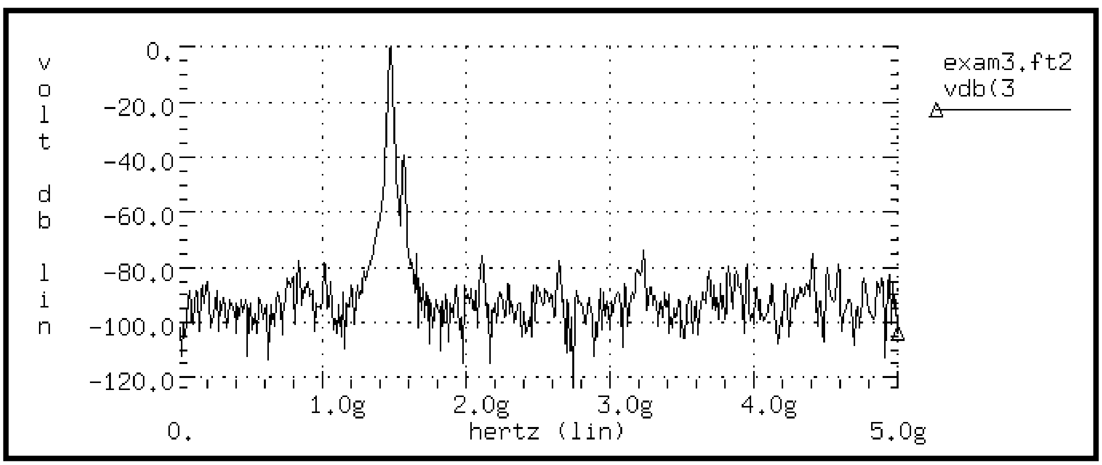

Figure 25-17: Mixer Output Spectrum, Hanning Window

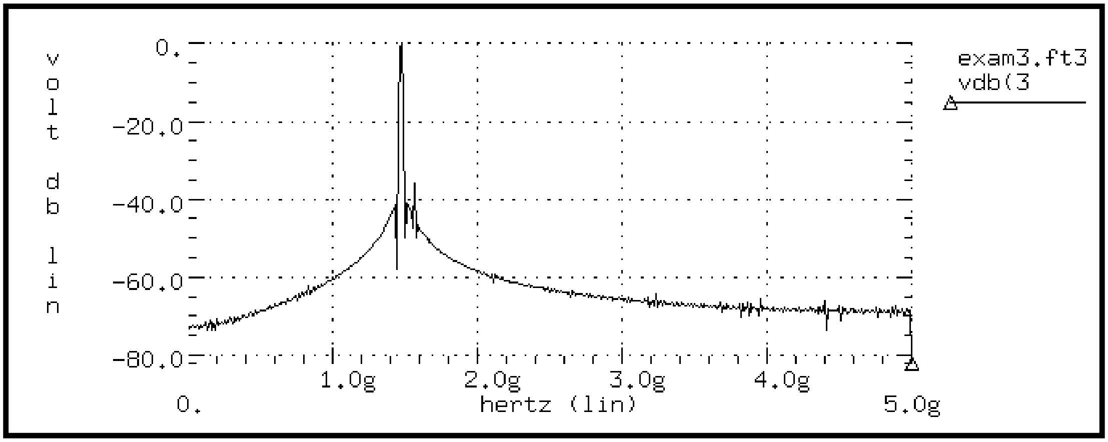

Figure 25-18: Mixer Output Spectrum, Hamming Window

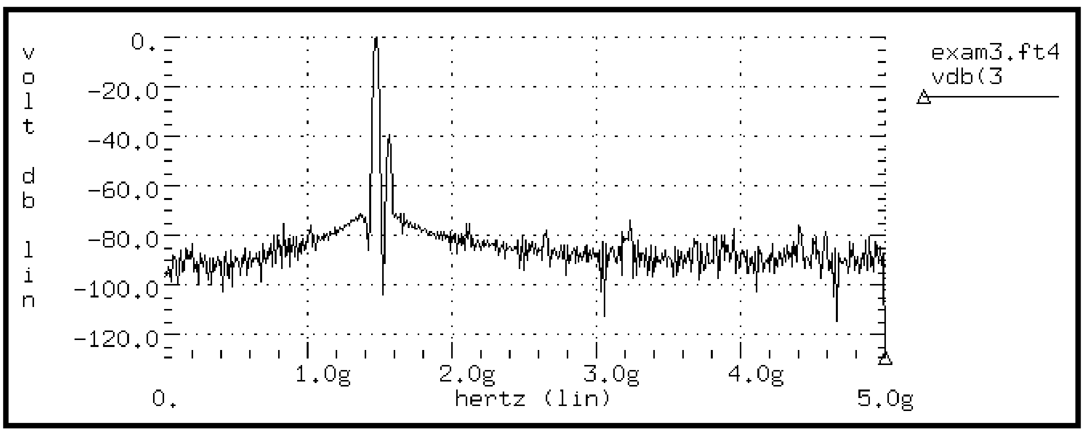

Figure 25-19: Mixer Output Spectrum, Blackman Window

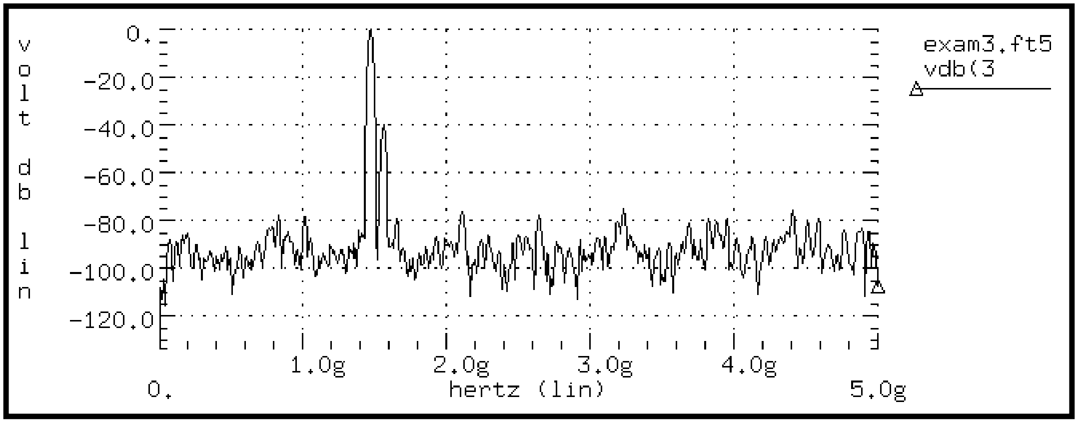

Figure 25-20: Mixer Output Spectrum, Blackman-Harris Window

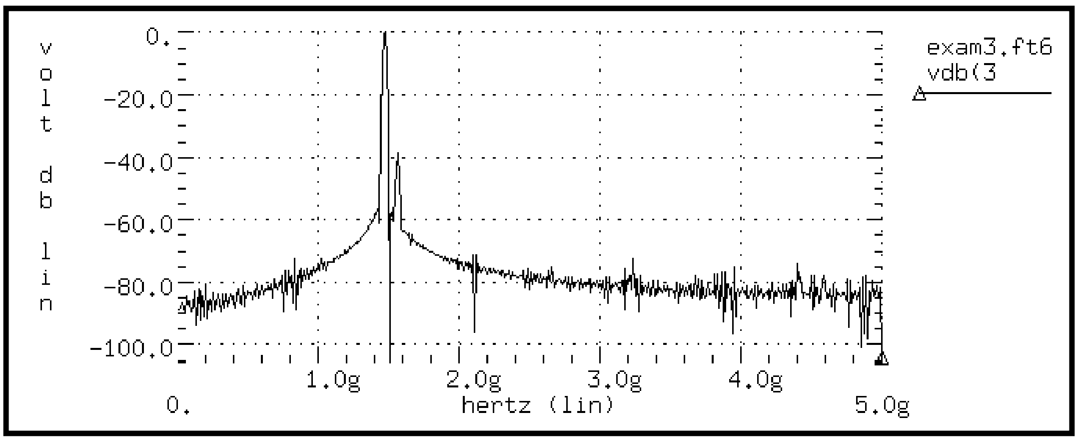

Figure 25-21: Mixer Output Spectrum, Gaussian Window

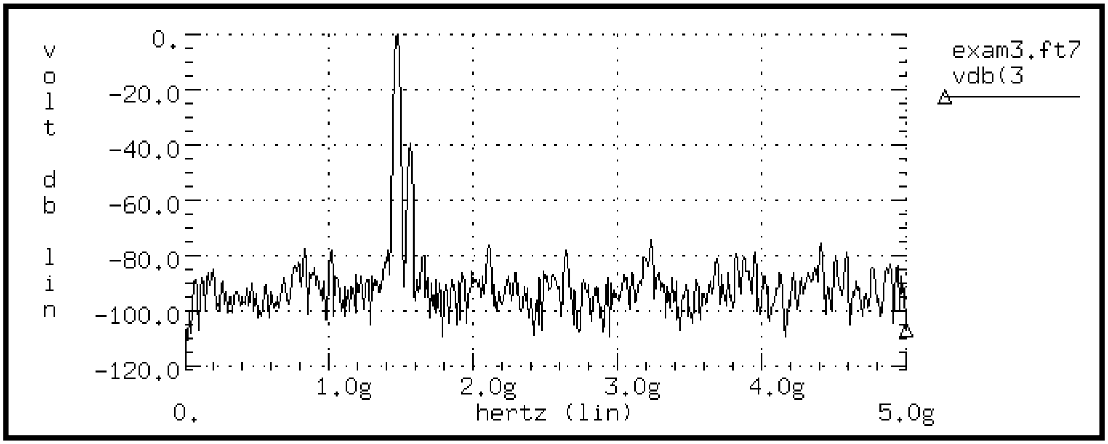

Figure 25-22: Mixer Output Spectrum, Kaiser-Bessel Window

References

1. For an excellent discussion of DFT windows, see Fredric J. Harris, “On the Use of Windows for Harmonic Analysis with Discrete Fourier Transform”, *Proceedings of the IEEE,* Vol. 66, No. 1, Jan. 1978.